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# Double Moving Average Control Chart for Autocorrelated Data

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## ABSTRACT

*The assumption of normality and independence is necessary for statistical inference of control charts. Misleading results could be obtained if the traditional control chart technique is applied to the autocorrelated data. A time series model is employed to produce optimum output when data is correlated. The objective of this current research is to create a new control chart methodology which takes the autocorrelation data observations into account. Charts of moving average, exponentially weighted, and cumulative sum perform better for the autocorrelation of data for small and moderate changes. The proposed methodology is highly skilled and receptive to minor, moderate, and major changes in the process. The proposed DMA chart increases the efficiency of the average run length (ARL) chart for moving average (MA) to detect the small to medium magnitude shifts in the mean. The simulation also demonstrated that the DMA chart with spans of w=10 and 15 generally performs better in terms of average run length (ARL) as compared to classical MA. This research might be extended to a multivariate autocorrelated statistical process control but it could also be used to recognize and categorize seven categories of traditional control chart patterns, such as downward, upward shift, normal trend, cyclic, systematic patterns, increasing and decreasing trend. In order to identify and categorise a set of subclasses of abnormal patterns, this model (multivariate autocorrelated statistical process control chart) should employ a multilayer feed-forward Artificial Neural Network (ANN) architecture controlled by a back-propagation learning rule.*

***Keywords:*** ARL, AR (1), independent identical distribution, serial correlation, time series model

## INTRODUCTION

The EWMA, CUSUM, and MA memory control charts are preferred to the Shewhart chart for detecting minor process changes because they used information for the full sequence of occurrences. Unlike the Shewhart chart, which simply used facts to provide the final samples. Memory control charts are more sensitive to minor process fluctuations in the mean, such as those of magnitude $1.5\sigma$ or less, as compared to the Shewhart chart [1]. Common causes are usual or predictable whereas assignable causes are odd or unpredictable in the variation system. The process with common causes could be described by a probability distribution. Time series control charts are used as an alternative approach for the independence of assumption if it gets violated. A concise overview of autocorrelated data control chart is presented below [2].

In SPC, the quality characteristics are frequently believed to be distributed normally. It is used to evaluate the control limits for this normal distribution [3]. The normally distributed statistics control limits commonly which cover 99.73% of all statistics, indicates that the control limits are at $\pm 3\sigma$ distance from the mean [4]. In control charts the control chart signals or a warning is announced when the tracking points are either above or below the limit of control [5].

The basis of CUSUM chart is the difference of cumulative sum between the observed and average values. The goal of this effective approach is to minimise the uncertainty and isolate sources of difficulties in the production process systematically. On the other hand, the MA and EWMA charts were used to separate weighted average methodologies. Several EWMA and CUSUM chart extensions have been created to date [6].

Planning and data collecting are the first steps in the implementation of the effective statistical process control. The PDSA cycle defined by Walter Shewart is the solution of any process improvement program [7]. To identify this issue and the potential cause for it, the quality control tools have a manual structure, in which problems and potential causes are found. Hence, formulated change to correct and improve the situation is required for the smooth process. To observe the effect of this situation on the recurring re changes, control charts have been employed to see the effcets of the adjusted and illustrated process.

If the product quality is good, it controls the change and continue to improve further. If the result is not satisfactory to this point, the future reseracher can look for other ways of improving the process or finding various reasons of the underlying problem. Statistical Process Control entails a number of processes, one of which is the use of a control chart as described in the **Figure 1** below.

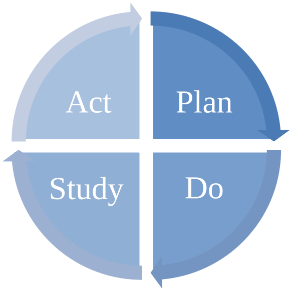


**Figure1.** PDSA cycle

An artificial neural network could learn control chart patterns from noisy data and recall them during pattern recognition operations in the real world. SPC tasks that were previously handled by quality engineers must be automated [8]. As a result, SPC has used a variety of artificial intelligence approaches. In real-time, neural networks (NNs) have exceptional noise tolerance and don't require assumptions concerning the statistical distribution of measured data [9]. Many guidelines for using a control chart to spot aberrant patterns within the control limits have been created. The recommendation, which used for various degrees include: geometric moving averages tests, zone tests, and runs tests, might imply the presence of an abnormal pattern but does not specify the existence of a specific pattern [10].

Usually, two basic assumptions for designing control charts have been made. Firstly, the distribution function behind the observation of qualitative interest findings is expected to be normal. Secondly, the assumption for the process data is believed to be distributed independently. In reality, one or both beliefs are consistently violated [11].

The SPC chart is designed to determine a specific value of $ARL_o$ for the process of in-control duration is the expected number of in-control observations until an out-of-control alarm (false) is triggered. The chart with smallest out-of-control average run length $ARL_1$ would be preferred. If the process that is monitored follows normal distribution with independent and identical variables, then control limits are determined according to the Shewhart and CUSUM tabular charts [12]. If we suppose that the process is normally distributed with $\mu$ as an average having standard deviation σ, both are identified then the control restrictions with a center line used in X-chart for independent observations are as follows in equation 1:

$$\begin{cases} \mathrm{UCL} = \mu + Z\alpha/_{2} \\ \mathrm{CL} = \mu \\ \mathrm{LCL} = \mu - Z\alpha/_{2} \end{cases} \tag{1}$$

Memory control charts such as the EWMA and MA perform better over the Shewhart control chart in detecting small process shifts. There have been several attempts to improve the detection speed of the EWMA chart by suggesting the DEWMA chart; however, no attempt has been made to improve the speed of an MA chart for the out-of-control state so far [13]. The new control chart technique is the proposed named of DMA which is compared to traditional MA control chart. The proposed DMA chart and MA chart performed well for detecting a small shift in the process, the proposed technique worked better to cover the qualities of the existing control chart.

The goal of this study is to suggest that the proposed DMA control chart is a better alternative for the MA chart for the autocorrelated data set. The proposed DMA chart is faster in detecting the out-of-control signals, precisely those having a smaller shift in the process, which is shown in the simulation results.

The article is divided into further subsections, section 1 describes a brief explanation of some important concepts of time series with discussion on the Shewhart control chart. In section 2, a review of the different types of memory control charts is discussed. While in section 3, we presented our proposed DMA chart with a brief discusssion of the results and than compare it with the clasical MA chart. Section 4 covers the conclusion and summarizes the findings of the current research drawn from the proposed study.

## 2. RELATED WORK

Many traditional univariate process control strategies rely on the concept of time-independent observations. However, this assumption is violated if the variables in the process are correlated with time because the autocorrelation effect the rate of detection and false alarm. Many authors have looked into this issue to further investigate and provide the relevant solutions [14-17]. Two general monitoring procedures were suggested to deal with this problem. The first method was the Shewhart, EWMA (exponentially-weighted moving average), and CUSUM (cumulative sum control) which involves fitting a time series model to the data and then applying classic control charts to the residuals from the time series model. The second method is used for the classical control chart to monitor autocorrelation using adjusted control limits.

Alwan and Roberts (1988) demonstrated that if the correct time series model is using the residuals from the time series model (ARIMA), the auto-correlated process could be made independent and identically distributed with mean 0 and variance $\sigma^2$ as Harris and Ross (1991) fitted a time series model to the univariate observations. In assuming normal iid process, Roberts (1959) compared the ARL properties of control-chart tests based on those averages with the test of ordinary moving averages and the standard control-chart test. The EWMA offers greater sensitivity for transition for relatively limited transformations. For the calculation of ARL's of EWMA chart Robinson and Ho (1978) have shown a numerical procedure [18]. According to [19] there should be a simple method for obtaining the best ARL performance for a variety of shifted values and autocorrelation levels. They conducted Monte Carlo simulations to assess an autoregressive process to monitor the average run length performance.

If this is the case, the process is considered in control, and the disparity between measurements is considered to be a reason caused by the intrinsic natural random fluctuation. However, when the discrepancy goes beyond the limits or has a number of points that are not normal, the procedure is considered out of control. The effects of such a method could be calculated, if a method is under control. There is no way to determine whether the outcomes reached the out-of-control phase or not. If the data is not distributed, and does not form a bell curve, it means the process might still be out-of-control and cannot give the expected values. Here we have to explore ways to monitor the process [20].

A moving average (MA) chart is the most effective way to detect minor changes [21]. It might also be used when the product characteristics are automatically determined or when the time to manufacture a unit is longer than the usual time [22]. The exponentially weighted moving average (EWMA) control charts might be helpful for the autocorrelated data as discussed that EWMA can also detect shifts [23]. For low and moderate changes EWMA chart performs better than the Shewhart chart. Schmid (1995, 1997a, 1997b) showed that if there is a large shift in the process [17]. The adjusted control limits for auto- correlated data [24] suggested that for the univariate case, using $\bar{X}$ chart depends on residuals which don't have similar properties as the $\bar{X}$ chart for an independent process. Autocorrelation data degrade the performance of control charts significantly [25].

The existence of several cause of changes are evidenced not only by a variation in the stochastic system behavior but also by the presence of specialised fluctuation. Many publications have explored the impact of autocorrelation on SPC chart performance. When the process autoregressive (AR) or moving average process MA model is used, Bagshawr and Johnson calculated an approximate run length distribution for the cumulative sum control chart. They suggested that to use Conventional CUSUM schemes for the better conclusions. The effect of autocorrelation on CUSUM & EWMA's efficiencies have been addressed in literature, furthermore, in the presence of autocorrelation, these charts were quite sensitive. Autocorrelated observations were characterized by ARIMA models and residuals based on ARIMA model forecast values have been monitored via unique control charts[26].

## 3. THE PROPOSED DMA CHART

The occurrence of autocorrelation during the manufacturing process was regular. In order to ensure efficient and relevant performance, the serial correlation should be included in the process. Even though the process is in control, the control charts often indicate false alarms because the quality characteristic are being monitored by neglecting the serial correlation. Therefore, the proposed chart provides the approach for tracking the quality trait of autocorrelation.

Here, we first evaluated the mean & variance for the proposed DMA chart. The first-order autoregressive model, AR (1) is given as in equation 2.

$$y_t = \alpha + \beta y_{t-1} + \epsilon_t \tag{2}$$

Taking the expectations of equation 2 on both sides

$$E(y_t) = E(\alpha + \beta y_{t-1} + \epsilon_t \ = \alpha + \beta E(y_{t-1}) + 0 = E(y_{t-1}) = \cdots = \mu, \tag{3}$$

So, the equation 3 becomes as:

$$\mu = \alpha + \beta\mu = \mu(1-\beta) = \alpha = \mu = \frac{\alpha}{1-\beta} \tag{4}$$

This is the mean of AR (1) model, and the variance of AR (1) could be obtained from the following equation 5.

$$Var(y_t) = Var(\alpha + \beta y_{t-1} + \epsilon_t) = \alpha + \beta^2 Var(y_{t-1}) + \sigma^2 \tag{5}$$

For stationary process the variance of AR (1) will become as shown in equation 6.

$$Var(y_t) = Var(y_{t-1}) = \cdots = \sigma^2 y_t \tag{6}$$

After putting value of equation 5 into equation 6, variance becomes as in equation 7 for AR (1) model.

$$\sigma^2 y_t = \beta^2 \sigma^2 y_t + \sigma^2 = \sigma^2 y_t (1-\beta^2) = \sigma^2 = \sigma^2 y_t = \frac{\sigma^2}{1-\beta^2} \tag{7}$$

In AR model the intercept is equal to α and for the simplicity we have put α=0 in simulations.

Now for the quality characteristic "$\boldsymbol{y_t}$" at span “w” and time “t” calculated for subgroups averaging $Y_t$, $Y_{t-1}$…, the MA statistic is as equation 8.

$$MA_t = \frac{Yt + Yt-1 + Yt-2 + \cdots + Yt-w+1}{w} \tag{8}$$

For specific in-control process, (over t ≥ w) the mean and variance of $MA_t$ will be:

$$E(MA_t) = \frac{E(Yt) + E(Yt-1) + E(Yt-2) + \cdots + E(Yt-w+1)}{w}$$

$$= \frac{\mu + \mu + \cdots + \mu}{w}$$

$$= \frac{w\mu}{w}$$

Finally the mean of $MA_t$ is ,

$$E(MA_t) = \mu \tag{9}$$

For variance, we have

$$Var(MA_t) = \frac{1}{w^2}[Var(y_t) + Var(y_{t-1}) + \cdots + Var(y_{t-w+1})]$$

$$= \frac{1}{w^2}[\sigma^2(y_t) + \sigma^2(y_t) + \cdots + \sigma^2(y_t)]$$

$$= \frac{w\sigma^2(y_t)}{w^2}$$

$$Var(MA_t) = \frac{\sigma^2_{yt}}{w} \quad (10)$$

Putting value of equation 7 into equation 10, the $\boldsymbol{\sigma^2_{yt}}$value we get,

$$\sigma^2 MA = \frac{\sigma^2}{w(1-\beta^2)} \quad (11)$$

DMA statistics is based on the calculation of the MA Chart subgroup twice of the averages. T across all the subgroup averages up to the interval "t," the MA statistic of span "w" at time "t" for the series of subgroup average is already calculated in equation 8. The DMA statistics is calculated as follows:

$$DMA = \frac{MA_t + MA_{t-1} + MA_{t-2} + \cdots + MA_{t-w+1}}{w} \quad (12)$$

Taking the expectation of equation 12 in order to calculate the mean.

$$E(DMA) = \frac{1}{w}E(MA_t + MA_{t-1} + MA_{t-2} + \cdots + MA_{t-w+1})$$

$$= \frac{1}{w}[E(MA_t) + E(MA_{t-1}) + E(MA_{t-2}) + \cdots + E(MA_{t-w+1})]$$

$$= \frac{1}{w}[\mu + \mu + \cdots + \mu]$$

$$= \frac{1}{w}(w) \quad (13)$$

Now the variance of DMA will be computed as below:

$$Var(DMA) = \frac{1}{w^2}[Var(MA_t) + Var(MA_{t-1}) + Var(MA_{t-2}) + \cdots + Var(MA_{t-w+1})]$$

$$= \frac{1}{w^2}[\sigma_{MA}^2 + \sigma_{MA}^2 + \cdots + \sigma_{MA}^2]$$

$$= \frac{1}{w^2} w\sigma_{MA}^2$$

So, $\sigma_{DMA}^2 = \frac{\sigma_{MA}^2}{w}$ (14)

Now put the equation 14 into equation 11 to calculate the value of Var(DMA) as follows:

$$\sigma_{DMA}^2 = \frac{1}{w} \cdot \frac{\sigma^2}{w} = \frac{\sigma^2}{w^2(1-\beta^2)} \tag{15}$$

and the standard deviation of DMA will be:

$$\sigma_{DMA} = \frac{\sigma}{w} \cdot \frac{1}{\sqrt{(1-\beta^2)}} \tag{16}$$

Now for double moving average chart based on the targeted value ($\boldsymbol{\mu_o}$ ) and the standard deviation of the statistic, the upper control, control, and lower control limits are as follows:

$$\begin{cases} UCL = \mu_o + K\sigma_{DMA} \\ \quad CL = \mu_o \\ LCL = \mu_o - K\sigma_{DMA} \end{cases} \tag{17}$$

After substituting the values of "$\boldsymbol{\sigma_{MA}}$" the control limits become

$$\begin{cases} UCL = \mu_o + K\frac{\sigma}{w\sqrt{1-\beta^2}} \\ \quad CL = \mu_o \\ LCL = \mu_o + K\frac{\sigma}{w\sqrt{1-\beta^2}} \end{cases} \tag{18}$$

## 4. RESULTS AND DISCUSSION

Here $\boldsymbol{\mu_o}$is the central value and $\boldsymbol{\sigma}$ is the standard deviation of DMA in AR(1) time series, “K” the parameter is the width in order to set the DMA control limit and |φ| the coefficient is < 1 at AR(1), in autoregressive model. The method should be tested whether the values of σ is within the control boundaries. On the other hand, if the mean value is moved from its goal (target value $\boldsymbol{\mu_o}$), the system goes out of control as it plots the observation outside the set bounds. In order to assess the DMA chart results, a simulation test is conducted by R Studio. In order to recognise minor changes in the process, the suggested DMA chart is ideal for the auto-correlated process. We compared the performance of the proposed DMA with the MA chart, and the "K" (control width) and "W" (span) specifications are modified according to the corresponding values of φ in order to get regulated with ARL close to 370.

**Table 1** showed that the proposed DMA detected the shifts from 0.25 to 7.00 with the corresponding standard of control width and the span more efficiently than the MA chart at φ=0.2. k=4.868, 6.002, 7.752, 8.89 for DMA and k=3.31, 3.341, 3.24, 3.1329 for MA under the span w=3, 5, 10, 15, respectively. When δ=0.25, the proposed DMA detects shift 206.870 is more efficiently than MA, which is 228.190 at span w=3 and moving over at δ=3.00,4.00,5.00,7.00 the proposed DMA chart turns to numeral 1 and the MA chart near to numeral 1 at δ= 3.00, 4.00. The DMA chart is better and more efficient as compared to the MA.

**Table 2** shows the evaluation between the proposed DMA and MA chart at φ =0.5 for detecting minor changes in the process at various levels of shifts δ=0.25, 0.50, 0.75, 1.00, 2.00, 3.00, 4.00, 5.00, 7.00. Instead of the usual ±3σ, we computed the limits with a width of ±kσ. Using simulations we obtained k= 5.98, 7.86, 10.66, 12.39 for proposed DMA and k=3.886, 4.14, 4.23, 4.164 for MA chart under the span w=3, 5, 10, 15. In the results, we observed that the proposed DMA chart performs better than classical MA in the detection of small shifts.

Hence, **Table 3** shows the comparison between proposed DMA and MA chart at φ =0.75. Under the span w=3, 5, 10, 15, the control width for the proposed DMA is k=6.852, 9.8, 14.712, 17.85 and that of MA chart is k=4.24, 4.866, 5.456, 5.617 which are adjusted to get the ARL at 370 when δ=0.00, when the mean of the in control process is shifted to some another value then span and control width are tune together to detect this shift faster and in an effective way. In comparison, the proposed DMA chart was significantly better at detecting out-of-control signals, especially when it required little alterations, the table demonstrates unambiguous identification performance as compared to the MA chart. In addition, the simulation results showed that the proposed DMA chart outperforms the MA chart.

**Table1:** ARL Comparison with Proposed DMA and MA at φ=0.2

| | **DMA $\varphi$=0.2** | | | | **MA $\varphi$=0.2** | | | |
|---|---|---|---|---|---|---|---|---|
| | **W=3** | **W=5** | **W=10** | **W=15** | **W=3** | **W=5** | **W=10** | **W=15** |
| $\delta\downarrow$ | **K=4.868** | **K=6.002** | **K=7.752** | **K=8.89** | **K=3.31** | **K=3.341** | **K=3.24** | **K=3.1329** |
| 0.0 | 370.196 | 370.275 | 370.217 | 370.513 | 370.832 | 370.078 | 370.0826 | 370.2917 |
| 0.25 | 206.870 | 176.336 | 129.179 | 104.620 | 228.190 | 196.108 | 152.5139 | 124.2773 |
| 0.50 | 79.8649 | 58.0857 | 33.3176 | 21.9764 | 94.9705 | 70.9453 | 44.5824 | 32.3492 |
| 0.75 | 33.7963 | 22.3659 | 10.2346 | 5.05415 | 42.4915 | 29.3773 | 16.11415 | 10.09735 |
| 1.00 | 16.2134 | 9.63505 | 3.2598 | 1.4893 | 20.897 | 13.8388 | 6.34725 | 3.2612 |
| 1.50 | 4.84515 | 2.3805 | 1.0526 | 1.0013 | 6.5332 | 3.857 | 1.4716 | 1.05615 |
| 2.00 | 2.0561 | 1.1479 | 1 | 1 | 2.8165 | 1.60025 | 1.01525 | 1.00015 |
| 3.00 | 1.0002 | 1.0001 | 1 | 1 | 1.0038 | 1.00615 | 1 | 1 |
| 4.00 | 1 | 1 | 1 | 1 | 1 | 1.00005 | 1 | 1 |
| 5.00 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 7.00 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 2:** ARL Comparison with Proposed DMA and MA at φ=0.5

| | DMA $\varphi$=0.5 | | | | MA $\varphi$=0.5 | | | |
|---|---|---|---|---|---|---|---|---|
| | **W=3** | **W=5** | **W=10** | **W=15** | **W=3** | **W=5** | **W=10** | **W=15** |
| $\delta\downarrow$ | **K=5.98** | **K=7.86** | **K=10.66** | **K=12.39** | **K=3.886** | **K=4.14** | **K=4.23** | **K=4.164** |
| 0.0 | 370.972 | 370.016 | 370.742 | 370.335 | 370.410 | 370.352 | 370.4981 | 370.3387 |
| 0.25 | 259.469 | 239.594 | 195.410 | 169.1563 | 269.782 | 253.212 | 216.3517 | 189.2367 |
| 0.50 | 124.799 | 102.117 | 70.554 | 53.0786 | 135.55 | 116.242 | 85.93545 | 67.7561 |
| 0.75 | 61.0349 | 46.9004 | 27.907 | 17.9511 | 69.1114 | 55.7500 | 36.63385 | 26.486 |
| 1.00 | 31.8028 | 23.1137 | 11.753 | 6.329 | 36.6915 | 28.382 | 17.10895 | 11.1095 |
| 1.50 | 10.7245 | 6.79165 | 2.3823 | 1.2724 | 12.6631 | 9.28285 | 4.372 | 2.27.115 |
| 2.00 | 4.2906 | 2.3917 | 1.0926 | 1.006 | 5.3819 | 3.50755 | 1.509 | 1.06865 |
| 3.00 | 1.3095 | 1.02955 | 1 | 1 | 1.54645 | 1.14985 | 1.00115 | 1 |
| 4.00 | 1.01295 | 1 | 1 | 1 | 1.03845 | 1.00235 | 1 | 1 |
| 5.00 | 1.00025 | 1 | 1 | 1 | 1.0085 | 1 | 1 | 1 |
| 7.00 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

**Table 3:** ARL Comparison with Proposed DMA and MA at φ=0.75

| | **DMA $\varphi$=0.75** | | | | **MA $\varphi$=0.75** | | | |
|---|---|---|---|---|---|---|---|---|
| | **W=3** | **W=5** | **W=10** | **W=15** | **W=3** | **W=5** | **W=10** | **W=15** |
| **δ ↓** | **K=6.82** | **K=9.8** | **K=14.2** | **K=17.85** | **K=4.24** | **K=4.866** | **K=5.456** | **K=5.617** |
| 0.0 | 370.466 | 370.432 | 370.732 | 370.0319 | 370.040 | 370.387 | 370.102 | 370.4323 |
| 0.25 | 286.531 | 279.671 | 260.761 | 243.9337 | 291.623 | 283.415 | 269.419 | 257.8407 |
| 0.50 | 167.012 | 156.531 | 131.066 | 112.233 | 171.538 | 164.012 | 143.273 | 127.544 |
| 0.75 | 94.8231 | 85.5259 | 65.6416 | 52.01255 | 99.4864 | 91.7109 | 74.565 | 63.13285 |
| 1.00 | 54.8382 | 47.4665 | 33.7057 | 24.6873 | 51.6778 | 52.1837 | 40.4022 | 32.37475 |
| 1.50 | 20.9307 | 16.9096 | 9.7841 | 5.60125 | 22.501 | 19.4364 | 13.2606 | 9.2455 |
| 2.00 | 8.85115 | 6.5013 | 3.02105 | 1.6139 | 9.795 | 7.96165 | 4.63585 | 2.78735 |
| 3.00 | 2.095 | 1.4703 | 1.0355 | 1.0013 | 2.3789 | 1.809 | 1.6405 | 1.02585 |
| 4.00 | 1.07445 | 1.0139 | 1 | 1 | 1.1242 | 1.04055 | 1.0016 | 1 |
| 5.00 | 1.0037 | 1.0002 | 1 | 1 | 1.0072 | 1.0013 | 1 | 1 |
| 7.00 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

The proposed DMA chart's average run length curves are drawn on various values of φ i.e 0.2, 0.5, 0.75 to observe its effect on ARL performance explicitly it is clear from the simulation results, **Figure 2**, showed the DMA perform more effective in the detection of smaller shifts. The ARL curve is below throughout in comparison with MA showing the improved and better presentation.

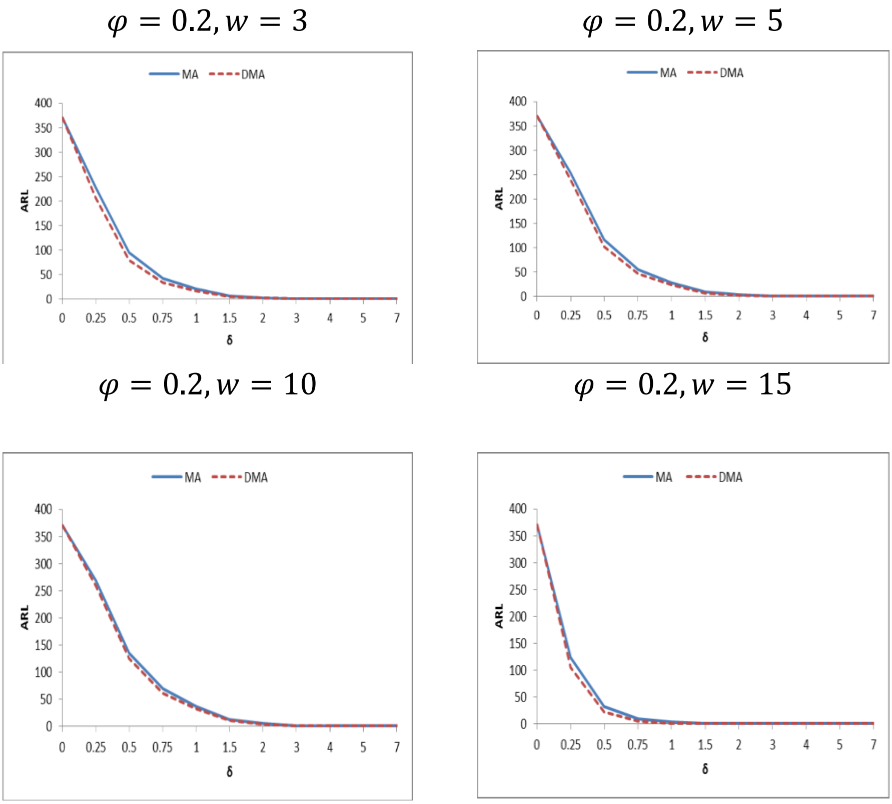


**Figure 2.** ARL graph DMA & MA, at φ=0.2 with “w”=3, 5, 10, 15.

When φ is at 0.2 the values of “K” are 4.868, 6.0025, 7.7522, 8.89 for the proposed DMA and 3.31, 3.341, 3.24, 3.1329 are for the classical MA, respectively. The graphs show the curve line of ARL values for the proposed DMA in comparison with MA. The proposed DMA chart works effectively as compared to MA, because it detected the small, medium or large shifts of the auto-correlated process. As the above graph depicts that our proposed DMA charts are continuous. Below is the curve line of MA which means that the proposed technique performed better for all the shifts.

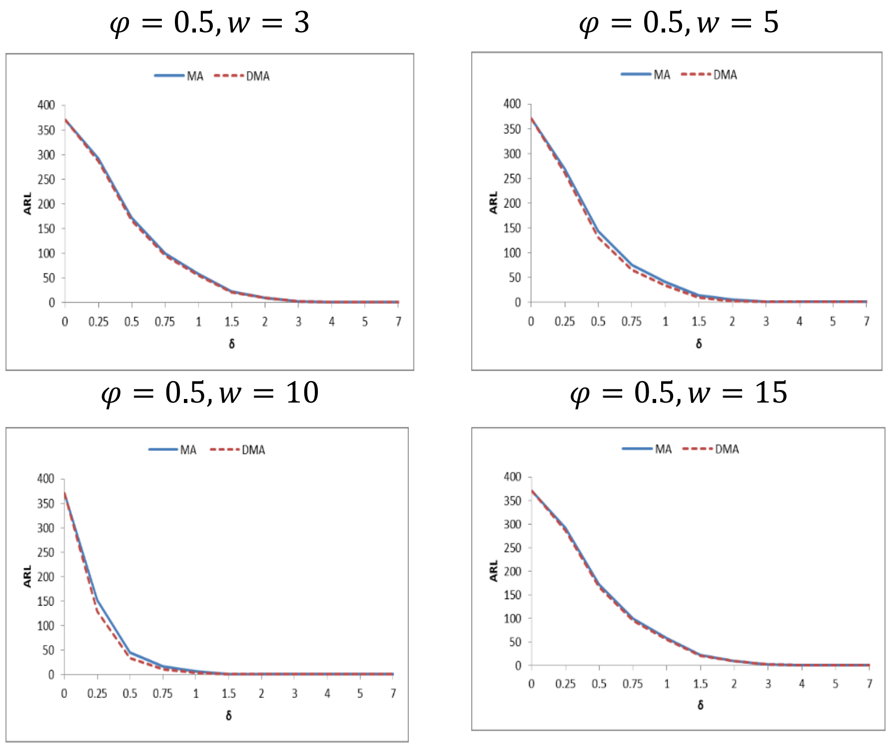


**Figure 3.** ARL graph DMA & MA, at φ=0.5 with w=3, 5, 10, 15.

When φ is at 0.5 the values of “K” are 5.98, 7.86, 10.665, 12.393 are set for the proposed DMA and 3.886, 4.143, 4.23, 4.164 are for MA, respectively. At “w” =10 with the control width set at point 10.665 for DMA and 4.164 for MA, it can visualize that the proposed chart is working much better as the curve line is below while in the remaining three graphs shows DMA efficiently works. DMA detects the minor shifts more quickly in comparison with MA, but the proposed DMA works well for small, medium and large shifts of the process. By looking at the above graphs, we can suggest that the proposed DMA chart is a control monitoring approach that is used in addition to the traditional MA chart.

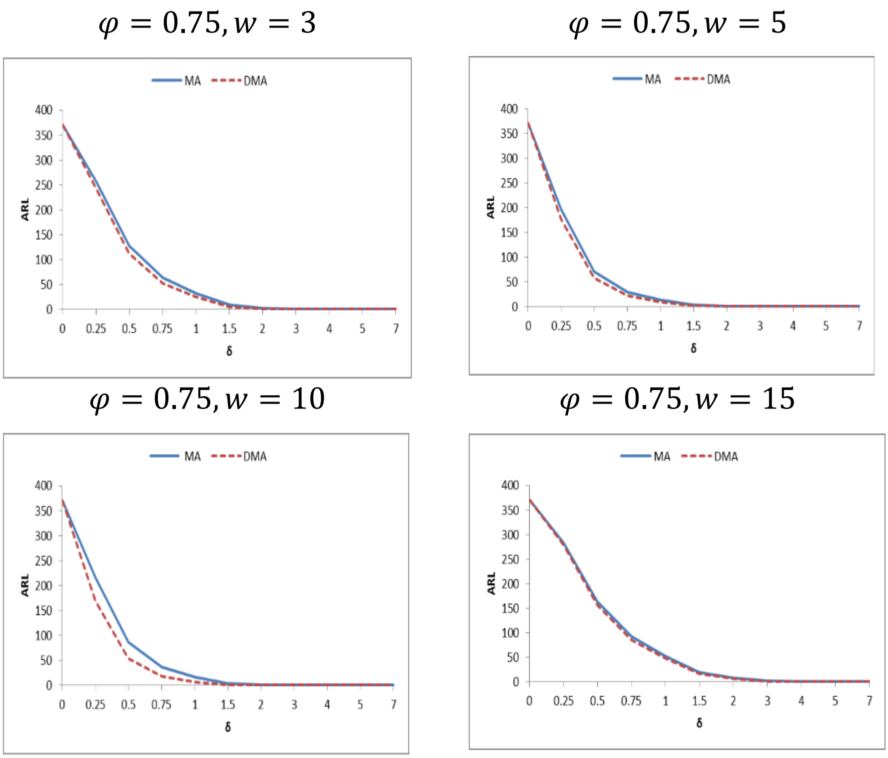


**Figure 4.** ARL graph DMA & MA, at φ=0.75 with w=3, 5, 10, 15.

When φ is at 0.75 the values of "K" for the proposed DMA are 6.852, 9.8, 14.712, and 17.85 and for MA, are 4.2483, 4.866, 5.4565, and 5.617, respectively. The above graphs show that our proposed chart working is effective and sensitive to detect the shifts of the process. The "K" control limits are changed for all of these charts so that the in-control ARL is near 370. The proposed DMA and MA chart's adjusted control limits parameter is 0.2, 0.5, and 0.75 against the values of $\varphi$, which showed that the curve of the proposed DMA is continuously below and the curve for the classical MA which means that the proposed technique is performing better for all the shifts in a given process.

## 5. CONCLUSIONS

The current research discusses the techniques of control charts applied only when the underlying independence assumption is not valid. Furthermore, this research suggested some suggestions which were made in order to motivate the future researchers to follow the time series control charts. The fundamental goal of control (SPC) is to increase the efficiency by reducing the variance. The most famous tool SPC is the technique for manipulating the chart. However, in practice its utility is restricted to various circumstances in which successive measurements could be presumed to be distributed independently, while most of the data sets which were found in practice display some sort of serial correlation. SPC is used where the data is naturally and autonomously distributed typically when EWMA, CUSUM, MA, and DOUBLE MA control diagram is used in violation of the standard time series. The key topic which is focused in this article was the model of AR (1), which could be used as the DMA chart for monitoring attributes. Instead of AR (1), the intended structure could be broadened for ARMA models. The combination of the control chart recognition model with a multivariate (Extension of Univariate) statistical process control approach could increase the process of monitoring and control performance. The current study, proposed technique with span w=10 & w=15 which provides a better result in comparison to the moving average control chart.

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