DCC-Garch Models Using Islamic Market and European Market Indices

Dr. Sabbah Gueddoudj
Researcher, ENSTA Paris Tech/BCL

Research Paper Information:

To cite this article

Access this article online
https://doi.org/10.32350/ibfr.2017.04.01

Contact Information
INSTITUTE OF ISLAMIC BANKING (IIB)
UNIVERSITY OF MANAGEMENT AND TECHNOLOGY
C-II, Johar Town, Lahore +92-42-3521-2801-10 (Ext – 3418)

This article is distributed under the terms of Creative Commons Attribution – Share Alike 4.0 International License.
DCC-Garch Models Using Islamic Market and European Market Indices

Dr. Sabbah Gueddoudj

Abstract
The last financial crisis (2007-2008) raises the question of how European stock shocks are distributed and transmitted from developed stock markets to Islamic stock markets. More precisely, the problem related to Islamic finance or any other alternative finance is, whether the shocks to the volatilities in the asset returns constitute substitute or complement in terms of risks strategies. A good understanding of volatilities of asset returns is necessary to analyze and forecast domestic and international investors’ portfolios. The cornerstone of the current paper is the analysis of the dynamic correlations between the European conventional financial indices (as a proxy for global benchmark) and Islamic indices. We have chosen European markets since most of the works on this topic have focused on the US market. We have used the Dynamic Conditional Correlations approach to detect any shifts in correlations between the different indices over a recent period (from 07/31/2007 to 08/25/2017). The period includes the most severe financial turbulences (2007-2011) in Europe. Two types of distribution have been tested namely Gaussian distributions versus t-distributions. The paper finds that European and Islamic indices are highly correlated. This point may be useful for the policymakers because of the contagion risk. The results are robust across different distributions and the model associated with t-distribution is more relevant.

Keywords: assets returns, Islamic market index, European conventional market index and dcc-garch model

Introduction
The dynamics of the financial markets are among the most complex economic phenomena. In fact, uncertainty controls this dynamic and plays a central role in most of the problems tackled by financial theory (Bollerslev et al., 1991). Therefore, modeling forecasting exercises cannot be considered without taking into account the countless factors that influence the market. The asymmetry of information and the significant number of agents intervening and interacting with the financial landscape are the main factors influencing the market and making financial modeling difficult.

---

1 Researcher, ENSTA Paris Tech/BCL.

This paper should not be reported as representing the views of the ENSTA-Paris Tech. The views expressed are those of the author and do not represent those of any institution the author are or have been affiliated with, such as Central Bank of Luxembourg, National Bank of Belgium, EDHEC, Jussieu University, Paris XI University (Orsay) or Paris I University (Sorbonne).

The author is very much grateful to the reviewers for their careful and meticulous reading of the paper, in particular to Dr. Rukhsana Kalim, Oliver Lohest and Yasmina Damil.
However, the high frequency of financial crises, especially during the last decade (2000-2011), raises questions about the reliability and relevance of existing financial models and their basic assumptions. These assumptions, in particular, the Brownian movement that conditions the normality of stock prices and the semi-strong efficiency hypothesis supported by Fama are restrictive assumptions.

Moreover, many hypotheses often accepted in financial theory are not relevant to answer the dynamics of financial time series. Firstly the assumption of normality is virtually rejected in most studies on financial assets (exchange rates, stock indices, macroeconomic aggregates, etc.). Many researchers have asserted empirically that the introduction of the standard Brownian motion generates an underestimation of risks. Consequently, several stock market shocks have occurred since the beginning of the 20th century although their probability of occurrence was practically zero. This is due to the shape of the Gaussian law (which characterizes the Brownian motion), extremely flattened at the extremities and whose tails are very thin, largely ignoring the extreme values (Walter and Véhel, 2002). All the basic underpinnings of these theories will be called into question because of their lack of realism and their incompatibility with the behavior of the financial series. Moreover, the inability of Gaussian modeling to predict the occurrence of crises and prevent the advent of extreme risks has called all the classic models of finance into question (Walter and Véhel, 2002; Wilson, 2004).

Another hypothesis empirically refuted is the properties of homoscedasticity. Homoscedasticity means that volatility is a constant variable over time. However, the fluctuations and upheavals that the financial landscape has experienced all along suggest a conditional volatility (ARCH) autoregressive effect present in the stochastic component of the financial series.

Given the imperfections in classical models, new mathematical models have been put forth to ensure optimal modeling of financial assets i.e. simple heteroscedastic conditional volatility models and generalized models (ARCH and GARCH) developed by Engle (1982). These models have the main advantage of taking into account essentially the variable temporal dynamics of volatility (heteroskedasticity) and also the leptokurticity of the returns, reflecting an excess of Kurtosis (coefficient measuring the flattening of distributions). This excess of Kurtosis is one of the indicators of non-normality.

New classes of models have been introduced, notably the extensions of the GARCH models, namely the GARCH exponential known as EGARCH and the fractional integrated GARCH known as FIGARCH.

ARCH and GARCH are symmetric models (in the sense that good and bad news have the same impact on future returns and therefore on volatility). Indeed, the asymmetric GARCH model or EGARCH was adopted when Black (1976) noticed that good and bad news have different impacts on volatility contrary to the GARCH model.
This phenomenon of asymmetry means that bad news tends to increase volatility to a greater extent than good news. This indicates the sensitivity of volatility to shocks.

The models described above are based on a univariate approach. There is no relationship between several financial assets. This may be limited to analyzing contagion effects during the financial crisis.

Currently, the improvement in econometric models is not sufficient to predict financial crises efficiently. The recurrence of crises added to the volume of enormous losses suffered by financial institutions around the world and raised questions about the way finance operates. The current financial system is the victim of large speculative drifts that have created explosive bubbles. The financial industry has also witnessed the birth of derivatives that generated excessive and reckless risk-taking.

Born in the 1970s, Islamic finance is currently gaining momentum throughout the world and is increasingly becoming a competitor of the conventional financial system. It has attracted the interest of a wide range of investors and also that of countries affected by the crisis who wish to revitalize their economies by allowing new avenues of financing. “The IMF estimates that the Islamic financial sector has grown by an average of 10 percent over the last decade and that the growth of the Islamic finance sector has averaged 15 percent since 2003, reaching $500 billion at the end of 2006” (Brack, 2007). In addition to banking, Islamic finance has been able to expand into market finance activities, including fund management and index management. The first Islamic index was launched in the market in 1998 known as Socially Aware Muslim Index. Since then, the range of Islamic indices has been extended and Islamic index providers now offer a variety of Sharia indices.

Despite the remarkable expansion of Islamic finance due to a number of factors, most notably the influx of petrodollar and excessive liquidity of the Gulf countries, studies and research works aiming at quantifying the volatility and risks related to Islamic finance are very rare, especially in Europe. In fact, there are few studies that have attempted to grasp the dynamics of Islamic stock markets. This field of research is of major interest since it makes possible to provide tools to assist investment decisions in new financial products that are hardly known. In fact, investment decisions are made as a result of market assessment. Risk assessment and asset valuation are based on statistical models. Volatility remains one of the most widely used quantitative tools for assessing a financial market and measuring its stability.

To study the interactions between traditional stock markets and Islamic stock market, we employ a DCC-Garch model.

The paper is structured as follows. The second section is devoted to literature review. The third section provides information on the Islamic index and on preliminary statistics. The fourth section describes the research methodology (DCC-Garch models). The fifth section presents the main results based on the DCC-Garch model taking into
account Dow Jones Islamic Market Index (DJIM), S&P Euro 75 (EUR) and S&P EUROPE 350 (EUR). The sixth section concludes the major findings of the study.

2. Literature Review

Islamic finance, which comprises banks, indices and funds, has often been ignored in the economic literature. There are several explanations of this fact such as the newness of the phenomenon and its size. Indeed, Islamic finance is a recent activity if we compare it with the traditional finance. According to some works, Islamic finance started during the 70’s with the development of financial organizations like the Dubai Islamic bank, and it skyrocketed during the 90’s (El Khalichi et al., 2014). The economic literature for Islamic finance can be divided in two distinct approaches. The first one is mainly qualitative. We can cite the studies of Elgari (1993), Alhabsi (1994), Anwar (1995) or Ahmad (1997). Their works focus on practices and regulation of Islamic finance. In 2000, Naughton and Naughton proposed a pioneering analysis of Islamic stock market but it remains at a qualitative level. These studies contribute greatly in understanding the Islamic finance. Nevertheless, they remain insufficient in evaluating the impacts of such activity on economic growth and the interdependence between traditional and Islamic finances. Consequently, a quantitative approach is more suitable.

Concerning economic growth model, there are few studies describing the linkages between the Islamic finance sphere and real economic sphere. According to Goaied and Sassi (2010), the relationship between Islamic banks and development is not significant. They based their analysis on a GMM panel approach for the Middle-East and North Africa (MENA) region for the period 1962-2006. Barjas et al. (2010) also explored the link between Islamic bank and growth and concluded the same results except for the oil exporting countries. This result is not surprising and is common in financial literature since the link between growth and finance is not trivial. In fact, Furqani and Mulyany (2009) thanks to a VEC model and causality tests from 1997 to 2004 for Malaysia, demonstrate that GDP Granger causes growth of Islamic banks. Moreover in 2012, Abduh and Omar using an ARDL\(^2\) framework from 2003 to 2010 for Indonesia have found a significant bi-directional relationship between finance and growth, which is in line with the traditional finance literature. Hakim Bm and Md Akther Uddin (2016) show that there is a solid and positive link between growth and finance for a recent period.

All these works focus on Islamic finance within Islamic countries and omit the cross-border linkages between Muslim and non-Muslim regions. To analyze the international impact, one of the most optimal variables is the stock market index.

The literature review comparing both Islamic and conventional equities is

\(^2\) Auto Regressive Distributed Lag.
mostly based on US stocks markets. The financial crisis has increased the interest in studying ethical finance. The Sharia finance seems to be an excellent alternative to conventional finance. Nevertheless, it is possible to find some interesting on this topic before the recent financial crisis. Atta (2000) compared the performance of Dow Jones Islamic Index (DJIM) to market index and no-risk rate. He found that the Islamic index does not outperform and it brings more yields compared to a free-risk rate asset. Several works compare Islamic indices performance to other indices (Hassan, 2001; Tilva and Tuli, 2002; Hakim and Rashidian, 2002, 2004). All these studies conclude that the Islamic indices carry a lower risk than other types of traditional index. They also notice no direct link between traditional and Islamic equities. This point reinforces the idea of Islamic finance as a “safe” investment and rational choice during financial turbulences. This conclusion was relatively obvious before 2007.

In 2010, some works focused on the existence (or the non-existence) of contagion effect (Abdul Karim et al., 2010). Using MGARCH DCC model, Rizvi and Arshad (2013) discovered that there is a weak relation between Sharia and conventional indices in the long run but Abu Bakar and Masih (2014) demonstrated that the Islamic indices are more correlated with the Western markets as compared to the Asian markets. To analyze the relationship between Sharia financial markets and US markets, Majdoub and Mansour (2014) ran different multivariate GARCH models and found a weak relation. Hengchao and Hamid (2015) found that the integration within the Asia-Pacific Islamic markets and conventional markets has arisen after the 2007 crisis. According to Saiti, Bacha, and Masih (2015), there is a contagion effect between conventional and Islamic stock index after the U.S. Subprime crisis.

During the recent decades, we have observed an increase in capital markets and this has been illustrated by a rise in financial assets correlations. This may at time raise the question of financial stability in Muslim countries since the risk increases with the degree of correlation between assets. Therefore, policy makers have to take into account this new challenge (Iqbal et al., 2007 and 2010).

Most of the works take into account U.S. stocks index and until now it is not possible to reach a consensus between all these different works. Nevertheless, the most recent analyses note a convergence process between Islamic indices and Western indices.

To conclude this section, the Islamic finance-growth nexus and the Islamic indices performances are still causing a heated discussion and the literature related to Muslim finance is blooming.

3. Data Collection

The aim of this section is to present the database and some preliminary statistics. We describe and justify the selected variables to introduce in the DCC-Garch model. We use several statistical tools to analyze the process of each variable. Unit root tests and
breakdown tests are applied to identify the variables’ nature. There are 4 financial variables (Dow Jones Islamic Market Index (DJIM), S&P Global Property (US), S&P Euro 75 (EUR) and, S&P EUROPE 350 (EUR)) but the MGarch model takes into account 3 of them since the property index causality test is not conclusive.

We emphasize on the Islamic finance index since this variable is not sufficiently known in Europe. Before presenting the statistical results, we present the history and the mechanism of an Islamic index.

Modern Islamic finance began to develop in the early 1970’s with the rise of religiosity in the Muslim world and the surge in oil prices. Islamic finance, based on the principles of Sharia which impose justice, fairness, and transparency, is distinguished from conventional financial practices by a different conception of the value of capital and labor (Chapra et al., 2008 and Imam and Kpodar, 2013).

Historically, the first Islamic index was launched in the market in 1998 which is Socially Aware Muslim Index (SAMI). Since then, the main suppliers of classical indices have expanded their range and now offer a wide range of Sharia indices to support the accelerated development of Islamic Finance, in particular, Sharia Compliant funds. Nowadays, agencies, such as Standards & Poors, offer more than 30 Sharia indices. The Dow Jones Islamic Market (DJIM) is based on hundred capitalizations. All geographical areas, sectors of activity and, levels of capitalization are covered. The Sukuk market and the Sharia ISR indices are available. The majority of Islamic indices are based on benchmarks and their constructions are a parent index filter. Islamic indices use several methods of screening financial assets in order to include them in their selection (Rahman et al., 2010; and Srairi, 2013). For instance, the Dow Jones Islamic Market Index (DJIM) is built on the prices of the 2700 companies, owned by the Dow Jones, but their activities regard the Sharia rules. The DJIM takes into account several criteria. The first one examines the firm’s debt ratios: “the debt to assets”. The second screening based on the level of non-exploitable interest income imposes a minimum level; “the Haram” share of income has to be withdrawn through a charity donation for example. In terms of liquidity, it is assumed that it is allowed for Muslim investors to buy the companies’ shares but the purchase ratio does not exceed 45% of the total assets. The DJIM screening also defines that investors cannot buy securities with a rate of return defined in advance and guaranteed capital. Moreover, they are not authorized to invest in the shares of companies whose main activity is illegal. Besides, the Sharia Board of the index recommends introductions of firms with pro-environmental policies concerns, and/or humanitarian activities. In 2006, the Dow Jones and the Sustainable Asset Management group launched the Dow Jones Islamic Sustainability Index. The main purpose of this index is to specify investment criteria in line with Islamic finance criteria and sustainable development. Among the other main indices used in the market we can also cite examples such as Global Islamic Index Series, S&P500 Sharia, and FTSE Global Islamic Index Series.
After the brief Islamic index presentation we select three other indices namely S&P Global Property (US), S&P Euro 75 (EUR) and S&P EUROPE 350 (EUR). We do not present them since they are famous but propound the rationality to introduce them in the model. The justification of analyzing the S&P Global Property (US) lays in the role of the real estate assets in the last financial crisis. We have also chosen to introduce European indices since most of studies on this topic are based on US indices (Rahman and Sidek, 2011; and Saiti et al., 2014) and we notice a great progression of Islamic finance in Europe, in particular in UK and Luxembourg. Indeed, the UK has been the most active market for Islamic finance for years. Luxembourg is one of the main financial places in Euro zone for the Islamic finance industry. Its popularity comes from a competitive pricing, several incentives and a great access to the European market. Since 2002, Luxembourg is the principal actor in listing Islamic funds. Indeed, it is one of the great Islamic finance centers in Europe. Remember that the Luxembourg Stock Exchange was the first to introduce Sukuk in 2002. The Grand-Duchy of Luxembourg is also the first European member of the International Islamic Liquidity Management. According to European analysis (Di Mauro et al., 2013), Islamic finance has been one of the fastest growing sector during the last few decades and the financial crisis amplified this trend.

The following Figures provide the return of the selected indices. Data have been based at 100 for the 7/31/2007 period. They are from the us.spindices website. All variables have been log-differentiated to get the stationary properties.

1-a/ DJIM

1-b/ S&P Global Property (US)

1-d/ S&P EUROPE 350 (EUR)

1-c/ S&P Euro 75 (EUR)

Figure 1. Indices’ Returns from 7/31/2007 to 8/28/2017 (daily frequency)

Source: Authors Calculations

The trajectory analysis demonstrates that the variable seems to be stationary. To assess this result, a Dickey-Fuller variant test is run. The following table 1 summarizes the unit roots results.

Table 1
Unit roots’ Results

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-54.20218</td>
<td>-41.02574</td>
<td>-46.43398</td>
<td>-46.28512</td>
</tr>
</tbody>
</table>

Notes: Test critical values at 1%, 5% and 10% are respectively -4.949133, -4.443649 and -4.193627.

The several indices’ studies show common breakdown trajectories for the period 10/13/2008-11/20/2008 (see table 2). Therefore, the dating estimations seem to be consistent with the financial stress indices.

Table 2
Break Dates for each Index

|--------------------------|-----------|---------------------------------------------|-------------------|----------------------|

These dates coincide with the Troubled Asset Relief Program (TARP) created by the Emergency Economic Stabilization Act1 (EESA) in October 20084 and the announcement of the Term Asset-backed Securities Facility (TALF) by the Federal Board in November 20085.

All mean returns are positive and the volatilities are higher for the S&P Euro 75 and DJIM. This latter result is not surprising since it is due to a less diversified portfolio on specific activities (Majid et al., 2007; and Majid et al., 2009). Moreover the mean returns are not high at all and small compared to the volatilities measured by standard

---

4 “EESA was passed by Congress and signed by President Bush to address an ongoing financial crisis that reached near-panic proportions in September 2008” (Source: https://fas.org/sgp/crs/misc/R41427.pdf).
deviations. The skewness values are negative except for the DJIM variable. This means that for the Islamic index, the positive returns are more frequent than negative returns. The Kurtosis and Jarque-Bera statistics show that the yields are not normally distributed (see table 3).

Table 3

Descriptive Statistics of Stocks Indices

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000267</td>
<td>0.000207</td>
<td>0.000102</td>
<td>0.000173</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.014352</td>
<td>0.012527</td>
<td>0.011736</td>
<td>0.014165</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.081474</td>
<td>-0.096070</td>
<td>-0.401385</td>
<td>0.021635</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.453163</td>
<td>10.98897</td>
<td>11.38589</td>
<td>12.56949</td>
</tr>
</tbody>
</table>

Table 4

Yields Correlation with Diversification Benefits

<table>
<thead>
<tr>
<th>Jarque-Bera</th>
<th>4371.863</th>
<th>6700.037</th>
<th>7445.676</th>
<th>9607.957</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

Note: Std. Dev. stands for Standard deviation.

Table 4 indicates that yields are highly correlated which means that there are no diversification benefits.

Table 5

Simple Correlations analysis

<table>
<thead>
<tr>
<th>Probability</th>
<th>S&amp;P Euro 75 (EUR)</th>
<th>S&amp;P EUROPE 350 (EUR)</th>
<th>S&amp;P Global Property (US)</th>
<th>DJIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P Euro 75</td>
<td>1.000000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P EUROPE 350</td>
<td>0.959652 (0.0000)</td>
<td>1.000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S&amp;P Global Property</td>
<td>0.626939 (0.0000)</td>
<td>0.641085 (0.0000)</td>
<td>1.000000</td>
<td></td>
</tr>
<tr>
<td>DJIM</td>
<td>0.859494 (0.0000)</td>
<td>0.881541 (0.0000)</td>
<td>0.658666 (0.0000)</td>
<td>1.000000</td>
</tr>
</tbody>
</table>

Notes: Numbers in brackets are p-values.
The correlation degree is a good preliminary proxy to forecast any contagion effect related to the economic and financial crisis. From 70’s to 90’s, some studies demonstrate that there is a low correlation between stock indices (Solnik, 1991; Lee and Kim, 1993; Arshanapalli and Doukas, 1993; Masih, 1997; Merci 1997; Masih, 2001; and Longin and Solnik. 2001). Recent analysis on the topic show that the indices are highly correlated (Yang et al., 2003; Narayan, 2004; Bley and Chen, 2006; Chuang and Tswei, 2007; Chapra, 2008; Alkulaib et al., 2009; Chang et al., 2010; Pop and Darne, 2011; and Walid et al., 2011). The financial liberalization and the finTech activities developments tend to reinforce this phenomenon (Kearney and Lucey, 2004).

To develop the linkage between indices, we perform a causality test (Granger) and we report only the relevant results6.

Table 6


<table>
<thead>
<tr>
<th>Null Hypothesis:</th>
<th>F-Statistic</th>
<th>P-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P EUROPE 350 does not Granger Cause S&amp;P Euro 75</td>
<td>0.39198</td>
<td>0.8545</td>
</tr>
<tr>
<td>S&amp;P Euro 75 does not Granger Cause S&amp;P EUROPE 350</td>
<td>0.27405</td>
<td>0.9274</td>
</tr>
<tr>
<td>DJIM does not Granger Cause S&amp;P Euro 75</td>
<td>0.35570</td>
<td>0.8787</td>
</tr>
<tr>
<td>S&amp;P Euro 75 does not Granger Cause DJIM</td>
<td>0.77228</td>
<td>0.5698</td>
</tr>
<tr>
<td>DJIM does not Granger Cause S&amp;P EUROPE350</td>
<td>0.68708</td>
<td>0.6333</td>
</tr>
<tr>
<td>S&amp;P EUROPE 350 does not Granger Cause DJIM</td>
<td>1.16112</td>
<td>0.3264</td>
</tr>
</tbody>
</table>

For the DCC-Garch model, we do not introduce the S&P Global Property index since the causality tests results are not conclusive.

The next section presents the methodology. The main results are presented in this section.

4. Model

This section is devoted to describe shortly multivariable Garch modeling (Engle, 1982), famous for their simplicity and explicable.

The main goal of the section is to present the most general architecture of a DCC-Garch model and the results of estimations. Generally named Hadamard model, it based on specific multi-dimensional conditionally heteroscedastic properties

---

6 The exhaustive results can be provided on request.
The first works on autoregressive conditional heteroscedasticity (ARCH) by Engle (1982) and Bollerslev (1986) deal with univariate volatility models, with some extensions aiming at greater flexibility (Nelson, 1991; Glosten, Jagannathan and Runkle, 1993; and Baillie, Bollerslev, and Mikkelsen, 1996).

Later, some improvements appear and provide several models associated with the multivariate approach. Often called multivariate Garch models and noted MGARCH. Amongst them we can cite: VECCH of Bollerslev, Engle and Wooldridge (1988); DVECH and extensions by Bollerslev’s (1990); CCCMGARCH, Engle and Kroner’s (1995); Baba-Engle-Kraft-Kroner (BEKK)-GARCH, Engle's (2002); dynamic conditional correlation DCC-Garch approach, conditional variance and covariance are related to past conditional variances and past squared innovation, past squared innovation and conditional variances of other assets (Bollerslev et al., 1994).

Let’s assume that $X_t$ is a $v$-dimensional conditionally heteroscedastic properties discrete-time process:

$$X_t = M_t^{1/2} \varepsilon_t$$  

$$\varepsilon_t \sim IN(0, I), t = 1, 2, ..., T$$  

With $\varepsilon_t$ a $m \times 1$ vector of normally distributed such that $E(\varepsilon_t) = 0$ and $E(\varepsilon_t, \varepsilon_t^T) = I$ at time $t$

$M_t^{1/2}$ a $m \times m$ matrix of conditional variances obtained by a Cholesky factorization at time $t$

$$M_t = D_t C_t D_t$$  

$D_t$ is the $m \times m$ diagonal matrix of standard deviations obtained from univariate GARCH model at time $t$. $X_t$ is uncorrelated in time, but there is a non-linear dependence.

$$R_t = \mu_t + X_t$$  

$R_t$ is a $m \times m$ conditional correlation matrix of $X_t$ at time $t$. $\mu_t$ is assumed to be constant or time series model.

For a DCC-Garch model, we have the following relations:
\[ C_t = Q_t^{-1} Q_t Q_t^{-1} \]  
\[ Q_t = (1 - a - b) \overline{Q} + a \varepsilon_{t-1} \varepsilon_{t-1}^T + b Q_{t-1} \]  
\[ Q = \sum_{t=1}^{T} \frac{\varepsilon_t \varepsilon_t^T}{T} \]

\[ Q_t^* = \begin{bmatrix} \sqrt{q_{11t}} & 0 & 0 \\ 0 & \sqrt{q_{22t}} \\ 0 & 0 & \sqrt{q_{nn}} \end{bmatrix} \]

\( Q_t \) is supposed to be positive. Moreover, we assume \( a \geq 0, b \geq 0 \) and \( a + b = 1 \).

Note that the different specifications of MGARCH models can be divided into several categories but for this short paper we put the emphasis on conditional variances and correlations models. Also, we will not describe this class of models since the empirical literature is flourishing and rich\(^7\).

For this paper, we consider two different distributions for the standardized error \( \varepsilon_t \): the multivariate Gaussian, and the multivariate Student’s \( t \)-distribution.

When the standardized errors (\( \varepsilon_t \)) are Gaussian, the joint distribution is given by:

\[ f(\varepsilon_t) = \prod_{t=1}^{T} \frac{1}{(2\pi)^{n/2}} e^{-\frac{1}{2} \varepsilon_t^T \Sigma_t \varepsilon_t} \]  

If the standardized errors (\( \varepsilon_t \)) are Student’s t-distributed, the following equation is selected:

\[ f(\varepsilon_t/\nu) = \prod_{t=1}^{T} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{\nu}{2}\right)^{1/2}} \left[1 + \frac{\varepsilon_t^T \varepsilon_t}{\nu} \right]^{-\frac{n+\nu}{2}} \]  

As all estimations are based on Garch processes, the choice of start values is extremely important which conditioned the results. Moreover, if the number of parameters to estimate is too large, the likelihood function becomes flat and often the model tends to local optima. To avoid this problem we run the estimation with different starting values. The starting values chosen are the combination of values that yield the

highest likelihood. Another problem of convergence may arise in the case of outliers. One of the solutions is to remove the outliers.

The goal of the next section is to expose and to discuss the empirical results of different DCC-Garch models outcomes based on Gauss and t distributions.

5. Results

We run a DCC-Garch model associated with Gaussian and t-densities. As already underlined above, we will take into account only three variables: DJIM, S&P 350 and S&P 75. DCC-Garch model results are summarized in table 7.

Table 7

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Gaussian density Estimate(Maximum-Likelihood/ BFGS)</th>
<th>t-density Estimate(Maximum-Likelihood/ BFGS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{11} )</td>
<td>0.91650*** (0.01663)</td>
<td>0.92125*** (0.015084)</td>
</tr>
<tr>
<td>( \lambda_{12} )</td>
<td>0.91999*** (0.015989)</td>
<td>0.93159*** (0.013450)</td>
</tr>
<tr>
<td>( \lambda_{13} )</td>
<td>0.90459*** (0.02230)</td>
<td>0.91989*** (0.011949)</td>
</tr>
<tr>
<td>( \lambda_{21} )</td>
<td>0.06450*** (0.013743)</td>
<td>0.05969*** (0.01779)</td>
</tr>
<tr>
<td>( \lambda_{22} )</td>
<td>0.06769*** (0.014117)</td>
<td>0.06199*** (0.014117)</td>
</tr>
<tr>
<td>( \lambda_{23} )</td>
<td>0.06376*** (0.02230)</td>
<td>0.05874*** (0.01959)</td>
</tr>
<tr>
<td>( \delta_1 )</td>
<td>0.93560</td>
<td>0.94165</td>
</tr>
<tr>
<td>( \delta_2 )</td>
<td>0.03699</td>
<td>0.03459</td>
</tr>
<tr>
<td>( \upsilon )</td>
<td>8.5924</td>
<td></td>
</tr>
<tr>
<td>Maximum likelihood Log-likelihood</td>
<td>-9 769.90</td>
<td>-9 621.90</td>
</tr>
<tr>
<td>AIC</td>
<td>17.928</td>
<td>17.465</td>
</tr>
<tr>
<td>SBC</td>
<td>18.024</td>
<td>17.834</td>
</tr>
</tbody>
</table>

---

8 Broyden-Fletcher-Goldfarb-Shannon. We use R package Bayes Dcc Garch for DCC-Garch model estimations coverage.
Notes: The results are based on a DCC-Garch model with gaussian and t-student densities for the period 2007-2017. The i=1,2,3 is respectively DJIM, S&P 350 and S&P 75. Numbers in brackets are standard errors. Test critical values at 1%, 5% and 10% are respectively ***,**, and *. \( \nu \) is the degree of freedom. \( \lambda \) coefficients are volatility parameters.

From table 7 we notice that all coefficients are highly significant for the parameters \( \lambda_{ij} \) for i=1,2 and j=1,2,3. The estimated coefficients \( \hat{\lambda}_{ij} \) are close to one, there is a gradual volatility decay for both models. We observe that the sums of \( \lambda_{ij} \) are less than one for each index suggesting that volatility shocks are transitory and not permanent. It is a usual outcome for high volatility variables (Saiti et al. (2014) as \( \sum \delta_{i} < 1 \), the models converge.

Moreover, the maximum log-likelihood is greater for the t-density model than the Gaussian model. The model with the lowest AIC and SBC is preferred. Given the AIC and SBC criteria, the Gaussian model underperforms the t-model (Pesaran, 2007; and Saidi et al., 2014). Therefore, we conclude that the DCC-Garch model associated with a t-distribution is more appropriate to model fat tailed variables. This is in line with the elementary statistics reported in table 3. The degree of freedom is far below the threshold limit (30).

We observe that there is a strong relationship between Islamic market and conventional European market (0.93). This is not surprising since the most recent empirical literature provides the same conclusion (Rua and Nunes, 2009; Saiti et al., 2014; and Bala and Taimoto, 2017). Hence, our findings reinforce the idea of the contagion effect possibility between Islamic finance and traditional finance. Indeed, the financial crisis has affected US market; EU market and Islamic market (see chart 1 and breakdown tests table 2). Despite the Islamic finance stability we cannot reject the hypothesis of imported crisis. Nevertheless, we observe that according to the skewness statistics, Islamic index outperforms US indices.

Islamic finance may be a good alternative to conventional finance especially during financial crisis thanks to its healthy management. As already underlined in section 1, Islamic finance is based on a sharing of profits & losses and some interdictions (speculation and usury) but given the interaction between markets, a contagion effect related to European or US crisis cannot be excluded. Nevertheless, we assess that the 2011 crisis did not impact Islamic finance. This assessment may be related to the crisis severity (Rizvi and Arshad, 2013). It may be interesting to develop models based on various degrees of crisis and measure the impact of imported economic turbulences.

Globally, the challenge for the Islamic finance is to find solutions to contain systemic risks linked to interdependent markets.
6. Conclusion

We have examined the stock market volatility contagion for different financial indices after the last financial crisis. We have taken into account an Islamic index, largely used by the empirical literature and different European indices namely Dow Jones Islamic Market Index (DJIM), S&P Global Property US, S&P Euro 75 (EUR) and, S&P EUROPE 350 (EUR). For each variable, we have provided some statistical analysis in order to run a DCC-Garch model. For the DCC-Garch model, we have used two densities: Gaussian and t-densities. The pre-tests led us to withdraw the real estate variable since the causality estimation is not conclusive. Both densities approaches provide significant and relevant results. But, if we compare the two models, the best model is one that takes into account t-densities. This conclusion is based on three criteria, the log-likelihood, the Akaike (AIC) and Schwartz Bayesian criteria (SBC).

The main results obtained for period (2007-2017) show that the financial indices are highly correlated. This result is not surprising due to financial integration and commercial globalization (Saiti, 2013; and Saiti et al., 2014). Nevertheless, in case of financial crisis, the contagion risk is not excluded and this assessment should be integrated into any stabilization policies (Hwahsin et al., 2006). Nowadays, the Islamic index cannot be perceived as an alternative asset but a supplementary asset. Indeed, the Islamic index yields are impacted by worldwide financial events. The financial crises are often imported. Nevertheless, we regret that we did not have a database before 2007 because of its availability. We cannot, therefore, develop models with long-term memories. 2/ A DCC-Garch model associated with a t-distribution are more performant than a model associated with a Gaussian distribution.

Despite the interesting results shown in the study some limitations have also been identified. For example, the size of the sample is relatively short (from 2007 to 2017). Similarly we have selected only 4 indices, more indices will improve the robustness of the model. We also did not introduce the skew-t distribution while this distribution is more adapted to financial assets returns (Bala and Takimoto, 2017). The next step is to increase the quality of the estimations according to the limits.

Nevertheless, the main results deduced from the estimation models are crucial for policy makers in the implementation of financial stabilization policies. The financial crises are not without implications for other markets due to the financial globalization (Arshanapalli and Doukas, 1993; Kearney, 1996; Charles et al., 2011; Cheng et al., 2008; Alkulaib et al., 2009; and Francis et al., 2008). Nowadays, it is inconceivable to present financial indices models without a linkage framework.
References


and Middle Eastern Finance and Management, 3(3), 228–240.


