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Analysis of Bridge Graph through K-Banhatti Sombor Invariants

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ABSTRACT

Graph invariant is a numeric quantity, which is often associative with a molecular graph. During the last few years countless mathematical graph invariants have been characterized and used for the prediction of different properties of chemical compounds. In any case, no solid assessment has been embraced to choose, how much these invariants are connected with a molecular graph. The current study introduce two new topological invariants, namely multiplicative k banhatti sombor index and multiplicative k banhatti reduce sombor index to calculate the results for three variants of bridge networks. Bridge graphs have a good potential for prediction in the field of computer science, mathematics, chemistry, pharmacy, informatics, and biology in relation to physical, chemical structures, and networks. These deduced results can be used for the computer network modelling, bio-informatics, and chemical compounds.

***Keywords*:** bridge graph, somor index, topological index

INTRODUCTION

Mansour and Schork in 2009, [1] introduced the idea of bridge graphs, which is a combination of networks bridged together. A bridge graph is a graph obtained from a number of graphs G1,G2, G3,...Gm by associating the vertices vi and vi + 1 by an edge∀, i = 1,2,..., m − 1 [2]. On the other hand, Gutman in 2021, defined the idea of Sombor indices. There are two variants of K-Banhatti Sombor indices, the first one is the K-Banhatti Sombor index and the second is its reduced version [3, 4].

Wiener's first notable application in chemistry was the investigation of paraffin edges of boiling. Different topological indices were presented by following this examination that clarified physico-synthetic properties. The headway of large-scale integrated circuit innovation has empowered the development of interconnected networks which are complex in nature. Graph theory gives a key apparatus for designing and evaluating such networks. Connected networks and graph theory gives a detailed

comprehension of these connected themes. Electric power organizations requirement to ceaselessly screen the condition of their frameworks on account of the voltage size at loads and the machine stage point at generators. In the electric power framework, a vertex addresses an electric hub and an edge addresses a transmission line joining two electrical points. Chemical graph theory is a part of numerical science wherein it applies devices of graph theory to demonstrate the chemical wonder numerically. In the architecture of a network interconnection point addresses processor and an edge addresses a network path through which transmission was completed. In network vertex is a network addresses hub like PC, switch, switch some other gadget and an edge addresses a network path through which transmission was completed. This theory contributes a noticeable job in the fields of chemical sciences [5]. A sub-atomic or chemical graph is a basic finite graph in which vertices indicate the molecules and edges signify the chemical bonds in the basic chemical construction. The topological indices are really a numeric amount related with chemical constitution indicating for connection of the chemical construction with numerous properties of physiochemical nature, reactivity of chemicals, and organic exercises. These topological indices or invariants are also numeric values which are related with the computer networks, their interconnections and their properties.

This examination gives a basis to understand the profound topologies of some important bridge networks and how these networks can be developed on the basis of best topological properties. This feature also gives potential assistance to scholars to contemplate networks characteristics better. For additional work, if the relating networks are replaced by different networks, this study can also calculate and get the comparing formulas. A topological record is actually planned by transforming a network structure into a number. Originally, the current study aims to introduce new computer architectures and networks with the assistance of topological indices. An interconnection network's structure can be mathematically demonstrated by a graph. The geography of a graph decides the way where vertices are associated by edges. From the geography of a network certain properties can easily be resolved. Maximum distance is resolved between any two hubs in the network. The level of hub is identified by the number of connections attached with it. Computer networks from intranet to world wide networks, electric power interconnection, social networks, sexual sickness of networks of transmission, and genome networks are comparable

with graph theory with the help of Complex Networks Analysis Apparatus (CNAA). All these networks are at peak level of their use and diversified. In this load of cases, this study computes boundaries called Topological Invariants (TIs) that mathematically depicted the connectedness designs (structure) between the hubs or entertainers in a network. Furthermore, to construct a brain shocking network of general sets of laws partner laws (hubs) that direct typical organic subjects for instance. QSAR and QSPR are providing the foundation for these models. A final remark regarding the utilization of the measurement in the network plane facilitates a quantitative evaluation of various geography safeguarding mapping algorithms.

To increase the effectiveness of the observed and conservation endeavours, it is of prime importance to foster sound quantitative techniques that are able to indicate, which key areas and landscape components play noticeable and critical job in the working of territory mosaics. In this article, new emerging study computes different indices of bridge networks. The results play a vital role in determining properties of these networks and its uses in computer industry, electronics, chemistry, and pharmacy. This study also performed index analysis on certain networks which may be beneficial for peoples who are working in the field of computer science, mathematics, and chemistry. In the end to formulate a mathematical formula to construct computer network or processor design or chemical compound, to check the properties of concerning before, and also to check the feasibility of one which are discussed earlier [6].

2. LITERATURE REVIEW

This study defined and calculated different kinds of indices in context with topology like degree based, distance based, and counting related topological indices. The molecular and atomic structures of lattice are similar to hexagonal structures; honey comb structures mesh structures, and grid structures networks. The ABC and GA correlate different physical and chemical properties of chemicals having above structures, like solidness and strain energy and boiling point. Group Theory and construction of graphs can use the Cayley graphs of groups for determining properties. In this paper, equations of General Randi' c index were computed for various upsides of α , first zagreb index, atom-bond connectivity (ABC) index, geometric arithmetic GA index, the fourth ABC index (ABC4) , fifth GA index (GA5) for certain groups of graphs[7]. These graphs provided the

basis for calculating many other families of graphs for chemical compounds and computer networks [8].

In the view of study, the headway of large-scale integrated circuit innovation has empathy development of interconnected networks which are complex in nature. Graph theory gives a key apparatus in designing and evaluating such networks. Connected networks and graph theory give a detailed comprehension of these connected themes. Chemical graph theory is a part of numerical science wherein they apply devices of graph theory to demonstrate the chemical wonder numerically. In network vertex is network addresses hub like PC, switch, switch some other gadget and an edge addresses a network path through which transmission completed. These topological indices or invariants are also numeric values related with computer networks, their interconnections and their properties. In this paper, researchers calculated the different topological indices of eccentricity-based for binary tree up to K-level. The results of a paper can be used for computer networks and chemical networks in topological characterization [9].

In this paper researchers calculated the irregularity indices of honey comb networks, hexagonal networks, oxide networks, and silicate networks. Researchers also compared the findings and results. The results were very helpful in understanding the behaviour of different computer networks and chemical networks. After understanding these formulas different researchers can construct their own best network in chemistry and computer [10].

This study elaborated the graph theory as a field through which the topological indices were calculated for finding the properties of different chemicals without performing any types of experiments on them. It may be understandable only in mathematical formulas or equations which are deduced for them. Furthermore, in this paper the researchers calculated the topological indices for M-Polynomial Block Shift networks which is a part of different chemical compounds with the help of division of edge [11].

Real world problem can be solved by applying network theory. There are lot of applications of chemical graph theory in bio chemistry and theoretical chemistry. This study discusses David derived networks used in different chemical compounds and computer science to calculate their topological indices for 1st type, 2nd type, and 3rd type. The results were

effective in preparation of new drugs and helpful in understanding the properties of chemical compounds [12].

According to the previous studies, chemical network is displayed numerically by topological indices in chemical graph theory; on the other hand specific physicochemical properties are correlated with basic chemical compounds. The graph assumes an indispensable part in displaying and planning any chemical network. Simonraj et al. determined other graphs, which were named as a third kind of hex-inferred networks. This paper, contemplated a recently framed third kind of hex-inferred networks. As they were significant and useful from numerous chemical perspectives just as for pharmaceutical sciences, these outcomes additionally give the premise to comprehend the profound basic geographies of the above networks. After wards they wanted to calculate the topological indices of irregular graphs, counted related, distance based, and polynomial of concerning networks in future [13].

For the analysis, chemical reaction network theory is a space of applied arithmetic that was undertaken to show the direct and genuine chemical structures. It has also drawn revenue from pure mathematicians in light of the captivating issues that rise up out of the numerical designs. In this report, the process as of late described topological indices; specifically, number-crunching mathematical index, SK index, 1 SK index, and 2 SK index of the octagonal network. They in like manner figure entire availability index and changed Randić index of subordinate network [14, 15]. This study indicated that topological indices and network polynomials are invariants of molecular graphs. Expected properties of structures of molecules can be studied with the help of invariants. In the current article, new topological portrayals of honeycomb networks are given a degree-based descriptors. They, for the first time, used some consistency indices and a few co-indices identified with this network without line graph and complement also. These indices of honeycomb networks are very much effective to understand the physiochemical properties of chemicals. These realities might be valuable for individuals working in software engineering and science who experienced honeycomb networks. An ideal level of a specific index can be acquired by putting a limitation on it [16, 17].

In accordance with the paper approach, the M-polynomial is one of the algebraic polynomials that are helpful in theoretical chemistry. It plays significant job in processing the specific articulations of numerous degree

based topological indices. They got numerous degree based topological indices for benzene ring installed in P-type-surface network and Tickysim SpiNNaker Model (TSM) sheet. Initially, they figured M-polynomial of these graphs and later recuperated numerous degree-based topological indices applying it. Furthermore, they have also shown the outcomes graphically. These outcomes can assume a significant part to imagine the geography of the aforesaid networks [18, 19].

In this study's view point, Recognition of Hand Gestures (HGR) is one of the primary space of research for Human Computer Interaction applications. Most existing methodologies depend on neighbourhood or mathematical properties of pixels. In any case, there are some genuine difficulties on HGR strategies like affectability to revolution, scale, brightening, bother, and impediment. In this paper, the researchers studied HGR from chart perspectives. They also used a distinct and wide-range dataset with 31 types of gesture of different quantities of fingers with raising isolation. These are total 2170 images. A large portion of the current datasets incorporate a predetermined number of gestures, while here they considered various disfigurements and varieties, for example, pivot in 2 and 3 dimensional space and scale additionally slight verbalization. They assessed our technique on SBU-1 and examined our strategy against scale, revolution, and clamour. They executed our calculation for this dataset and got the mean precision up to 90%. They additionally contrasted the outcomes and some datasets that mostly share the pictures of our dataset. The validity of screaming our calculation in presence of commotion and freedom of it to scale and turn both theoretically and tentatively. Since the GNG diagram isn't special, they additionally tried the affectability of our algorithm to this graph [20, 21].

Readings of the work showed how a high density of catalytic sites joined with specific spatial directions of those sites which can cause a supra molecular catalytic machine that works deliberately to work with the development of pertinent progress states and, thus, upgrade response rates. The surviving variety of MOF topologies proposes that these permeable systems would end up being helpful organizational structures for a rich assortment of multisite-catalysed compound responses [22].

The current research investigated, physicochemical properties and topological indices which are discussed and examined comprehensively in the QSAR/QSPR, for example, Randić, Zagreb, and ABC index are utilized

to foresee bioactivity of the synthetic mixtures. Chart theory has discovered significant use in science, especially in demonstrating substance structures. Topological indices are planned fundamentally by changing a molecular graph into a number. In this paper researchers computed the Randić, Zagreb, and ABC index of Silicate, honeycomb and hexagonal networks. Assembled recursively utilizing the hexagon decoration, honeycomb networks are broadly utilized in computer illustrations, phone base stations, picture preparing, and in science as the portrayal of benzenoid hydrocarbons [23]. This likewise gives possible assistance to researchers to consider better characteristics of networks [24, 25].

This study demonstrated how computers are becoming more and more commonplace as a tool for gathering information around the globe. To such an extent, today, it is hard to track down any space, logical or other, which isn't dependent on their application. Indeed, even general sets of laws have been affected by novel processing and data strategies. A considerable lot of these QSAR procedures depend on the utilization of structural boundaries, which are a mathematical arrangement that classify helpful structural data and empathy relationships among structure and natural properties [26].

In every one of these cases, they can compute boundaries called Topological Indices (TIs) that mathematically depict the connectedness designs between the nodes or entertainers in a network. Thus, TIs are helpful as contributions for QSPR models at all the structural levels. Indeed, even general sets of laws might be moved toward utilizing and figuring the data strategies like networks. So, they can be used to develop an unpredictable network of overall sets of laws associating laws (nodes) that manage basic organic points. A systematic judicial framework is expected to give proper and pertinent direction to addressing different registering procedures as applied to logical research [27].

The current study concluded that graphs are considered as a stunning modelling apparatus which can be utilized to show and clarify diverse sort of relations between actual issues. A great deal of issues can be investigated exhaustively by employing graph theory. In this paper, scientists concentrated on various types of issues identified with graph theory and their executions and suggestions in the field of software engineering to exhibit the adequacy of graph theory. These applications are acquainted especially with stretch out graph theory and to display its objective and significance in software engineering designing. This study benefits the

under investigation of software engineering to get significance data on graph theory and its significance with different subjects like working frameworks, networks, databases, and programming. This paper also focused on various uses of graph theory that have congruity to the field of software engineering and applications [28].

The hypothetical thoughts about graphs were utilized by software engineering applications, such as, information mining, picture division, grouping, picture catching, and networking. For example, an information design can be arranged as tree utilizing vertices and edges. Additionally, modelling of network geographies ought to be conceivable using graph thoughts. Also the fundamental thought of graph concealing is utilized in asset distribution and booking. Furthermore, ways, walks, and circuits in graph theory are used in enormous applications to deal with versatile issues, information base arrangement thoughts, and resource networking. This prompts the headway of new estimations and new hypotheses that can be used in gigantic applications. This paper consists of two sections. Section 1 gives the certain establishment of graph theory and a couple of uses in booking. Section 2 underlines how graph theory is utilized in various applications. Research material of this paper explains that the field of math accepts vital part in various fields [29]. Thus, these fields have fortified the improvement of various new chart hypothetical thoughts and incited many testing graph theory issues. They also identified that graph theory and various spaces of use would provoke critical new developments. The critical occupation of graph theory in PC applications is the improvement of graph estimations [30].

Topological invariants empower to accumulate information about algebraic structures and gives us a mathematical technique to guess the hidden properties of different structures[31-33]. Numerous methods in the present history are used to check the quality of a topological index. There are two main clashes of topological indices, first one is the degree based topological and the second class is known as distance based topological indices. The topological invariant was advanced by Wiener in 1947 [34], when he was attempting to find the edges of boiling over of Alkane. This disclosure prompts the start of recent fad in science. The first degree based topological index was advanced by Randic in 1975 [35]. There are hundreds of such invariants in the present history [36-40]. K-Banhatti Sombor index and its reduced version has good potential of prediction in the field of

computer science, mathematics, chemistry, pharmacy, informatics, and biology in context with physical and chemical structures and networks.

3. MATERIALS AND METHODS

Methodology is based on quantitative inquiry. The purpose of this research is to explore and develop understanding about critical concerns error free, failure free, best performance and having advance capabilities computer networks, interconnection network of processors, and power interconnection networks.

3.1 Objectives

The main objective of this study is to investigate the topological invariants of bridge computer networks. The study found out the intensity of seriousness of topological indices in certain networks like computer networks, interconnection network of processors, power interconnection networks, and chemical structures. This study also explained K-banhatti sombor indices, its reduced form, and their benefits for the application. Furthermore, this article also explains recent uses of K-banhatti sombor indices. Its prime objective is to develop formulas, so that it can check the topology, performance of certain networks without doing / performing experiments and also before their manufacturing. The work deduced some results which were used in the modelling of certain computer networks, interconnection networks, power interconnection networks, and chemical structures.

3.2 Significance

The study is very significant in these days because it creates awareness about topological invariants of certain networks like computer networks, interconnection network of processors, power interconnection networks, and chemical structures. It is also discovering new and significant solutions or formulas for modelling certain networks.

3.3 Hypothesis

The current study considers the following hypothesis for the development of certain networks like computer networks, interconnection network of processors, power interconnection networks, and chemical structures because venders and manufacturer need to understand the complexity and intensity of performance and failure free products. Moreover, the analysis provides the strength to developed error free, failure free and best performer

computer networks, interconnection network of processors, and power interconnection networks.

3.4 Research Methodology

In this systematic study it will take an existing Bridge network, associate it with graph and solve the topology of graph with the help of K-Banhatti Sombor indices and its reduced form. These deduced results will be applicable on many other networks in the fields of computer networks, processor interconnection networks, memory interconnection networks, power interconnection networks, and image processing afterward.

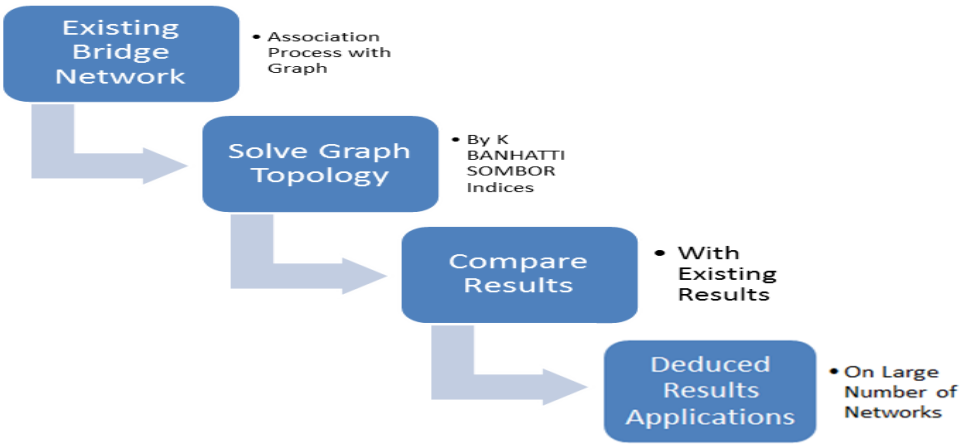

Figure 1. Research methodology

Figure 1 shows the flow of systematic study of the current research, which take existing bridge networks and associate it with graph theory, solve the topology of graph by K-Banhatti Sombor Indices and compare the results, and deduced results would be used for modelling of certain networks.

4. RESULTS AND DISCUSSION

4.1 Computational Results

The results and discussion may be presented separately, or in one combined section, and may optionally be divided into headed subsections. A bridge graph is a graph obtained from a number of graphs $G_1, G_2, G_3, \ldots G_m$ by associating the vertices v_i and v_{i+1} by an edge$\forall$, $i = 1,2,\ldots, m-1$ [2]. K-

Banhatti Sombor indices have two variants, K-Banhatti Sombor index and its reduced version [3]. Here, we introduced multiplicative K-Banhatti Sombor and reduced Sombor indices.

$$MBSO(G) = \prod_{ue}\sqrt{d_u{}^2 + d_\varrho{}^2} \tag{3.1}$$

Eq. 3.1 shows the K-Banhatti Sombor Index which would be used for the solution of bridge networks mention in the figures 2, 4, and 6.

$$MBSO_{r\varrho d}(G) = \prod_{ue}\sqrt{(d_u - 1)^2 + (d_\varrho - 1)^2} \tag{3.2}$$

Eq. 3.2 shows the multiplicative K-Banhatti Sombor Reduced Index which would be used for the solution of bridge networks mention in the **Figures 2**, **4**, and **6**.

Table 1: Edge Partition of G_R ($P_{S,\,V}$) Over P_S

E	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	ε(1 ,2)	1	ε(1 ,1)	t
ε_2	ε(2 ,2)	2	ε(2 ,2)	3t + 2
ε_3	ε(2 ,3)	3	ε(2 ,3)	t
ε_4	ε(3 ,3)	4	ε(3 ,4)	t - 3

de = du + dv – 2

Table 1 describes the edge partitions of graph G_r ($P_{s,\,v}$) Over P_s of bridge graph given in **Figure 2**.

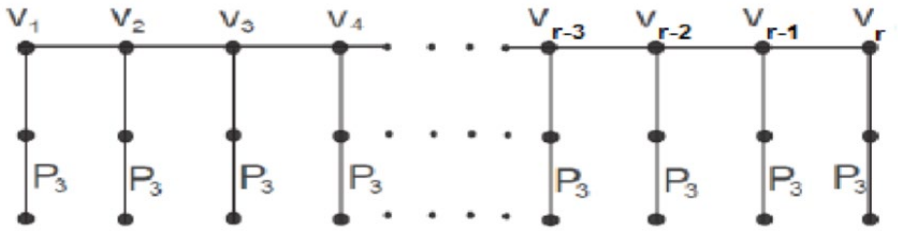

Figure 2. Gt (Ps, v) over Ps

Figure 2 shows the bridge networks in which bus networks and star networks bridge in a tree like structure.

4.2 Bridge Graph Gt (Ps, v) Over Path

If the vertex set is *V* then by the observation of **Figure 2**, it can order this

vertex set into four subsets*V1,V2, V3,* and*V4*, Such that $V = V1 + V2 + V3 + V4$. If E represents the edge set. The **Figure 2** shows that there are four distinct kinds of edges present in the graph of Bridge graph $G_r(P_s, v)$ over path of hybrid networks. **Table 1**, explains the details of the edges partition.

Theorem 4.1. Let G be a graph of $G_t(P_s, v)$ over P_s, then, MBSO and $MBSO_{red}$ indices are

1. $MBSO(G) = 20\sqrt{13}t^2(t-3)(3t+2)$
2. $MBSO_{red}(G) = \sqrt{130}(t-3)(3t+2)$

Proof: Let,

1. $MBSO(G) = \prod_{ue}\sqrt{d_u{}^2 + d_\varrho{}^2}$

$MBSO(G) = \sqrt{1^2+1^2}\,(t) \times \sqrt{2^2+2^2}\,(3t+2) \times \sqrt{2^2+3^2}\,(t) \times \sqrt{3^2+4^2}\,(t-3)$

$MBSO(G) = 100\sqrt{13}r^2(t-3)(3t+2)$

2. $MBSO_{red}(G) = \prod_{ue}\sqrt{(d_u-1)^2 + (d_\varrho-1)^2}$

$MBSO_{red}(G) = \sqrt{(1-1)^2+(1-1)^2}\,t \times \sqrt{(2-1)^2+(2-1)^2}\,(3t+2)$
$\times \sqrt{(2-1)^2+(3-1)^2}\,t \times \sqrt{(3-1)^2+(4-1)^2}(t-3)$

$MBSO_{red}(G) = \sqrt{130}(t-3)(3t+2)$

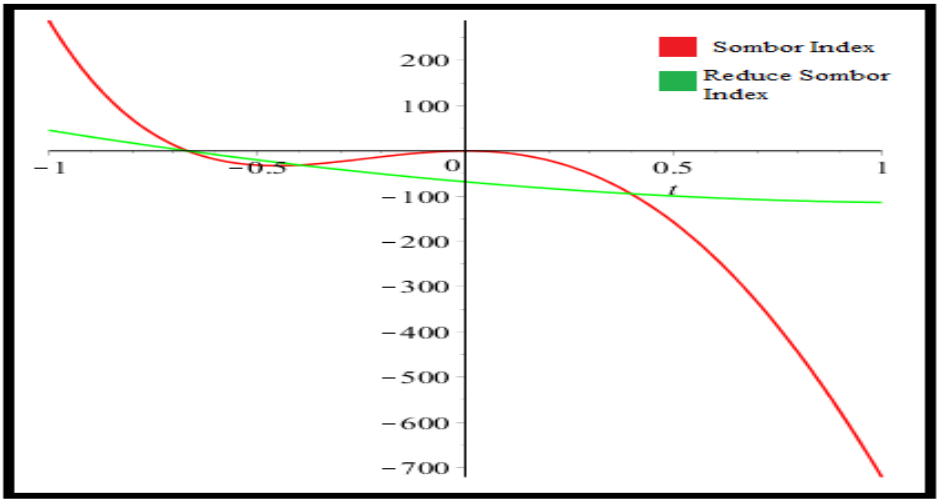

Figure 3. Graphical Representation of *MBSO* and $MBSO_{red}$ for of $G_t(P_s, v)$

Table 2: Edge Partition of G_R ($K_{s,v}$) Over K_s

ε	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	$\varepsilon_{(2,2)}$	2	$\varepsilon_{(2,2)}$	ts – 2t
ε_2	$\varepsilon_{(2,3)}$	3	$\varepsilon_{(2,3)}$	4
ε_3	$\varepsilon_{(2,4)}$	4	$\varepsilon_{(2,4)}$	2t – 4
ε_4	$\varepsilon_{(3,4)}$	5	$\varepsilon_{(3,5)}$	2
ε_5	$\varepsilon_{(4,4)}$	6	$\varepsilon_{(4,6)}$	t-3

Table 2 describes the edge partitions of graph G_r ($K_{s,v}$) over K_s of bridge graph given in **Figure 4** with frequencies.

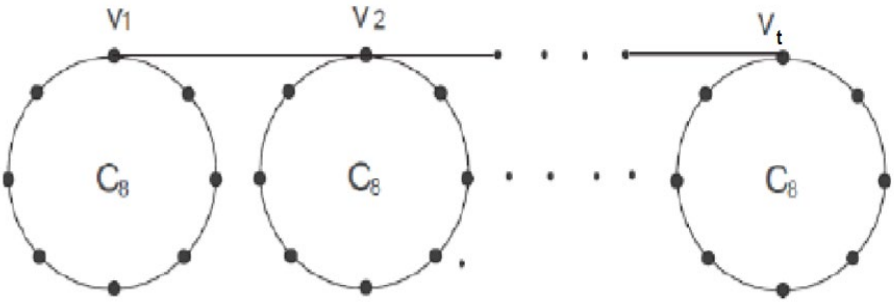

Figure 3. Gt (Cs, v) over Cs

Figure 4 shows the bridge networks in which bus networks and ring networks bridge together.

4.3 Bridge Graph Gt (Cs, v) Over Cycle

Assuming V is the arrangement of vertices saw in **Figure 4**, this arrangement of vertices can be parted into four subsets *V1, V2, V3,* and *V4* to *V = V1 + V2 + V3 + V4*. When *ε (D2 (m))* addresses an edge set. **Figure 4** shows a half network cycle with five distinct kinds of edges in the network graph of the bridge graph *Gt (Cs, v).* **Table 2** provides a detailed description of the edge set.

Theorem 4.2. Let G be a graph of G_t (C_s, v) over C_s Then *KBSO* and KBSO_{red} are

1. $MBSO\ (G) = 3328\sqrt{85}t(s-2)(t-2)(t-3)$

2. $MBSO_{red}(G) = 320\sqrt{170}t(s-2)(t-2)(t-3)$

Proof:

1. $MBSO(G) = \prod_{ue}\sqrt{d_u{}^2 + d_\varrho{}^2}$

$MBSO(G) = \sqrt{2^2+2^2}\,(ts-2t) \times \sqrt{2^2+3^2}\,(4) \times \sqrt{2^2+4^2}\,(2t-4)$
$\times \sqrt{3^2+5^2}\,(2) \times \sqrt{4^2+6^2}\,(t-3)$

$MBSO(G) = 3328\sqrt{85}t(s-2)(t-2)(t-3)$

2. $MBSO_{red}(G) = \prod_{ue}\sqrt{(d_u-1)^2+(d_\varrho-1)^2}$

$MBSO_{red}(G) = \sqrt{(2-1)^2+(2-1)^2}\,(ts-2t) \times \sqrt{(2-1)^2+(3-1)^2}\,(4) \times \sqrt{(2-1)^2+(4-1)^2}$
$(2t-4) \times \sqrt{(3-1)^2+(5-1)^2}\,(2) \times \sqrt{(4-1)^2+(6-1)^2}\,(t-3)$

$MBSO_{red}(G) = 320\sqrt{170}t(s-2)(t-2)(t-3).$

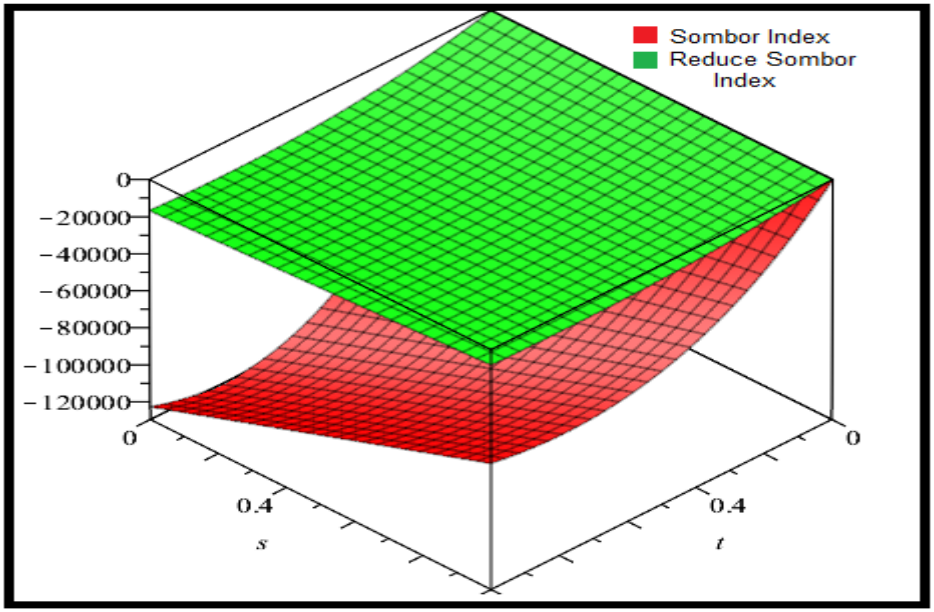

Figure 5. Graphical Representation of *MBSO* and $MBSO_{red}$ for Gt (Cs, v)

Table 3: Edge Partition of G_R ($C_{S,V}$) Over C_S

ε	ε (du , dv)	de	ε(du , de)	Recurrence
ε_1	$\varepsilon_{(2,2)}$	2	$\varepsilon_{(2,2)}$	rs – 2r

ε	ε (du , dv)	de	ε(du , de)	Recurrence
ε_2	$\varepsilon_{(2,3)}$	3	$\varepsilon_{(2,3)}$	4
ε_3	$\varepsilon_{(2,4)}$	4	$\varepsilon_{(2,4)}$	2r – 4
ε_4	$\varepsilon_{(3,4)}$	5	$\varepsilon_{(3,5)}$	2
ε_5	$\varepsilon_{(4,4)}$	6	$\varepsilon_{(4,6)}$	r-3

Table 3 describes the edge partitions of graph G_r ($C_{s,v}$) Over C_s of bridge graph given in **Figure 6** with number of occurrences.

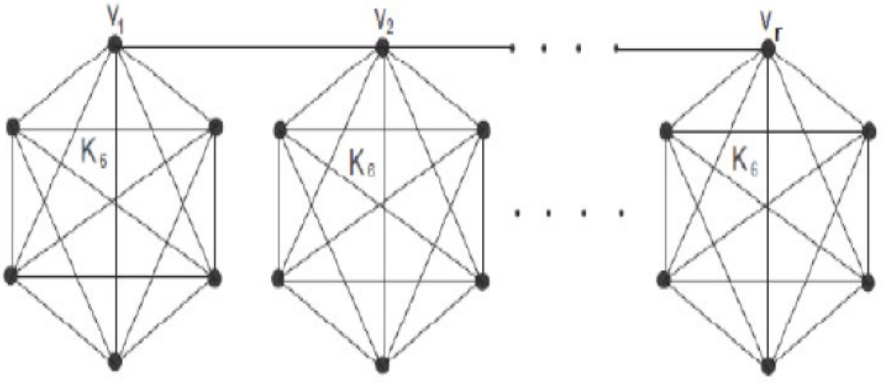

Figure 4. Gr (Ks, v) over Ks

Figure 6 shows the bridge networks in which bus networks and fully connected networks are bridge together.

4.4 Bridge Graph Gr (Ks, v) Over Complete Graph

Assuming that vertices set is V, understanding **Figure 6** allows us to sort this set of vertices into three subsets V1, V2, and V3 so that V = V1 + V2 + V3. If E shows the edge set. **Figure 6** shows the bridge graph Gr (Ks, v) of the complete graph of the hybrid network. Bridge graph of the network graph has five different edges. **Table 3** provides a detailed description of the edge set.

Theorem 4.3. Let G be a graph of *G_r (K_s, v)* over K_s. Then, MBSO and $MBSO_{red}$ indices are;

1. *MBSO (G)* = $2\sqrt{5785}(r-2)^2(s^2+2s+17)(s^2+2s+29)(5s^2+14s+17)((rs^2-rs-2s-2)$

2. $MBSO_{red}(G) = 30\sqrt{13}\sqrt{s^2+9}(r-2)^2(s^2+2s+17)(5s^2+16s+13)((rs^2-rs-2s-2)$

Proof:

1. $MBSO(G) = \prod_{ue}\sqrt{d_u{}^2 + d_\varrho{}^2}$

$MBSO\ (G) = \sqrt{4^2+7^2}\ (2) \times \sqrt{4^2+(s+1)^2}\ (2) \times \sqrt{5^2+8^2}\ (r-2) \times \sqrt{5^2+(s+2)^2}\ (r-2)$

$\sqrt{(s-1)^2+(2s+4)^2}(\frac{rs(s-1)-2(s+1)}{2})$

$MBSO\ (G) = 2\sqrt{5785}(r-2)^2(s^2+2s+17)(s^2+2s+29)(5s^2+14s+17)((rs^2-rs-2s-2)$

2. $MBSO_{r\varrho\underline{d}}(G) = \prod_{ue}\sqrt{(d_u-1)^2+(d_\varrho-1)^2}$

$MBSO_{red}\ (G) = \sqrt{(4-1)^2+(7-1)^2}\ (2) \times \sqrt{(4-1)^2+(s)^2}\ (2) \times \sqrt{(5-1)^2+(8-1)^2}\ (r-2)$

$\times \sqrt{(5-1)^2+(s+1)^2}\ (r-2) \times \sqrt{(s-2)^2+(2s+3)^2}(\frac{rs(s-1)-2(s+1)}{2})$

$MBSO_{red}\ (G) = 30\sqrt{13}\sqrt{s^2+9}(r-2)^2(s^2+2s+17)(5s^2+16s+13)((rs^2-rs-2s-2).$

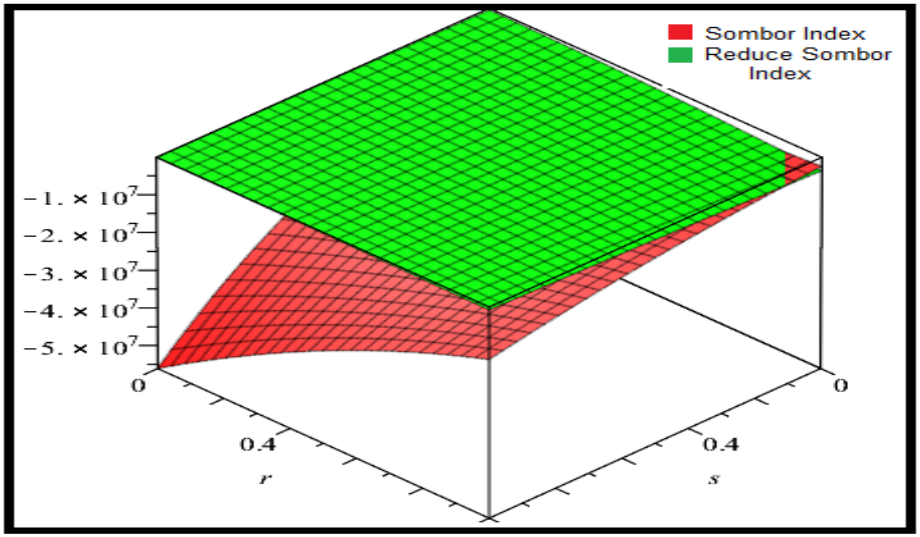

Figure 7. Graphical Representation of *MBSO* and $MBSO_{red}$ for G_r (K_s, *v*)

Figures 3, **5**, and **7** show the graphical representation of bridge graph over path, cycle and complete graph, respectively. Where, red colour is fixed for multiplicative k-banhatti sombor index and green colour is fixed for multiplicative k-banhatti sombor reduce index.

4.5 Conclusion

TIs have lots of uses and implementations in different fields of computer science, chemistry, biology, informatics, arithmetic, material sciences, and many more. However, the utmost significant application is in the non-exact Quantitative Structure-Property Relationships (QSPR) and Quantitative Structure - Activity Relationships (QSAR). TIs are associated with the structure of networks, backbone of internet, local area networks, and chemical structure. Whereas the current research, discusses the multiplicative K-Banhatti Sombor invariants which are have numerous prediction quality for different variants of bridge graphs or networks, for instance G_r (P_s, v), G_r (C_s, v) and G_r (K_s, v). These deduced results would be used for the modelling of computer networks (like LAN, MAN, WAN, and Backbone of internet), interconnection networks, power generation networks, chemical structures, and image processing, bio-informatics, and memory interconnection networks.

Data Availability

The data used in this paper can be requested from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this work.

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