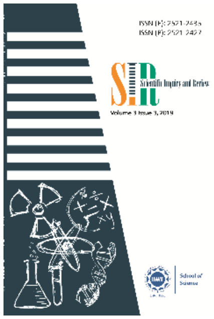

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Idealistic Soft Topological Hyperrings

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Abstract

The aim of this article is to introduce the concept of an idealistic soft topological hyperring over a hyperring. Some structural properties of this concept are also studied. Moreover, this study investigates the relationship between the idealistic soft topological hyperrings and the idealistic soft hyperrings. Finally, the restricted (extended) intersection and ⋀-intersection of the family of the idealistic soft topological hyperrings are examined.

Keywords: idealistic soft topological ring, soft hyperring, soft set, idealistic soft topological hyperring

Introduction

The hyperstructure theory was initiated in 1934 when Marty proposed the concept of a hypergroup [1]. Different hyperstructures such as hypergroups, hyperrings, and hyperfields have been widely studied by many mathematicians in both theoretical and applied fields. These hyperstructures have been combined with the various theories modeling uncertainty such as fuzzy sets, rough sets and the soft set theory [2, 3]. Hyperrings, one of these superstructures, were introduced by M. Krasner and their connection with the soft set theory was investigated by different researchers [4, 5]. Selvachandran defined the notions of soft hyperrings and soft hyperring homomorphism [6]. The definitions of the idealistic soft hyperrings, soft subhyperrings and soft hyperideals were presented by Wang et al. Recently, some connections between topological hyperstructures and soft sets were examined [7]. The concept of the soft topological polygroups was studied by Oguz [8].

Soft set theory defined by Molodstov is a powerful mathematical tool for modeling uncertainties [9]. Successful results have been obtained using the soft set theory to solve complex problems in many fields such as economics, engineering, social sciences, medical science and others. Indeed, this theory has been applied with different perspectives in diverse areas such as algebra, topology, analysis and geometry by many mathematicians [10, 11, 12, 13, 14, 15].

In this study, the notion of the idealistic soft topological hyperrings is expounded and studied as a new topological construction acting between the soft sets and the hyperrings. Examples of this notion are presented and its several characteristic properties are investigated.

2. Preliminaries

This section provides some preliminary definitions and results about soft sets, topological hyperrings and idealistic soft hyperrings for the sake of convenience and completeness.

Let's suppose $\mathbb{W}$ as an initial universe set and $\mathfrak{G}$ as a set of parameters. Also, let $\mathcal{P}(\mathbb{W})$ symbolize the power set of $\mathbb{W}$ and $\mathcal{V} \subset \mathfrak{G}$. The description of a soft set proposed by Molodtsov is as follows:

2.1. Definition [9]

A pair $(\mathcal{S}, \mathcal{V})$ over $\mathbb{W}$ is termed as a soft set, where $\mathcal{S}$ is a mapping defined as

$$\mathcal{S}: \mathcal{V} \to \mathcal{P}(\mathbb{W})$$

Notice that a soft set over $\mathbb{W}$ can be regarded as a parameterized family of the subsets of the universe$\mathbb{W}$.

2.2. Definition [15]

The support of a soft set $(\mathcal{S}, \mathcal{V})$ is defined as a set

$$Supp(\mathcal{S}, \mathcal{V}) = \{\alpha \in \mathcal{V} : \mathcal{S}(\alpha) \neq \emptyset\}$$

If $Supp(\mathcal{S}, \mathcal{V})$ is not equal to the empty set, then $(\mathcal{S}, \mathcal{V})$ is said to be non-null.

Below are stated some general characterizations for the non-empty family. $\{(\mathcal{S}_{\mathrm{k}}, \mathcal{V}_k) \mid \mathrm{k} \in \mathcal{J}\}$ of soft sets over the common universe $\mathbb{W}$ such that $\mathcal{J}$ is an index set.

2.3. Definition [16]

The *restricted intersection* of the family $\{(\mathcal{S}_k, \mathcal{V}_k) \mid k \in \mathcal{J}\}$ is defined by a soft set $(\mathcal{S}, \mathcal{V}) = \widetilde{\bigcap}_{k \in \mathcal{J}} (\mathcal{S}_k, \mathcal{V}_k)$ such that $\mathcal{V} = \bigcap_{k \in \mathcal{J}} \mathcal{V}_k \neq \emptyset$ and $\mathcal{S}(\alpha) = \bigcap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha)$ for all $\alpha \in \mathcal{V}_k$.

2.4. Definition [16]

The *extended intersection* of the family $\{(\mathcal{S}_k, \mathcal{V}_k) \mid k \in \mathcal{J}\}$ is a soft set $(\mathcal{S}, \mathcal{V}) = (\cap_{\mathcal{S}})_{k \in \mathcal{J}}(\mathcal{S}_k, \mathcal{V}_k)$ such that $\mathcal{V} = \cup_{k \in \mathcal{J}} \mathcal{V}_k \neq \emptyset$ and $\mathcal{S}(\alpha) = \cap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha)$ for all $\alpha \in \mathcal{V}_k$.

2.5. Definition [16]

The $\wedge$ *-intersection* of the family $\{(\mathcal{S}_k, \mathcal{V}_k) \mid k \in \mathcal{J}\}$is defined by a soft set $(\mathcal{S}, \mathcal{V}) = \widetilde{\wedge}_{k \in \mathcal{J}}(\mathcal{S}_k, \mathcal{V}_k)$ *such that* $\mathcal{V} = \prod_{k \in \mathcal{J}} \mathcal{V}_k$ and $((\alpha_k)_{k \in \mathcal{J}}) = \cap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha_k)$.

Here, the concepts of topological hyperrings, idealistic soft topological rings and soft hyperrings will be reviewed.

2.6. Definition [17]

Let $(\mathcal{S}, \mathcal{V})$ be a non-null soft set on a commutative ring Ꝁ endowed with the topology $\mathcal{T}$ In this case, the system $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is considered as an idealistic soft topological ring over Ꝁ if the axioms given below are fulfilled for all $\alpha \in \mathcal{V}$:

i. $\mathcal{S}(\alpha)$ is an ideal of Ꝁ for all $\alpha \in \mathcal{V}$.

ii. The mapping $\mathcal{S}(\alpha) \times \mathcal{S}(\alpha) \longrightarrow \mathcal{S}(\alpha)$ defined by $(x, y) \longmapsto x - y$ is continuous.

iii. The mapping Ꝁ $\times \mathcal{S}(\alpha) \longrightarrow \mathcal{S}(\alpha)$ defined by $(r, y) \longmapsto r \cdot y$ is continuous.

Before stating the definition of a hyperring, let's present the definition of a hypergroup.

2.7. Definition [4]

Let $\mathcal{N}$ be a non-empty set and $\mathcal{P}^*(\mathcal{N})$ denote the family of non-empty subsets of $\mathcal{N}$. Then, the mapping $\cdot : \mathcal{N} \times \mathcal{N} \longrightarrow \mathcal{P}^*(\mathcal{N})$ is said to be a hyperoperation. The pair $(\mathcal{N}, \cdot)$ is also considered to be a hypergroupoid.

2.8. Definition [4]

A hypergroup is a hypergroupoid $(\mathcal{N}, \cdot)$ if it holds the following conditions:

i. $x \cdot (y \cdot z) = (x \cdot y) \cdot z$ for all $x, y, z \in \mathcal{N}$

ii. $x \cdot \mathcal{N} = \mathcal{N} \cdot x$ for all $x \in \mathcal{N}$.

2.9. Definition [4]

An algebraic structure $(\mathcal{N}, +, \cdot)$ is called a hyperring if it satisfies the following three axioms:

i. $(\mathcal{N}, +)$ is a commutative hypergroup.

ii. $(\mathcal{N}, \cdot)$ is a semi-hypergroup.

iii. The hyperoperation '·' is distributive with respect to the hyperoperation '+'.

Note that a non-empty subset $\mathcal{N}_1$ of a hyperring $(\mathcal{N}, +, \cdot)$ is said to be hyperideal if $a, b \in \mathcal{N}_1$ and $r \in \mathcal{N}$ implies $a - b \in \mathcal{N}_1$ and $r \cdot x \in \mathcal{N}_1$.

2.10. Definition [18]

Let $(\mathcal{N}, \mathcal{T})$ be a topological space and $\mathcal{P}^*(\mathcal{N})$denote the family of the non-empty subsets of $\mathcal{N}$. Then, the collection $\mathcal{B}$ consisting of all sets $\mathcal{S}_Q = \{W \in \mathcal{P}^*(\mathcal{N}) : W \subseteq Q, W \in \mathcal{T}\}$ is a base for the topology $\mathcal{T}^*$on $\mathcal{P}^*(\mathcal{N})$ denoted by $\mathcal{T}^*$.

2.11. Definition [10]

Let $(\mathcal{N}, +, \cdot)$ be a hyperring and $(\mathcal{N}, \mathcal{T})$ be a topological space. In this case, algebraic hyperstructure $(\mathcal{N}, +, \cdot, \mathcal{T})$ is said to be a topological hyperring if three hyperoperations $'+'$, $'\cdot'$ and $'/'$ are continuous.

Example [18]. Every topological ring is a topological hyperring by trivial hyperoperations.

2.12. Definition [7]

Let's suppose $(\mathcal{S}, \mathcal{V})$ as a non-null soft set over the hyperring $\mathcal{N}$. Then, the pair $(\mathcal{S}, \mathcal{V})$ is termed as an idealistic soft hyperring over $\mathcal{N}$ if $\mathcal{S}(\alpha)$ is a hyperideal of $\mathcal{N}$ for all $\alpha \in Supp(\mathcal{S}, \mathcal{V})$.

Example [7]. Choose a hyperring $(\mathcal{N}, +, \cdot)$ with the associated hyperoperations as follows:

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

$\cdot$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	0	0	0

Define a soft set $(\mathcal{S},\mathcal{V})$ over the hyperring $\mathcal{N} = \{0,1,2,3\}$, where $\mathcal{V} = \mathcal{N}$, by $\mathcal{S}(0) = \mathcal{S}(3) = \{0,3\}$, $\mathcal{S}(1) = \mathcal{S}(2) = \{0,1,2,3\}$. It is easy to check that $\mathcal{S}(0)$, $\mathcal{S}(1)$, $\mathcal{S}(2)$ and $\mathcal{S}(3)$ are hyperideals of $\mathcal{N}$. Therefore, $(\mathcal{S},\mathcal{V})$ is an idealistic soft hyperring over $\mathcal{N}$.

3. Idealistic Soft Topological Hyperrings

In this section, a definition is proposed for the idealistic soft topological hyperrings. Also, the related structural features are examined.

3.1. Definition

Let's suppose $\mathcal{T}$ as a topology and $(\mathcal{S},\mathcal{V})$ as a non-null soft set over on the hyperring $\mathcal{N}$. Then, the triplet $(\mathcal{S},\mathcal{V},\mathcal{T})$ is termed as an idealistic soft topological hyperring over $\mathcal{N}$, given that the followings conditions are satisfied for all $\alpha \in Supp\,(\mathcal{S},\mathcal{V})$.

i. $\mathcal{S}(\alpha)$ is a hyperideal of $\mathcal{N}$.

ii. The hyperoperations $+,\ /\ : \mathcal{S}(\alpha)\times\mathcal{S}(\alpha) \longrightarrow \mathcal{P}^*(\mathcal{S}(\alpha)$ and $\cdot : \mathcal{N}\times\mathcal{S}(\alpha) \longrightarrow \mathcal{P}^*(\mathcal{S}(\alpha)$ are continuous in reference to the topologies induced by $\mathcal{T}\times\mathcal{T}$ and $\mathcal{T}^*$.

As a consequence of the above definition, we can say that if $\mathcal{N}$ is a topological hyperring and the condition ***i.*** is hold, the system $(\mathcal{S},\mathcal{V},\mathcal{T})$ is also called an idealistic soft topological hyperring.

Example. Every idealistic soft topological ring is an idealistic soft topological hyperring.

Example. Let $(\mathcal{S},\mathcal{V},\mathcal{T})$ be an idealistic soft topological hyperring over $\mathcal{N}$ and $\mathcal{M}\subseteq\mathcal{V}$. It follows that $(\mathcal{S}|_{\mathcal{M}},\mathcal{M},\tau)$ is an idealistic soft topological hyperring over $\mathcal{N}$ if it is non-null.

3.2. Definition

Let's suppose $(\mathcal{S},\mathcal{V},\mathcal{T})$ as an idealistic soft topological hyperring over $\mathcal{N}$.

Then $(\mathcal{S},\mathcal{V},\mathcal{T})$ is said to be

i. an identity idealistic soft topological hyperring if $\mathcal{S}(\alpha) = \{0\}$ for all $\alpha \in \mathcal{V}$.

ii. an absolute idealistic soft topological hyperring if $\mathcal{S}(\alpha) = \mathcal{N}$ for all $\alpha \in \mathcal{V}$.

Example. Assume that $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is an idealistic soft topological hyperring over $\mathcal{N}$ such that $\mathcal{V} = \mathcal{N}$ and $\mathcal{S}(\alpha) = \{\beta \in \mathcal{N} : \alpha + \beta = \{\alpha\}\}$ for all $\alpha \in \mathcal{V}$. Then, it is clear that $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is an identity idealistic soft topological hyperring over $\mathcal{N}$.

Here, we examine the relationship between the idealistic soft hyperrings and the idealistic soft topological hyperrings.

3.3. Theorem

Every idealistic soft hyperring on a topological hyperring $\mathcal{N}$ is an idealistic soft topological hyperring.

Proof. Suppose that $(\mathcal{S}, \mathcal{V}, \tau)$ is an idealistic soft hyperring over the topological hyperring $\mathcal{N}$ with the topology τ. Since $\mathcal{S}(\alpha)$ is a hyperideal of $\mathcal{N}$ for all $\alpha \in \mathcal{V}$, we have three hyperoperations $+, \ /: \mathcal{S}(\alpha) \times \mathcal{S}(\alpha) \longrightarrow \mathcal{P}^*(\mathcal{S}(\alpha))$ and $\cdot : \mathcal{N} \times \mathcal{S}(\alpha) \longrightarrow \mathcal{P}^*(\mathcal{S}(\alpha))$. These are continuous operations with reference to the topologies induced by $\mathcal{T} \times \mathcal{T}$ and $\mathcal{T}^*$ for all $\alpha \in \mathcal{V}$. So, $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is an idealistic soft topological hyperhyperring over $\mathcal{N}$.

3.4. Remark

Each idealistic soft hyperring $\mathcal{N}$ can be transformed into an idealistic soft topological hyperring by equipping both $\mathcal{N}$ and $\mathcal{P}^*(\mathcal{N})$ with either indiscrete topology or discrete topology. On the other hand, every idealistic soft hyperring over a hyperring is not an idealistic soft topological hyperring.

Let us present some generalizations for a non-empty family of idealistic soft topological hyperrings.

3.5. Theorem

Let $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ be a non-empty family of idealistic soft topological hyperrings over $\mathcal{N}$.

i. *The restricted intersection of the family* $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ *with* $\bigcap_{k \in \mathcal{J}} \mathcal{V}_k \neq \emptyset$ *is an idealistic soft topological hyperring over* $\mathcal{N}$ *if it is non-null.*

ii. *The extended intersection of the family* $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ *is an idealistic soft topological hyperring over* $\mathcal{N}$ *if it is non-null.*

Proofs of Theorem:

i. The restricted intersection of the family $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ with $\bigcap_{k \in \mathcal{J}} \mathcal{V}_k \neq \emptyset$ is defined by the soft set $\widetilde{\bigcap}_{k \in \mathcal{J}} (\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) = (\mathcal{S}, \mathcal{V}, \mathcal{T})$ such that $\bigcap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha)$ for all $\alpha \in \mathcal{V}$. Choose $\alpha \in Supp\ (\mathcal{S}, \mathcal{V})$. Together with the hypothesis, $\bigcap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha) \neq \emptyset$ implies that $\mathcal{S}_k(\alpha) \neq \emptyset$ for all $k \in \mathcal{J}$. Since $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ is a non-empty family of idealistic soft topological hyperrings over $\mathcal{N}$, it is then easy to see that $\mathcal{S}_k(\alpha)$ is a hyperideal of $\mathcal{N}$ for all $k \in \mathcal{J}$. Moreover, $\bigcap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha)$ is too a hyperideal of $\mathcal{N}$. On the other hand, it can be easily verified that the condition ***ii.*** of the Definition 3.1 holds. Consequently, $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is an idealistic soft topological hyperring over $\mathcal{N}$.

ii. It is similar to the proof of the previous case.

3.6. Theorem

Let's consider $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ as a non-empty family of idealistic soft topological hyperrings over $\mathcal{N}$. Then, the $\wedge -$ intersection $\widetilde{\bigwedge}_{k \in \mathcal{J}} (\mathcal{S}_k, \mathcal{V}_k, \mathcal{T})$ is an idealistic soft topological hyperring over $\mathcal{N}$, given that it is non-null.

Proof. Consider $(\mathcal{S}, \mathcal{V}, \mathcal{T}) = \widetilde{\bigwedge}_{k \in \mathcal{J}} (\mathcal{S}_k, \mathcal{V}_k, \mathcal{T})$ for a non-empty family $\{(\mathcal{S}_k, \mathcal{V}_k, \mathcal{T}) \mid k \in \mathcal{J}\}$ of idealistic soft topological hyperrings over $\mathcal{N}$. Let $\alpha \in Supp(\mathcal{S}, \mathcal{V})$. It holds from the assumption $\bigcap_{k \in \mathcal{J}} \mathcal{S}_k(\alpha_k) \neq \emptyset$ that $\mathcal{S}_k \alpha_k \neq \emptyset$ for all $k \in \mathcal{J}$ and $(\alpha_k)_{k \in \mathcal{J}} \in \mathcal{V}_k$. Hence, $\mathcal{S}_k(\alpha_k)$ is a hyperideal of $\mathcal{N}$ for all $k \in \mathcal{J}$, so that their intersection is too a hyperideal of $\mathcal{N}$. Furthermore, the condition ***ii.*** of the Definition 3.1 is also satisfied. Therefore, $(\mathcal{S}, \mathcal{V}, \mathcal{T})$ is an idealistic soft topological hyperring over $\mathcal{N}$. This completes the proof.

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