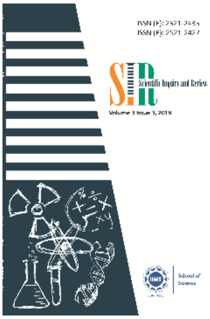

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Heat Transportation of Ferrofluid in a Micro Annulus

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Abstract

This article investigated the effect of magnetic field on heat-absorbent ferrofluid in a vertical loop consisting of a pair of concentric cylinders with surface sliding and temperature jumping. For this puprose, the control equation was converted into the dimensional form using non-dimensional quantities and parameters. The system of equations was solved analytically using an integration technique. Subsequently, the solution was obtained in the form of first and second type of Bessel functions. The results are presented graphically and show that velocity increases with the increase in the nanoparticle size. Resultantly, the rate of heat transfer of the fluid in the inner cylinder reduces.

***Keywords*:** generating/absorbing ferrofluid, inner and outer cylinder, magnetic field, vertical annulus

Introduction

A nanofluid (NF) is a liquid carrying nanometer-sized particles called nanoparticles. The nanoparticles (NPs) utilized in NF are made up of metals, oxides, and carbides. Choi [1] was the first to introduce a new type of fluid called NF. Gorla et al. [2] have investigated NF regular convection borderline layer flow via a permeable channel on a perpendicular cone. Kuznetsov and Nield [3] have performed an analysis on natural NF flow passed through a perpendicular plate. The effect of radiation on the natural convection flow of NF has been discussed over a static perpendicular plate by Das and Jana [4]. Pak and Cho [5] demonstrated convective heat exchange in flow regime taking Al_2O_3 and TiO_2 with H_2O. Nanjundappa et al [6] analyzed the effect of MHD viscosity in a ferrofluid (FF) permeable layer. A few studies have been directed at imposing convection fluid flow in microchannels. Due to its significance, such as designing and optimizing micro-machines when shipping micro-electromechanical devices, convective flow in a vertical micro-annulus has drawn a good deal of interest in industrial and technological applications. The study of the flow field at an extreme level helped to develop a method of combining fluid for micro-scale use. Jha et al [7] researched the completely advanced fluid flow in a micro-channel. Jha et al. [8] analyzed the fully advanced combined convection flow of heat-producing fluid in a vertical micro-concentric annulus cylinder. Jha and Aina [9] created a model for analyzing the impact of absorption or infusion on fully established stable laminar convection flow in a micro-annulus. Dawood et al. [10] expressed the aspects of convection flow in an upright annulus. More experimental and numerical testing can be found in the mixed convection flow by Mohammed et al. [11]. In a microchannel, Day and Stone [12] examined a rotating cylinder and reported that the cylinder and channel walls have small pores between them. The flow of stokes around a microchannel is determined by Yoon and Jeong [13]. Chen and Weng [14] numerically studied the impact of microflow on heat exchange rate. Magnetohydrodynamics (MHD) is the mutual contact of magnetic field and fluid flow. The marvels of MHD offer a few critical applications in innovative and producing fields, including MHD generators, quickening agents and pumps, stream meters and heading. Lorentz force is for the most part used to control the stream in such conditions. Sheikholeslami et al. [15] examined the magnetic impacts of finite-element methods on NF flow and thermal transport in a semi-circular yard. Sheikholeslami and Gorji-Bandpy [16] examined the mathematical solution of FF flowing. The MHD natural convection flow of NF flowing through the region between a cold external square cylinder and an inner circular heated internal is examined by Sheikholeslami et al. [17].

Several researchers studied the fluid flow in a vertical cylinder. Yucel [18] regarded the combination of heat and mass exchange in permeable media around a vertical cylinder. Jamil and Fetecau [19] described Maxwell fluid flows at the border between coaxial cylinders with specified shear stress. With the presence of a static radial magnetic field, Sheikholeslami et al. [20] researched the natural convection in a concentrated annulus. Aldoss and Ali [21] looked at the combined convection from a horizontal cylinder with suction/infusion as well as a magnetic field. Lin and Shih [22] regarded the laminar boundary layer and heat exchange with constant velocity along horizontally and vertically moving cylinders and discovered that similar solutions could not be achieved owing to the curvature impact of the cylinder. The demand for heat dissipation from electronic components has been increasing for the last forty years. Tuckerman and Pease [23] are the pioneers of microchannel heat convection. They demonstrated that for small volumes microchannels have high heat flux. The impact of viscous dissipation in microtubes and microchannels was studied by Kleinstruer and Koo [24]. Sabry [25] demonstrated the impact of thermal efficiency on microchannel designs. In a microchannel, Day and Stone [26] examined a rotating cylinder, the cylinder and channel walls have small pores between them. Yoon and Jeong [27] determined the flow of stokes around a micro-channel. Chen and Weng [28] studied numerically the impact of microflow on heat exchange rate. Jha et al. [29] demonstrated the impact of MHD flow in vertical micro-channel. The natural convection flow in a vertical micro-channel was demonstrated by Jha et al. [30]. Munawar et al. [31] deal with vacillate stretching cylinders. Malvandi [32] regarded NPs in the rotating sphere to report the time-dependent factor. The slip condition was used by Abbas et al. [33] to analyze unstable stretching / shrinking cylinders. Ahmad B [35,36] solve the partial differential equation (PDE) with help of the spline technique.

2. Problem Formulation

We assume a steady, laminar, fully developed mixed convection incompressible viscous flow in a vertical concentric annulus. We considered the infinite length of the micro-annulus. The r-axis is in the radial direction and z-axis is along upward direction. The radius of the inner and outer cylinder are r_1 and r_2. The temperature of the inner and outer cylinder is T_1 and T_2. B_0 is the magnetic field. Fe_3O_4 nanoparticles are considered EG as a base fluid with uniform shape and size. We considered that the Fe_3O_4 is Newtonian and incompressible.

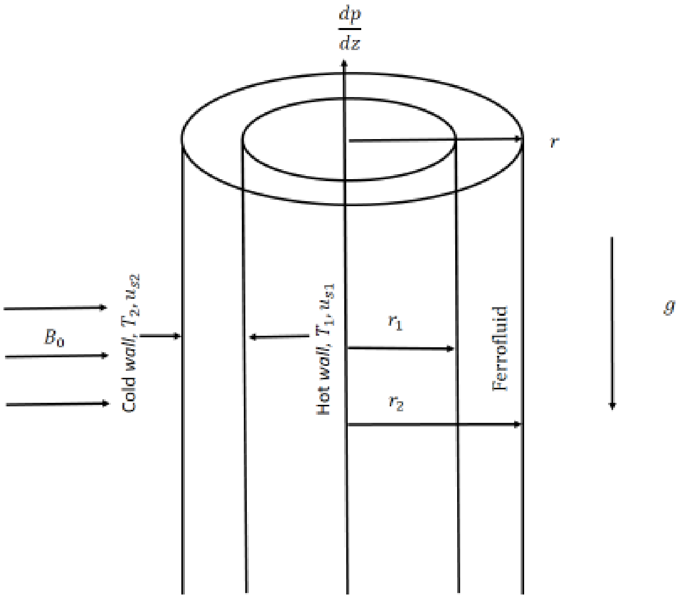


Figure1. Geometry of the Annulus

The fluid is supposed to be single phase continuum. By assuming the Boussineq model the governing equation for laminar, two dimensional flow are

$$\frac{\mu_{nf}}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)+\beta_{nf}\,g\,\rho_{nf}\left(T-T_0\right)=\frac{dp}{dz}+\sigma_{nf}B_0^2u, \tag{1}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)\pm\frac{Q_0\left(T-T_0\right)}{k_{nf}}=0. \tag{2}$$

The reference temperature is characterized in any cross sectional area of the duct as

$$T_0=\frac{2}{r_2^2-r_1^2}\int_{r_1}^{r_2}T\left(r\right)rdr. \tag{3}$$

The B.Cs are as following:

$$u=u_{s_1}\quad \text{at } r=r_1,$$

$$u = u_{s_2} \text{ at } r = r_2,$$
$$T = T_{s_1} \text{ at } r = r_1,$$
$$T = T_{s_2} \text{ at } r = r_2. \tag{4}$$

The velocity slip is defined as

$$u_{s_1,s_2} = \pm \frac{2-F_v}{F_v} \lambda u_r \Big|_{r=r_1,r_2} \tag{5}$$

where the temperature jump is defined as

$$T_{s_1,s_2} - T_w = \pm \frac{2-F_t}{F_t} \frac{2\gamma}{\gamma+1} \frac{\lambda}{(\Pr)_{nf}} T_r \Big|_{r=r_1,r_2} \tag{6}$$

Presenting the accompanying dimensionless parameters characterized as

$$R = \frac{r}{r_2}, \quad r^* = \frac{r_1}{r_2}, \quad Z = \frac{z}{\mathrm{Re}\, D_h}, \quad D_h = 2(r_2 - r_1),$$
$$\theta = \frac{T - T_0}{T_1 - T_2}, \quad U = \frac{u}{u_0}, \quad \Pr = \frac{\mu C_p}{k}, \quad \beta_v = \frac{2-F_v}{F_v}, \tag{7}$$
$$Kn = \frac{\lambda}{D_h}, \quad Gr = \frac{g\beta\Delta T D_h^3}{\mu^2}, \quad \beta_t = \frac{2-F_t}{F_t} \frac{2\gamma}{\gamma+1} \frac{1}{\Pr},$$
$$w = \frac{T_1 - T_0}{T_1 - T_2}, \quad \mathrm{Re} = \frac{u_0 D_h}{\mu}, \quad H^2 = \frac{Q_0 r_2^2}{k}, \quad \mathrm{P} = \frac{p}{\rho_0 u_0^2}.$$

The expressions for μ_{nf}, ρ_{nf}, α_{nf}, β_{nf}, σ_{nf}, $(\rho c_p)_{nf}$ and k_{nf} are expressed as

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \quad \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}},$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_s, \tag{8}$$

$$\beta_{nf} = (1-\phi)\beta_f + \phi\beta_s, \quad \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3\phi\left(\frac{\sigma_s}{\sigma_f} - 1\right)}{\left(\frac{\sigma_s}{\sigma_f} + 2\right) - \phi\left(\frac{\sigma_s}{\sigma_f} - 1\right)}.$$

Table 1. Thermo-physical Properties of NFs and NPs .

Liquids and nano particles	$\rho(Kg/m^3)$	$c_p(k/kg)$	$k(wK/m)$	$\beta * 10^{-5}(K^{-1})$	$\sigma(S/m)$
EG	113.2	2410	0.252	1.89	$1.07*10^{-6}$
Fe_3o_4	5200	670	6	0.5	25000

Using quantities given in (7)&(8) Eqs. (1) and (2) in dimensionless form are

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{dU}{dR}\right)+\frac{\phi_1}{A}\frac{Gr}{\mathrm{Re}}\theta-\frac{1}{A}\frac{dP}{dZ}-\phi_4 A_3 M^2 U=0, \tag{9}$$

$$\frac{1}{R}\frac{d}{dR}\left(R\frac{d\theta}{dR}\right)\pm\frac{1}{A_1}H^2\theta=0, \tag{10}$$

The BCs are

$$U=2\beta_v Kn\left(1-r^*\right)\frac{dU}{dR} \qquad \text{at} \qquad R=r^*, \tag{11}$$

$$U=-2\beta_v Kn\left(1-r^*\right)\frac{dU}{dR} \qquad \text{at} \qquad R=1, \tag{12}$$

$$\theta=\frac{T_{s_1}-T_0}{T_1-T_2}=w+2A_2\beta_T Kn\left(1-r^*\right)\frac{d\theta}{dR} \qquad \text{at} \qquad R=r^*, \tag{13}$$

$$\theta=\frac{T_{s_2}-T_0}{T_2-T_0}=w-1-2A_2\beta_T Kn\left(1-r^*\right)\frac{d\theta}{dR} \qquad \text{at} \qquad R=1, \tag{14}$$

Where $$A_2=\frac{\phi_1 A_1}{\phi_3}.$$

From Eqs. (5) and (10), we obtained the following temperature constraints

$$\int_{r^*}^{1}\theta\left(R\right)RdR=0 \tag{15}$$

2.1. Analytical Solution

It can be noted that the solution for θ is different for $+H$ in Eq.(10) from $-H$ in Eq.(10). Two cases of solutions are formed.

2.1.1. Case I: Heat Generating Fluid (source)

The closed form solution for Eq. (10) with BCs given in Eqs. (13), (14) and (15) is

$$\theta(R)=\frac{A_5Y_0(E_1R)-A_6J_0(E_1R)}{(x_2-x_4)A_5-(x_1-x_3)A_6}. \tag{16}$$

By using Eq. (16) into Eq. (9) and integrating any applying B.Cs shown in Eq. (11) and Eq. (12)

$$U(R)=E_3\left(\phi_1\frac{Gr}{\mathrm{Re}}\left(\frac{A_5Y_0(E_1R)-A_6J_0(E_1R)}{(x_2-x_4)A_5-(x_1-x_3)A_6}\right)-\frac{dP}{dZ}\right)+\mathrm{C}_1\,\mathrm{l}_0(E_2R)+C_2K_0(E_2R). \tag{17}$$

The mean velocity u_0 for any cross sectional area in the channel is

$$u_0=\frac{\int_{r^*}^{1}U(R)RdR}{\int_{r^*}^{1}RdR}=1. \tag{18}$$

The dimensionless θ_b can be defined as

$$\theta_b=\frac{T_b-T_0}{T_1-T_2}=\frac{2}{(1-r^{*2})}\int_{r^*}^{1}RU(R)\theta(R)dR \tag{19}$$

By using Eq. (16) the convective heat exchange for inner/ outer cylinder is

$$h=\frac{-k\left.\frac{dT}{dr}\right|_{r=r_1}}{T_1-T_b}. \tag{20}$$

$$h=\frac{-k\left.\frac{dT}{dr}\right|_{r=r_2}}{T_2-T_b}. \tag{21}$$

Nu_1 at the outer/inner surfaces of inner/outer cylinder is

$$Nu_1=\frac{2(1-r^*)\left.\frac{d\theta}{dR}\right|_{R=r^*}}{\theta_b-w}. \tag{22}$$

$$Nu_2=\frac{2(1-r^*)\left.\frac{d\theta}{dR}\right|_{R=1}}{\theta_b+1-w}. \tag{23}$$

The velocity of the outer and inner surface

$$\left.\frac{dU}{dR}\right|_{R=r^*}=0 \quad \text{and} \quad \left.\frac{dU}{dR}\right|_{R=1}=0. \tag{24}$$

The Gr/Re for $+H$ from Eq. (24) for the outer/inner surface of the outer/inner cylinder is given as

$$\left.\frac{Gr}{\mathrm{Re}}\right|_{R=r^*}=\frac{x_{34}}{x_{33}} \quad \text{and} \quad \left.\frac{Gr}{\mathrm{Re}}\right|_{R=1}=\frac{x_{37}}{x_{36}}. \tag{25}$$

2.1.2. Case II: Heat Absorbing Fluid (sink)

The closed form solution for Eq. (10) with BCs given in Eqs. (13), (14) and (15) is .

$$\theta(R)=\frac{A_7 I_0(E_1R)-A_8K_0(E_1R)}{(x_{40}-x_{41})A_8-(x_{38}-x_{39})A_7}. \tag{26}$$

The $U(R)$ can be written as

$$U(R)=E_3\left(\phi_1\frac{Gr}{\mathrm{Re}}\left(\frac{A_7I_0(E_1R)-A_8K_0(E_1R)}{(x_{40}-x_{41})A_8-(x_{38}-x_{39})A_7}\right)-\frac{dP}{dZ}\right)+\mathrm{C}_3\,\mathrm{I}_0(E_2R)+C_4K_0(E_2R). \tag{27}$$

At last Gr/Re for the case of sink from Eq. (24) is as under

$$\left.\frac{Gr}{\mathrm{Re}}\right|_{R=r^*}=\frac{x_{63}}{x_{62}} \quad \text{and} \quad \left.\frac{Gr}{\mathrm{Re}}\right|_{R=1}=\frac{x_{70}}{x_{69}}. \tag{28}$$

3. Results and Discussion

The results for $U(R)$, $\theta(R)$, Nu are discussed and the graphs are shown in Figures 2 - 11 so that the effect of each parameter can be seen.

From Figure 2 we examined the impact of ϕ on $\theta(R)$ for the case of $+H$ and $-H$. In Figure 2 for case $+H$ and $-H$ the $\theta(R)$ enhances with enhance in ϕ. Figure 3 shows the influence of $+H$ and $-H$ on θ. From the Figure, we can see that $\theta(R)$ increases by increasing $+H$ and decreases in the case of $-H$. Figure 4 shows the effect of ϕ on $U(R)$. We see from the fig. that $U(R)$ increases for ϕ in both cases $+H$ and $-H$. Figure 5 is graphed to show the influence of Kn on $U(R)$. From the Figures, we see that velocity enhances with increase in Kn for both cases $+H$ and $-H$. It is noted that the u_{s_1,s_2} on surface of the cylinders increases for both $+H$ and $-H$ case, as the amount of Kn rises, the retarding impact of the cylinder reduces. Figure 6 shows the effect of M on $U(R)$.It can be noted that the velocity enhances with increase in M for both cases of $+H$ and $-H$. The variation in Gr/Re is shown in Figure 7. we can see that the $U(R)$ increases with increase in Gr/Re for both cases $+H$ and $-H$. It is noted that for $\pm H$, enhancing Gr/Re leads to an increase in $U(R)$ on the outer surface of the inner cylinder where the result is simply increased on the outer cylinder’s inner surface.

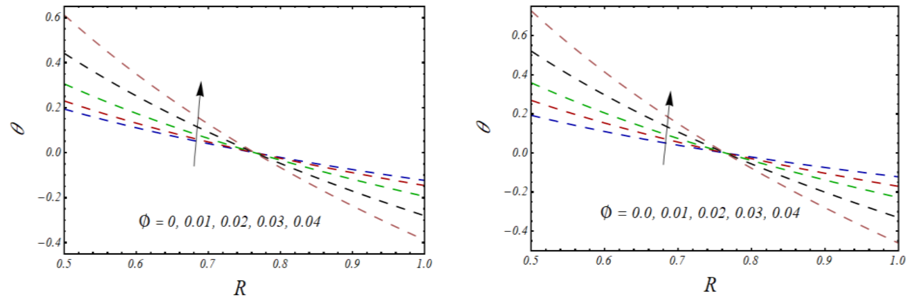
Figure 8 shows the effect of H on Nu_1.we can see that the Nu_1 increases with increase in H for both cases of (a) and (b). Figure 9 shows the impact of H on Nu_2. We can see that the Nu_2 increases with increase in H for both cases of (a) and (b). The variation in ϕ is shown in Figure. 10 for the case of Nu_1. The figure shows that the Nu_1 decreases for both cases in ϕ. The variation in ϕ is shown in Figure 11 for the case of Nu_2. The Figure shows that Nu_2 increases with increase in ϕ for both cases.

4. Conclusions

This paper presented the analysis of heat exchange for ferrofluid in a micro annulus. The impact of the M ,ϕ, H, Gr/Re, Kn, r^* on the $U(R)$, $\theta(R)$ and Nu are expressed as:

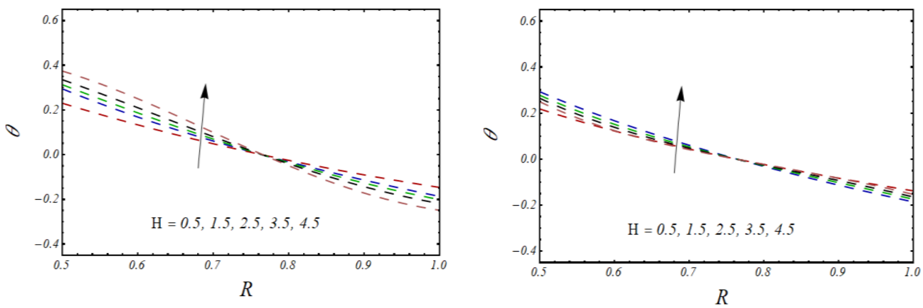
- $\theta(R)$ increases when ϕ increases for heat generating/absorbing fluid.
- $\theta(R)$ increases when H increases.
- $U(R)$ increases when ϕ increases for heat generating/absorbing fluid.
- $U(R)$ increases when Kn increases for heat generating/absorbing fluid.
- $U(R)$ increases when M increases for heat generating/absorbing fluid.
- $U(R)$ increases when Gr/Re increases for heat generating/absorbing fluid.
- Nu for inner cylinder decreases when ϕ increases for heat generating/absorbing fluid.

• Nu increases when ϕ increases for both cases heat generating/absorbing fluid.



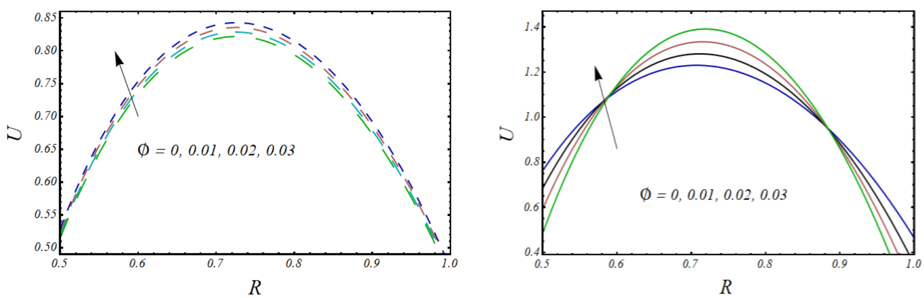
(a) (b)

Figure 2. (a) θ (b) θ for ϕ.



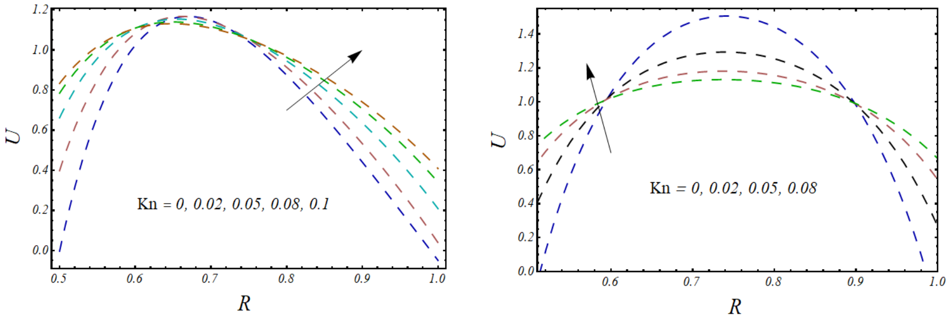
(a) (b)

Figure 3. (a) θ (b) θ for H.



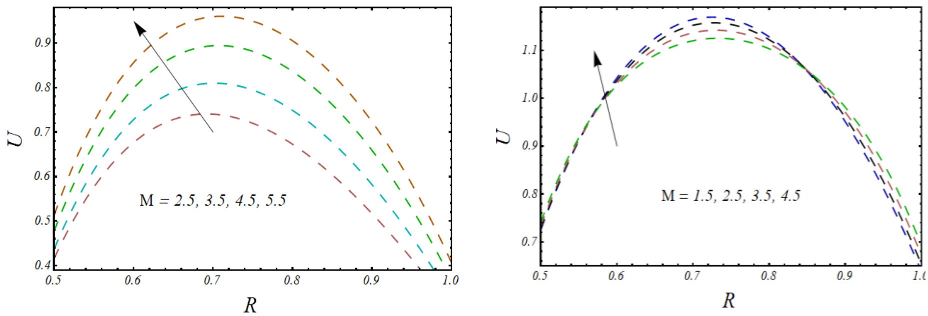
(a) (b)

Figure 4. (a) U (b) U for ϕ.



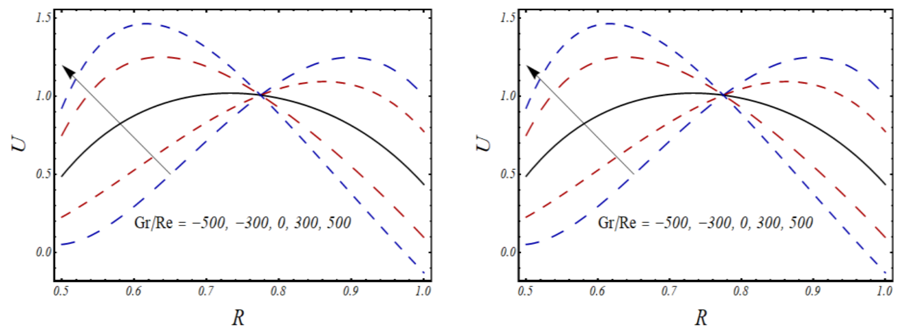
(a) (b)

Figure 5. (a) U (b) U for Kn.



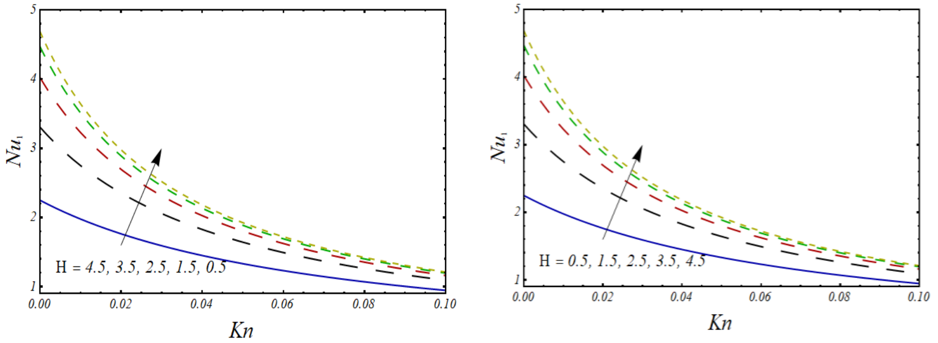
(a) (b)

Figure 6. (a) U (b) U for M.



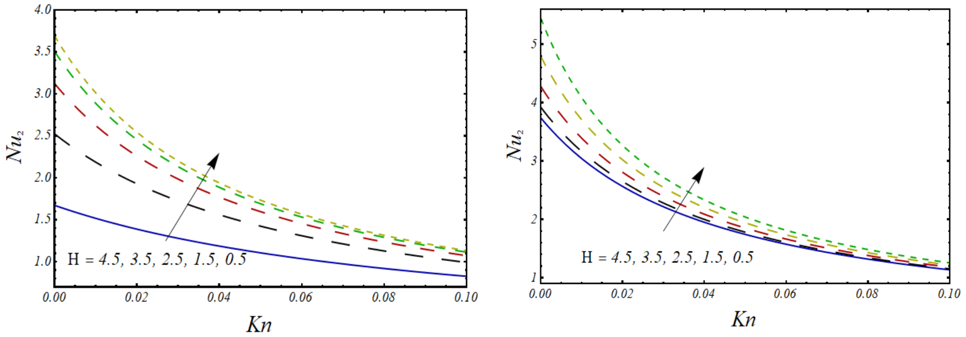
(a) (b)

Figure 7. (a) U (b) U for Gr/Re.



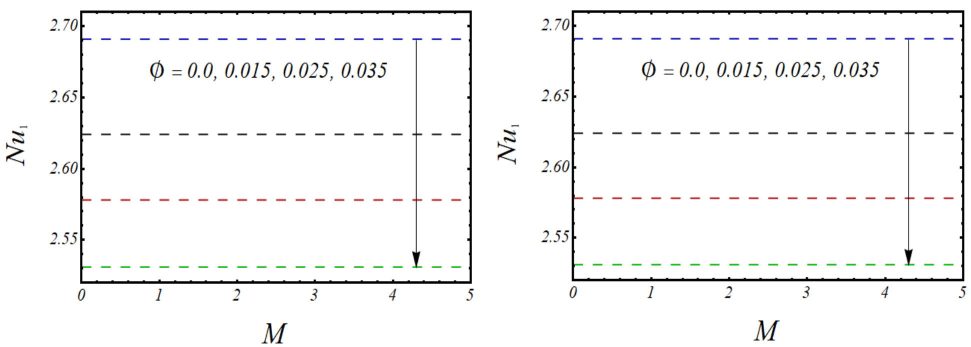
(a) (b)

Figure 8. (a) Nu_1 (b) Nu_1 for H.



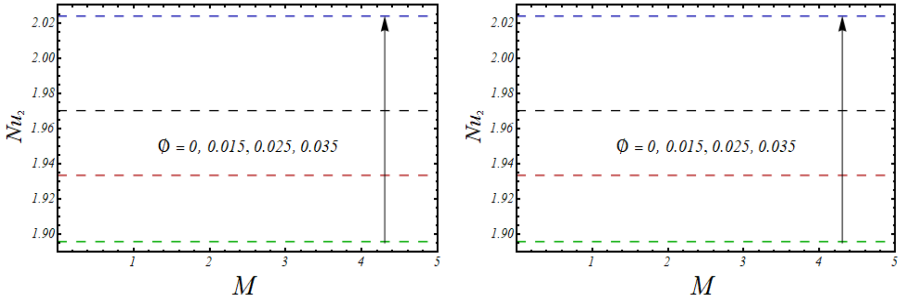
(a) (b)

Figure 9. (a) Nu_2 (b) Nu_2 for H.



(a) (b)

Figure 10. (a) Nu_1 (b) Nu_1 for ϕ.



(a) (b)

Figure 11. (a) Nu_2 (b) Nu_2 for ϕ.

Competing Interests: The authors declare no conflict of interest.

References

[1] Shail R. On laminar two-phase flows in magnetohydrodynamics. *International Journal of Engineering Science*. 1973;11(10):1103-8.

[2] Malashetty MS, Umavathi JC. Two-phase magnetohydrodynamic flow and heat transfer in an inclined channel. *International Journal of Multiphase Flow*. 1997;23(3):545-60.

[3] Abbas Z, Hasnain J, Sajid M. Hydromagnetic mixed convective two-phase flow of couple stress and viscous fluids in an inclined channel. *Zeitschrift für Naturforschung A*. 2014;69(10-11):553-61.

[4] Hasnain J, Abbas Z, Sajid M. Effects of porosity and mixed convection on MHD two phase fluid flow in an inclined channel. *PloS one*. 2015;10(3):e0119913.

[5] Hasnain J, Abbas Z. Hydromagnetic convection flow in two immiscible fluids through a porous medium in an inclined annulus. *Journal of Porous Media*. 2017;20(11):977-987.

[6] Dey D, Baruah AJ. Free convective flow of two immiscible memory fluids in an inclined channel with energy dissipation. *Model Measur Control B*. 2018;87(2):63-7.

[7] Govindarajan A, Lakshmi Priya S. Effect of magnetic and heat generation on MHD convective flow of two viscous fluids. In*AIP*

Conference Proceedings 2019 (Vol. 2112, No. 1, p. 020108). AIP Publishing LLC.

[8] Choi SU, Eastman JA. *Enhancing thermal conductivity of fluids with nanoparticles. Argonne National Lab*.(ANL), Argonne, IL (United States); 1995 Oct 1.

[9] Choi SU, Zhang ZG, Yu W, Lockwood FE, Grulke EA. Anomalous thermal conductivity enhancement in nanotube suspensions. *Applied Physics Letters*. 2001;79(14):2252-4.

[10] Das S, Jana RN, Makinde OD. Mixed convective magnetohydrodynamic flow in a vertical channel filled with nanofluids. *Engineering Science and Technology, an International Journal*. 2015;18(2):244-55.

[11] Abbas Z, Hasnain J. Two-phase magnetoconvection flow of magnetite (Fe3O4) nanoparticles in a horizontal composite porous annulus. *Results in Physics*. 2017;7:574-80.

[12] Hasnain J, Abbas Z. Entropy generation analysis on two-phase micropolar nanofluids flow in an inclined channel with convective heat transfer. *Thermal Science*. 2019;23(3 Part B):1765-77.

[13] Salehpour A, Salehi S, Salehpour S, Ashjaee M. Thermal and hydrodynamic performances of MHD ferrofluid flow inside a porous channel. *Experimental Thermal and Fluid Science*. 2018;90:1-3.

[14] Khan I, Alqahtani AM. MHD nanofluids in a permeable channel with porosity. *Symmetry*. 2019;11(3):378.

[15] Gupta PS, Gupta AS. Radiation effect on hydromagnetic convection in a vertical channel. *International Journal of Heat and Mass Transfer*. 1974;17(12):1437-42.

[16] Sanyal DC, Samanta SK. Effect of radiation on hydromagnetic vertical channel flow. *Czechoslovak Journal of Physics B*. 1989;39(4):384-91.

[17] Hayat T, Awais M, Alsaedi A, Safdar A. On computations for thermal radiation in MHD channel flow with heat and mass transfer. *Plos one*. 2014;9(1):e86695.

[18] Srinivas J, Murthy JV, Bég OA. Entropy generation analysis of radiative heat transfer effects on channel flow of two immiscible couple stress fluids. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*. 2017;39(6):2191-202.

[19] Kumar BR, Basha HT, Sivaraj R, Sandeep N. Effect of thermal radiation on chemically reacting magnetohydrodynamic dusty viscous fluid flow in a porous channel. *InIOP Conference Series: Materials Science and Engineering 2017* (Vol. 263, No. 6, p. 062028). IOP Publishing.

[20] Tufail MN, Saleem M, Chaudhry QA. An oscillation effect on MHD radiative Casson fluid flows in an asymmetric channel through group theoretical analysis. *Canadian Journal of Physics*. 2020;98(1):81-8.

[21] Munawar S, Mehmood A, Ali A. Three-dimensional squeezing flow in a rotating channel of lower stretching porous wall. *Computers & Mathematics with Applications*. 2012;64(6):1575-86.

[22] Sri Ramachandra Murty P, Balaji Prakash G. MHD two-fluid flow and heat transfer between two inclined parallel plates in a rotating system. *International Scholarly Research Notices*. 2014;2014.

[23] Murty PS, Raju TL. MHD two-phase flow and heat transfer between two parallel porous walls in a rotating system. *Science Domain International*. 2014;4(13):1894-907.

[24] Murty PS, Prakash GB. Heat transfer aspects on rotating MHD two-phase convective flow through an inclined channel in the presence of electric field. *Physical Science International Journal*. 2014;4(9):1260.

[25] Chutia M, Nath T, Jyoti P. Numerical Solution of Unsteady Hydromagnetic Couette Flow in a Rotating System Bounded by Porous Plates with Hall Effects. *International Journal of Computer Applications.* 2017;171:1-6.

[26] Kandelousi MS. Effect of spatially variable magnetic field on ferrofluid flow and heat transfer considering constant heat flux boundary condition. *The European Physical Journal Plus*. 2014;129(11):1-2.

[27] Rosseland S. *Astrophysics and nuclear - theoretical foundations*. Springer-Verlag, Berlin, 1931:41-44.

[28] Ashorynejad HR, Sheikholeslami M, Pop I, Ganji DD. Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field. *Heat and Mass Transfer*. 2013;49(3):427-36.

[29] Abbas Z, Hasnain J, Sajid M. MHD two-phase fluid flow and heat transfer with partial slip in an inclined channel. *Thermal Science*. 2016;20(5):1435-46.

[30] Adesanya SO. Free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump. *Ain Shams Engineering Journal*. 2015;6(3):1045-52.

[31] Rao IJ, Rajagopal KR. The effect of the slip boundary condition on the flow of fluids in a channel. *Acta Mechanica*. 1999;135(3):113-26.

[32] Sanyal DC, Sanyal MK. Hydromagnetic slip flow with heat transfer in an inclined channel. *Czechoslovak Journal of Physics B*. 1989;39(5):529-36.

[33] Abbas Z, Rahim T, Hasnain J. Slip flow of magnetite-water nanomaterial in an inclined channel with thermal radiation. *International Journal of Mechanical Sciences*. 2017;122:288-96.

[34] Murty PS, Prakash GB. Magnetohydrodynamic two-fluid flow and heat transfer in an inclined channel containing porous and fluid layers in a rotating system. *Maejo International Journal of Science and Technology*. 2016;10(1):25.

[35] Ahmad B, Perviz A, Ahmad MO, Dayan F. Numerical Solution with Non-Polynomial Cubic Spline Technique of Order Four Homogeneous Parabolic Partial Differential Equations. Scientific Inquiry and Review. 2021 Dec 27;5(4):1-15. https://doi.org/10.32350/ sir.54.03

[36] Ahmad B, Perviz A, Ozair Ahmad M, Dayan F. Solution of Parabolic Partial Differential Equations Via Non-Polynomial Cubic Spline Technique. *Sci Inquiry Rev*. 2021;5(3):1-5. https://doi.org/ 10.32350/sir.53.05

Appendix

$$A=4\left(1-r^*\right)^2,\ \phi_1=(1-\phi)^{2.5}\left(1-\phi+\frac{\phi\rho_s}{\rho_f}\right),\ \phi_2=1-\phi+\frac{\phi\rho_s}{\rho_f},$$

$$\phi_3=1-\phi+\frac{(\rho c_p)_s}{(\rho c_p)_f},\ \phi_4=(1-\phi)^{2.5},\ A_1=\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}$$

$$A_3=\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)},\ A=4\left(1-r^*\right)^2,$$

$$\phi_1=(1-\phi)^{2.5}\left(1-\phi+\frac{\phi\rho_s}{\rho_f}\right),\ \phi_2=1-\phi+\frac{\phi\rho_s}{\rho_f},\ \ \phi_3=1-\phi+\frac{(\rho c_p)_s}{(\rho c_p)_f},$$

$$\phi_4=(1-\phi)^{2.5},\ A_1=\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}$$

$$A_3=\frac{\sigma_{nf}}{\sigma_f}=1+\frac{3\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}{\left(\frac{\sigma_s}{\sigma_f}+2\right)-\phi\left(\frac{\sigma_s}{\sigma_f}-1\right)}.$$

$$E_1^2=\frac{H^2}{A_1},\ E_2^2=\phi_4 A_3 M^2,\ C_1=\frac{1}{x_{19}}\left(E_3x_{18}\frac{dP}{dZ}+E_3E_4x_{17}\frac{Gr}{\mathrm{Re}}\right),$$

$$C_2=\frac{1}{x_{22}}\left(E_3x_{21}\frac{dP}{dZ}+E_3E_4x_{20}\frac{Gr}{\mathrm{Re}}\right),\ \frac{dP}{dZ}=\frac{Gr}{\mathrm{Re}}\frac{x_{26}}{x_{27}}-\frac{1}{E_3x_{27}},$$

$$A_5=J_1(E_1)-r^*J_1\left(E_1r^*\right),\ \ A_6=Y_1(E_1)-r^*Y_1\left(E_1r^*\right),$$

$$x_1=J_0\left(E_1r^*\right)+2A_2\beta_tk_n(1-r^*)E_1J_1\left(E_1r^*\right),$$

$$x_2=Y_0\left(E_1r^*\right)+2A_2\beta_tk_n(1-r^*)E_1Y_1\left(E_1r^*\right),$$

$$x_3=J_0(E_1)-2A_2\beta_tk_n(1-r^*)E_1J_1(E_1),$$

$$x_4=Y_0(E_1)-2A_2\beta_tk_n(1-r^*)E_1Y_1(E_1),$$

$$x_5 = J_1\left(E_1\right) - r^* J_1\left(E_1 r^*\right),\ x_6 = Y_1\left(E_1\right) - r^* Y_1\left(E_1 r^*\right),$$
$$x_7 = I_0\left(E_2 r^*\right) - 2\beta_v k_n (1-r^*) E_2 I_1\left(E_2 r^*\right),$$
$$x_8 = K_0\left(E_2 r^*\right) + 2\beta_v k_n (1-r^*) E_2 K_1\left(E_2 r^*\right),$$
$$x_9 = Y_0\left(E_1 r^*\right) + 2\beta_v k_n (1-r^*) E_1 Y_1\left(E_1 r^*\right),$$
$$x_{10} = J_0\left(E_1 r^*\right) + 2\beta_v k_n (1-r^*) E_1 J_1\left(E_1 r^*\right),$$
$$x_{11} = I_0\left(E_2\right) + 2\beta_v k_n (1-r^*) E_2 I_1\left(E_2\right),$$
$$x_{12} = K_0\left(E_2\right) - 2\beta_v k_n (1-r^*) E_2 K_1\left(E_2\right),$$
$$x_{13} = Y_0\left(E_1\right) - 2\beta_v k_n (1-r^*) E_1 Y_1\left(E_1\right),$$
$$x_{14} = J_0\left(E_1\right) - 2\beta_v k_n (1-r^*) E_1 J_1\left(E_1\right),\ x_{15} = x_5 x_9 - x_6 x_{10},\ x_{16} = x_5 x_{13} - x_6 x_{14},$$
$$x_{17} = x_8 x_6 - x_{12} x_{15},\ x_{18} = x_{12} - x_8,\ x_{19} = x_7 x_{12} - x_8 x_{11},\ x_{20} = x_7 x_{16} - x_{11} x_{15},$$
$$x_{21} = x_{11} - x_7,\ x_{22} = x_8 x_{11} - x_7 x_{12},\ x_{23} = I_1\left(E_2\right) - r^* I_1\left(E_2 r^*\right),$$
$$x_{24} = K_1\left(E_2\right) - r^* K_1\left(E_2 r^*\right),\ x_{25} = \frac{2}{E_2\left(1-r^{*2}\right)},$$
$$x_{26} = \frac{E_4 x_{25}\left(x_{17} x_{22} x_{23} - x_{19} x_{20} x_{24}\right)}{x_{19} x_{22}},\ x_{27} = \frac{x_{19} x_{22} - x_{25}\left(x_{18} x_{22} x_{23} - x_{19} x_{20} x_{24}\right)}{x_{19} x_{22}},$$
$$x_{28} = E_2\left(\frac{E_4 x_{17}}{x_{19}} + \frac{x_{18} x_{26}}{x_{19} x_{27}}\right),\ x_{29} = E_2\left(\frac{E_4 x_{20}}{x_{22}} + \frac{x_{21} x_{26}}{x_{22} x_{27}}\right)\ x_{30} = \frac{E_2 x_{18}}{E_3 x_{19} x_{27}},$$
$$x_{31} = \frac{E_2 x_{21}}{E_3 x_{22} x_{27}},$$
$$x_{32} = x_5 Y_1\left(E_1 r^*\right) - x_6 J_1\left(E_1 r^*\right),\ x_{33} = x_{28} I_1\left(E_2 r^*\right) - x_{29} K_1\left(E_2 r^*\right) - E_1 E_4 x_{32},$$
$$x_{34} = x_{30} I_1\left(E_2 r^*\right) - x_{31} K_1\left(E_2 r^*\right),\ x_{35} = x_5 Y_1\left(E_1\right) - x_6 J_1\left(E_1\right),$$
$$x_{36} = x_{28} I_1\left(E_2\right) - x_{29} K_1\left(E_2\right) - E_1 E_4 x_{35},\ x_{37} = x_{30} I_1\left(E_2\right) - x_{31} K_1\left(E_2\right),$$
$$E_3 = \frac{1}{A E_2^2},\ E_4 = \frac{\phi_1}{x},\ E_5 = \frac{\phi_1}{x_{44}},\ C_3 = \frac{1}{x_{49}}\left(E_3 x_{51} \frac{dP}{dZ} + E_3 E_5 x_{50} \frac{Gr}{\mathrm{Re}}\right),$$

$$C_4 = \frac{1}{x_{52}}\left(E_3 x_{54}\frac{dP}{dZ} + E_3 E_5 x_{53}\frac{Gr}{\mathrm{Re}}\right),\ \frac{dP}{dZ} = \frac{Gr}{\mathrm{Re}}\frac{x_{55}}{x_{56}} - \frac{1}{E_3 x_{56}},$$

$$A_7 = K_1\left(E_1\right) - r^* K_1\left(E_1 r^*\right),\ A_8 = I_1\left(E_1\right) - r^* I_1\left(E_1 r^*\right),$$

$$x_{38} = I_0\left(E_1 r^*\right) - 2A_2\beta_t k_n(1-r^*)E_1 I_1\left(E_1 r^*\right),$$

$$x_{39} = I_0\left(E_1\right) + 2A_2\beta_t k_n(1-r^*)E_1 I_1\left(E_1\right),$$

$$x_{40} = K_0\left(E_1 r^*\right) + 2A_2\beta_t k_n(1-r^*)E_1 K_1\left(E_1 r^*\right),$$

$$x_{41} = K_0\left(E_1\right) - 2A_2\beta_t k_n(1-r^*)E_1 K_1\left(E_1\right),\ x_{42} = I_1\left(E_1\right) - r^* I_1\left(E_1 r^*\right),$$

$$x_{43} = K_1\left(E_1\right) - r^* K_1\left(E_1 r^*\right),\ x_{45} = I_0\left(E_1 r^*\right) - 2\beta_v k_n(1-r^*)E_1 I_1\left(E_2 r^*\right),$$

$$x_{46} = K_0\left(E_1 r^*\right) + 2\beta_v k_n(1-r^*)E_1 K_1\left(E_1 r^*\right),$$

$$x_{47} = I_0\left(E_1\right) + 2\beta_v k_n(1-r^*)E_1 I_1\left(E_1\right),$$

$$x_{48} = K_0\left(E_1\right) - 2\beta_v k_n(1-r^*)E_1 K_1\left(E_1\right),\quad x_{49} = x_7 x_{12} - x_8 x_{11},$$

$$x_{50} = x_8\left(x_{43}x_{47} + x_{42}x_{48}\right) - x_{12}\left(x_{43}x_{45} - x_{42}x_{46}\right),\ x_{51} = x_{12} - x_8,$$

$$x_{52} = x_8 x_{11} - x_7 x_{12},\ x_{53} = x_7\left(x_{43}x_{47} + x_{42}x_{48}\right) - x_{11}\left(x_{43}x_{45} - x_{42}x_{46}\right),$$

$$x_{54} = x_{11} - x_7,\ x_{55} = \frac{E_5 x_{25}\left(x_{23}x_{50}x_{52} - x_{24}x_{49}x_{53}\right)}{x_{49}x_{52}},$$

$$x_{56} = 1 - \frac{x_{25}\left(x_{23}x_{51}x_{52} - x_{24}x_{49}x_{54}\right)}{x_{49}x_{52}},\ x_{57} = E_2\left(\frac{E_5 x_{50}}{x_{49}} + \frac{x_{51}x_{55}}{x_{49}x_{56}}\right),$$

$$x_{58} = E_2\left(\frac{E_5 x_{53}}{x_{52}} + \frac{x_{54}x_{55}}{x_{52}x_{56}}\right),\ x_{59} = \frac{E_2 x_{51}}{E_3 x_{49}x_{56}},\ x_{60} = \frac{E_2 x_{54}}{E_3 x_{52}x_{56}},$$

$$x_{61} = x_{43}I_1\left(E_1 r^*\right) - x_{42}K_1\left(E_1 r^*\right),\ x_{62} = x_{57}I_1\left(E_2 r^*\right) - x_{58}K_1\left(E_2 r^*\right) - E_1 E_5 x_{61},$$

$$x_{63} = x_{59}I_1\left(E_2 r^*\right) - x_{60}K_1\left(E_2 r^*\right),\ x_{64} = x_{43}I_1\left(E_1\right) - x_{42}K_1\left(E_1\right),$$

$$x_{65} = \frac{x_{51}}{E_3 x_{49}x_{56}},\ x_{66} = \frac{x_{54}}{E_3 x_{52}x_{56}},\ x_{67} = x_{57}I_1\left(E_2\right) - x_{58}K_1\left(E_2\right) + E_1 E_5 x_{64},$$

$$x_{68} = x_{65}I_1\left(E_2\right) - x_{66}K_1\left(E_2\right).$$

Nomenclature

b	Body force	U	Dimensionless axial velocity
C_p	Specific heat at constant pressure	V	Velocity field
D_h	Hydraulic diameter	w	Ratio of wall temperature
F_v	Tangential momentum accommodation coefficient	z, r	Axial and radial coordinate, respectively
F_t	Thermal accommodation coefficient	Z, R	Dimensionless axial and radial coordinate,
g	Gravitational acceleration		**Greeks letters**
Gr	Grashof number	τ	Cauchy stress tensor
Gr/Re	Mixed convection parameter	α	Thermal diffusivity of the fluid
h	Convective heat transfer coeffcient	β	Coefficient of thermal expansion
$\pm H$	Dimensionless heat generation or absorption parameter	β_v, β_t	Dimensionless variables
k	Thermal conductivity of the fluids	γ	Ratio of specific heat
Kn	Knudsen number	λ	Molecular mean free path
Nu_1	Rate of heat transfer at the outer surface of the inner cylinder	μ	Dynamic viscosities of the phases
Nu_2	Rate of heat transfer at the inner surface of the outer cylinder	ρ	Density
P	Pressure difference	ν	Fluid kinematic viscosity fluid kinematic viscosity
Pr	Prandtl number	θ	Dimensionless temperature
Q_0	Dimensional heat generation parameter	θ_b	Bulk temperature
Re	Reynolds number	ϕ	Dimensionless concentration
r^*	Ration of radiuses	σ	Electrical conductivity
r_1	Radius of the inner cylinder	μ_{nf}	Effective dynamic viscosity
r_2	Radius of the outer cylinder	$(\rho C_p)_{nf}$	Heat capacitance
s_1	Fluid properties on the inner cylinder	k_{nf}	Thermal conductivity
s_2	Fluid properties on the outer cylinder	ρ_{nf}	Effective density
T	Temperature of the fluid	α_{nf}	Thermal diffusivity
T_0	Reference temperature of the fluid	μ_f	Dynamic viscosity

T_1	Temperature at outer surface of the inner cylinder	ρ_f	Density for ferrofluid
T_2	Temperature at inner surface of the outer cylinder	ρ_s	Density for base fluid
T_w	Wall temperature	k_s	Thermal conductivity of solid particle
u_0	Mean velocity	k_f	Thermal conductivity of fluid
u	Axial velocity		