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Author(s)	**Fazal Dayan** **Muhammad Javaid** **Muhammad Aziz ur Rehman**
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On Leap Reduced Reciprocal Randic and Leap Reduced Second Zagreb Indices of Some Graphs

Fazal Dayan[1]*, Muhammad Javaid[1], Muhammad Aziz ur Rehman[1]

[1]Department of Mathematics, School of Science,
University of Management and Technology, Lahore, Pakistan

*fazaldayan1@gmail.com

Abstract

Naji et al. introduced the leap Zagreb indices of a graph in 2017 which are new distance-degree-based topological indices conceived depending on the second degree of vertices. In this paper, we have defined the first and second leap reduced reciprocal Randic index and leap reduced second Zagreb index for selected wheel related graphs.

***Keywords*:** leap indices, reduced reciprocal Randic index, reduced second Zagreb index, flower graph, gear graph, helm graph, sunflower graph, wheel graph

1. Introduction

In the current chemical and mathematical literature, a large number of vertex degree based graph invariants have been studied. Among them, the first and second Zagreb indices are, by far, the most extensively investigated. They were introduced more than forty years ago [1].

The properties of the two Zagreb indices can be seen in [2, 3, 4, 5, 6]. Many novel variants of the Zagreb indices have been studied in recent years. Some of these are multiplicative Zagreb indices [7, 8, 6]. Zagreb coincides [9, 10], multiplicative Zagreb coincides [11] and the sum Zagreb index [12, 6]. Recently, leap Zagreb indices of a graph have been introduced by Naji et al. [13], which are new distance-degree-based topological indices conceived depending on the second degree of vertices (number of their second neighbors). Some basic properties of these new indices have been established as well. A. M. Naji and N. D. Soner [14] presented the exact expressions for the first Zagreb index of selected graph operations containing the corona product, cartesian product, composition, dis junction and symmetric difference between graphs. Shiladhar P. et al. [15] computed leap Zagreb indices of selected wheel related graphs. Fazal Dayan et al. [16] defined leap Gourava indices and computed the exact values of leap Gourava indices for some wheel related graphs. In recent years, many researchers have worked on computing topological indices [17, 18, 19].

2. Definitions and Preliminaries

Definition. The reduced reciprocal Randic index is defined as follows:-

$$RRR(G) = \sum_{uv \in G} \sqrt{(d\,(u)\text{-}1)(d\,(u)\text{-}1)}$$

Definition. The reduced second Zagreb index is defined as follows:-

$$RM_2(G) = \sum_{uv \in G} (d\,(u)\text{-}1)(d\,(u)\text{-}1)$$

Definition. The leap reduced reciprocal Randic and leap reduced second Zagreb indices are defined as follows:-

$$LRRR(G) = \sum_{uv \in G} \sqrt{\left(d_{\,2}(u)\text{-}1\right)\left(d_{\,2}(u)\text{-}1\right)} \qquad (1)$$

and

$$LRM_2(G) = \sum_{uv \in G} \left(d_2\,(u)\text{-}1\right)\left(d_{\,2}(u)\text{-}1\right) \qquad (2)$$

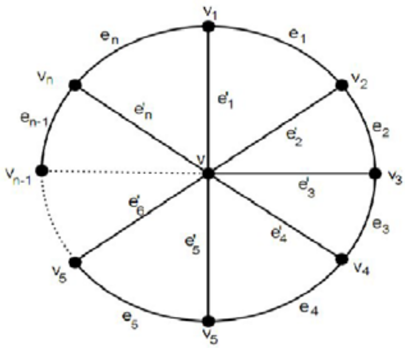

Figure 1. Wheel graph W_n

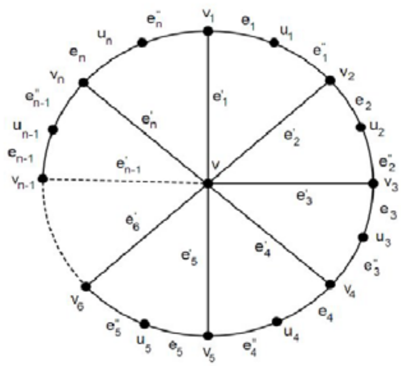

Figure 2. Gear graph G_n

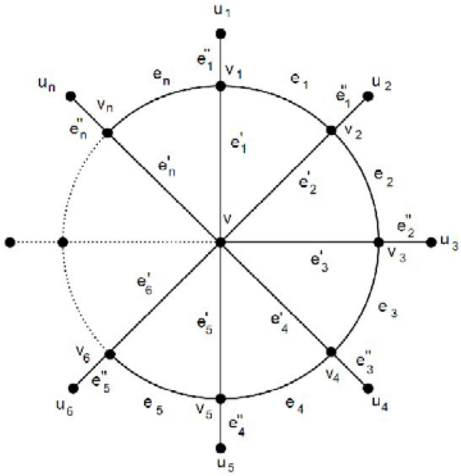

Figure 3. Helm graph H_n

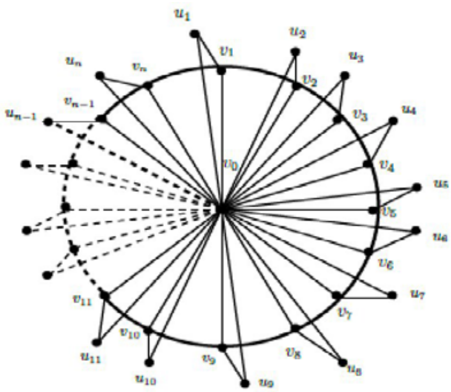

Figure 4. Flower graph Fl_n

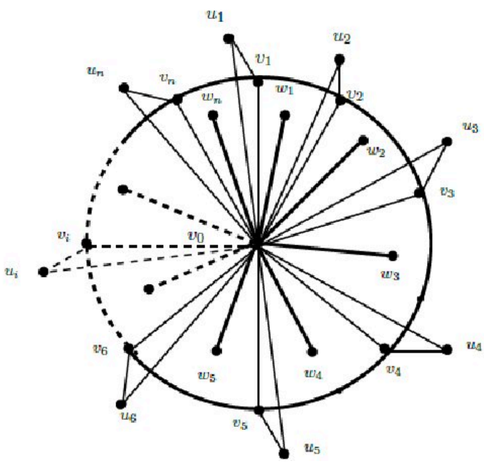

Figure 5. Sunflower graph Sf_n

Figures 1-5 show the wheel graph, gear graph, helm graph, flower graph and sunflower graph, respectively.

3. Main Results

Theorem

Let W_n *be a wheel graph with* $n+1$ *vertices, its leap reduced reciprocal Randic and leap reduced second Zagreb indices are given below:-*

$$LRRR(G) = n\left[\sqrt{4\text{-}n} + n(n\text{-}4)\right]$$

$$LRM_2(G) = n(n\text{-}4)(n\text{-}5)$$

Proof: We compute the required result by using definitions 1 and 2 given in the previous section as follows:-

$$LRRR(G) = \sum_{uv\in G}\sqrt{\left(d_2(u)\text{-}1\right)\left(d_2(u)\text{-}1\right)}$$

$$= n\sqrt{[(0\text{-}1)(n\text{-}3\text{-}1)]} + n\sqrt{[(n\text{-}3\text{-}1)(n\text{-}3\text{-}1)]}$$

$$= n\sqrt{[(\text{-}1)(n\text{-}4)]} + n\sqrt{[(n\text{-}4)]^2}$$

$$= n\sqrt{4\text{-}n} + n(n\text{-}4)$$

$$= n\left[\sqrt{4\text{-}n} + n(n\text{-}4)\right]$$

and

$$LRM_2(G) = \sum_{uv\in G}\left(d_2\,(u)\text{-}1\right)\left(d_2(u)\text{-}1\right)$$

$$LRM_2(G) = n[(0\text{-}1)(n\text{-}3\text{-}1)] + n[(n\text{-}3\text{-}1)(n\text{-}3\text{-}1)]$$

$$= n(4\text{-}n) + n(4\text{-}n)^2$$

$$= n(4\text{-}n + n^2\text{-}8n + 16)$$

$$= n(n^2\text{-}9n + 20)$$

$$= n(n\text{-}4)(n\text{-}5)$$

Theorem

Let G_n *be a gear graph with* $2n+1$ *vertices, its leap reduced reciprocal Randic and leap reduced second Zagreb indices are given below:-*

$$LRRR(G) = n(n\text{-}2)\left[\sqrt{n\text{-}1} + 2\sqrt{2}\right]$$

$$LRM_2(G) = n(n\text{-}2)(5n\text{-}1)$$

Proof: We compute the required result by using definitions 1 and 2 given in the previous section as follows:-

$$\text{LRRR}(G) = \sum_{uv\in G} \sqrt{(d_2(u)\text{-}1)(d_2(u)\text{-}1)}$$
$$= n\sqrt{[(n\text{-}1)(n\text{-}1\text{-}1)]} + 2n\sqrt{[(n\text{-}1\text{-}1)(3\text{-}1)]}$$
$$= n\sqrt{[(n\text{-}1)(n\text{-}2)]} + 2n\sqrt{2(n\text{-}2)}$$
$$= n(n\text{-}2)\left[\sqrt{n\text{-}1} + 2\sqrt{2}\right]$$

and

$$\text{LRM}_2(G) = \sum_{uv\in G} (d_2(u)\text{-}1)(d_2(u)\text{-}1)$$
$$= n[(n\text{-}1)(n\text{-}1\text{-}1)] + 2n[(n\text{-}1\text{-}1)(3\text{-}1)]$$
$$= n[(n\text{-}1)(n\text{-}2)] + 2n[2(n\text{-}2)]$$
$$= n(n\text{-}2)[n\text{-}1 + 4n]$$
$$= n(n\text{-}2)(5n\text{-}1)$$

Theorem

Let H_n *be a helm graph with* $2n + 1$ *vertices, its leap reduced reciprocal Randic and leap reduced second Zagreb indices are below:-*

$$\text{LRRR}(G) = n(n\text{-}2)\left[\sqrt{n\text{-}1} + \sqrt{(n\text{-}2)} + \sqrt{2}\right]$$
$$\text{LRM}_2(G) = n(n\text{-}2)(2n\text{-}1)$$

Proof: We compute the required result by using definitions 1 and 2 given in the previous section as follows:-

$$\text{LRRR}(G) = \sum_{uv\in G} \sqrt{(d_2(u)\text{-}1)(d_2(u)\text{-}1)}$$
$$= n\sqrt{[(n\text{-}1)(n\text{-}1\text{-}1)]} + n\sqrt{[(n\text{-}1\text{-}1)(n\text{-}1\text{-}1)]} + n\sqrt{[(n\text{-}1\text{-}1)(3\text{-}1)]}$$
$$= n\sqrt{[(n\text{-}1)(n\text{-}2)]} + n\sqrt{(n\text{-}2)^2} + n\sqrt{2(n\text{-}2)}$$
$$= n(n\text{-}2)\left[\sqrt{n\text{-}1} + \sqrt{(n\text{-}2)} + \sqrt{2}\right]$$

and

$$\text{LRM}_2(G) = \sum_{uv\in G} (d_2(u)\text{-}1)(d_2(u)\text{-}1)$$
$$= n[(n\text{-}1)(n\text{-}1\text{-}1)] + n[(n\text{-}1\text{-}1)(n\text{-}1\text{-}1)] + n[(n\text{-}1\text{-}1)(3\text{-}1)]$$
$$= n[(n\text{-}1)(n\text{-}2)] + n[(n\text{-}2)(n\text{-}2)] + 2n[(n\text{-}2)]$$
$$= n(n\text{-}2)(n\text{-}1 + n\text{-}2 + 2)$$
$$= n(n\text{-}2)(2n\text{-}1)$$

Theorem

Let F_n *be a flower graph with* $2n+1$ *vertices, its leap reduced reciprocal Randic and leap reduced second Zagreb indices are given below:-*

$$LRRR(G) = n\left[\sqrt{[(2n\text{-}3)(2n\text{-}5)]} + \sqrt{3\text{-}2n} + \sqrt{5\text{-}2n} + (2n\text{-}5)\right]$$
$$LRM_2(G) = 8n(n^2\text{-}5n + 6)$$

Proof: We compute the required result by using definitions 1 and 2 given in the previous section as follows:-

$$LRRR(G) = \sum_{uv\in G} \sqrt{\left(d_2(u)\text{-}1\right)\left(d_2(u)\text{-}1\right)}$$
$$= n\sqrt{[(2n\text{-}2\text{-}1)(2n\text{-}4\text{-}1)]} + n\sqrt{[(2n\text{-}2\text{-}1)(0\text{-}1)]}$$
$$+ n\sqrt{[(2n\text{-}4\text{-}1)(0\text{-}1)] + n\sqrt{[(2n\text{-}4\text{-}1)(2n\text{-}4\text{-}1)]}}$$
$$= n\sqrt{(2n\text{-}3)(2n\text{-}5)} + n\sqrt{3\text{-}2n} + n\sqrt{5\text{-}2n} + n(2n\text{-}5) + n(2n\text{-}5)$$
$$= n\left[\sqrt{[(2n\text{-}3)(2n\text{-}5)]} + \sqrt{3\text{-}2n} + \sqrt{5\text{-}2n} + (2n\text{-}5)\right]$$

and

$$LRM_2(G) = \sum_{uv\in G} \left(d_2\,(u)\text{-}1\right)\left(d_2(u)\text{-}1\right)$$
$$= n[(2n\text{-}2\text{-}1)(2n\text{-}4\text{-}1)] + n[(2n\text{-}2\text{-}1)(0\text{-}1)] + n[(2n\text{-}4\text{-}1)(0\text{-}1)]$$
$$+ n[(2n\text{-}4\text{-}1)(2n\text{-}4\text{-}1)]$$
$$= n[(2n\text{-}3)(2n\text{-}5)] + n[(2n\text{-}3)(\text{-}1)] + n[(2n\text{-}5)(\text{-}1)] + n(2n\text{-}5)^2$$
$$= n(8n^2\text{-}40n + 48)$$
$$= 8n(n^2\text{-}5n + 6)$$

Theorem

Let Sf_n *be a sunflower graph with* $3n+1$ *vertices, its leap reduced reciprocal Randic and leap reduced second Zagreb indices are given below:-*

$$LRRR(G) = n\left[\sqrt{2\text{-}3n} + \sqrt{3\text{-}3n} + \sqrt{5\text{-}3n} + \sqrt{[(3n\text{-}3)(3n\text{-}5)]}\right.$$
$$\left. + (3n\text{-}5)\right]$$
$$LRM_2(G) = n(18n^2\text{-}63n + 50)$$

Proof: We compute the required result by using definitions 1 and 2 given in the previous section as follows:-

$$\mathrm{LRRR}(G) = \sum_{uv\in G} \sqrt{(d_2(u)\text{-}1)(d_2(u)\text{-}1)}$$

$$= n\sqrt{[(3n\text{-}1\text{-}1)(0\text{-}1)]} + n\sqrt{[(3n\text{-}2\text{-}1)(0\text{-}1)]} + n\sqrt{[(3n\text{-}4\text{-}1)(0\text{-}1)]}$$
$$+ n\sqrt{[(3n\text{-}2\text{-}1)(3n\text{-}4\text{-}1)]} + n\sqrt{[(3n\text{-}4\text{-}1)(3n\text{-}4\text{-}1)]}$$
$$= n\sqrt{2\text{-}3n} + n\sqrt{3\text{-}3n} + n\sqrt{5\text{-}3n} + n\sqrt{[(3n\text{-}3)(3n\text{-}5)]} + n(3n\text{-}5)$$
$$= n\left[\sqrt{2\text{-}3n} + \sqrt{3\text{-}3n} + \sqrt{5\text{-}3n} + \sqrt{[(3n\text{-}3)(3n\text{-}5)]} + (3n\text{-}5)\right]$$

$$\mathrm{LRM}_2(G) = \sum_{uv\in G} (d_2(u)\text{-}1)(d_2(u)\text{-}1)$$

$$= n[(3n\text{-}1\text{-}1)(0\text{-}1)] + n[(3n\text{-}2\text{-}1)(0\text{-}1)] + n[(3n\text{-}4\text{-}1)(0\text{-}1)]$$
$$+ n[(3n\text{-}2\text{-}1)(3n\text{-}4\text{-}1)] + n[(3n\text{-}4\text{-}1)(3n\text{-}4\text{-}1)]$$
$$= n(2\text{-}3n) + n(3\text{-}3n) + n(5\text{-}3n) + n[(3n\text{-}3)(3n\text{-}5)] + n(3n\text{-}5)^2$$
$$= n(18n^2\text{-}63n + 50)$$

4. Conclusion

In this paper, we have extended the work initiated by Naji et al. In this regard, we have defined the leap reduced reciprocal Randic and leap reduced second Zagreb indices and we have computed the exact values of these newly defined indices for selected graphs including wheel graph, gear graph, helm graph, flower graph and sunflower graph.

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