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Comparison of Computation Period for Some Newtonian Fluid Flow Problems by Elzaki, Sumudu and Laplace Transform

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Abstract

Newtonian Fluid Flow Problems (NFFPs) of Stokes' first theorem for suddenly started and suddenly stopped plate are solved by Elzaki Transform Method and the results are identical to those of Laplace Transform and Sumudu Transform. It shows that the said method is very effective due to smaller computation period. The results are verified by graphing the outcome of all the transforms and by calculating their computation period, so that NFFP can be easily solved by Elzaki Transform Method to avoid lengthy calculations.

Keywords: Newtonian fluid flow problems (NFFPs), stokes' first problem, Elzaki transform, Laplace transform, Sumudu transform, exact solutions, error function

1. Introduction

Integral Transforms (IT) play a significant role in several arenas of technology. ITs are extensively used in physics, engineering, arithmetic, optics etc. [1]. They are also used to a considerable extent in Variational Iteration Method (VIM), Homotopy Perturbation (HP) approach and Differential Transform (DT) approach. Laplace Transform (LT) is widely used to solve Differential Equations (DE) [2]. It has been applied successfully to solve both linear and non-linear equations of the type of ordinary and partial differentials [3]. The above stated Newtonian Fluid Flow Problem (NFFP) is also solved through its application and it gives the same results as in the literature [4, 5]. Laplace Transform reduces a linear differential equation to an algebraic equation which can be solved by applying the rules of algebra. Sumudu Transform (ST) was presented for the first time by Watagula in his research [6]. ST is very useful in solving differential and integral equations [7]. Some complicated problems proved difficult to solve with Laplace and Sumudu transforms. To overcome such differential and integral problems. Tarig. M proposed a new transform to solve them known as Elzaki Transform (ET) which is useful to solve Partial Differential Equations (PDEs), Ordinary Differential Equations (ODEs), and the system of ordinary and partial differential equations [8, 9, 10]. Since its development, Elzaki Transform has become an excellent tool for solving DEs which cannot be solved by Sumudu Transform. Hassan Eltayeb proposed a transform which was helpful in solving the linear ODEs with some constant and non-constant coefficients and compared the solutions by using two different transforms [11]. The relationship between Laplace, Elzaki and Aboodh transforms is discussed by Abdelbagy A. Alshikh et al. and the results reveal that Laplace Transform is closely connected with other transforms [12]. For the sake of comparison, LT was applied to solve differential problems of second order in some special cases [11]. Saqlain. M. et al. discussed the convergence of Elzaki and Differential Transform Method (DTM) and the results reveal that DTM converges more quickly than ETM [13]. The DEs in the steady heat-transfer problem were solved by Xiao-Jun YANG in his research [14]. Elzaki turned his attention toward the application of the Elzaki Transform to solve some non-homogeneous fractional ODEs [15]. To resolve nonlinear PDE's, Khalid et al. suggested that ET can be applied to get approximate analytical solutions of the proposed problem and compared the numerical results with exact solutions [16]. Indeed, Elzaki Transform is more appropriate than Sumudu Transform and Laplace Transform to solve ODEs with non-constant coefficients [17].

In this study, Laplace Transform, Sumudu Transform and Elzaki Transform are used to solve two NFFPs suddenly started and suddenly stopped plates.

2. Preliminaries

Definition 1: Consider function in the following set X defined by [11]

$$X = \left\{ f(t) \mid \exists\, M, k_1, k_2 > 0 \mid f(t) < Me^{\frac{|t|}{k_i}}, if\ t \in (-1)^i X[0,\infty) \right\}$$

Then Elzaki Transform is defined as follows,

$$E[f(t)] = u^2 \int_0^\infty f(ut)e^{-t}dt = T(u)\,, \qquad u \in (k_1 k_2) \tag{1}$$

Definition 2: Laplace Transform of a function f(t), defined for all real numbers t ≥ 0, is the function F(s), which is a unilateral transform defined by [1].

$$F(s) = \int_0^\infty f(t)e^{-st}dt$$

where s is a complex number frequency parameter $s = \sigma + i\omega$, with real numbers σ and ω. An alternate notation for Laplace Transform is $L\{f\}$ of F.

Definition 3: Over the set of functions

$$A = \{ f(t) \mid \exists M, \tau_1, \tau_2 > 0, \ | f(t) < Me^{|t|/\tau_j} \ , \ if \ t \in (-1)^j \times [0, \infty) \}$$

Sumudu Transform is defined by [19]

$$G(u) = S[f(t)] = \int_0^\infty f(ut)\, e^{-t} dt, \quad u \in (-\tau_1, \tau_2) \tag{2}$$

3. Calculations

3.1. Stokes' First Problem

Consider the unsteady motion of a flat plate in an infinite fluid. Since streamlines are parallel to the plate, therefore the velocity components $V = W = 0$ and the equation of continuity reduces to $\frac{\partial U}{\partial x} = 0$, so that the velocity component U will be the only function of "y" and "t". Thus

$$U = U(y, t)\ , \ V = 0 \ , \ W = 0, \tag{3}$$

Hence, the only non-zero velocity component is $U\ (y, t)$, where $y - axis$ is perpendicular to the plate. The pressure will be independent of x, that is, the pressure gradient will be zero everywhere because the plate is situated in an infinite fluid where pressure remains constant everywhere. Using these properties of fluid flow field, the governing equation reduces PDEs to

$$\frac{\partial U}{\partial y} = \vartheta \frac{\partial^2 U}{\partial y^2} \tag{4}$$

We consider the following cases.

3.2. Flow over a suddenly accelerated flat plate

Let a stationary viscous incompressible fluid over the upper half plane $y = 0$ (that is $xz - plane$). Suddenly, the plate is jerked into motion in its own plane with a constant velocity $U_o i$ and continues to traverse with this velocity for $t > 0$. Since the fluid is viscous, it is expected that with the passage of time the motion of the plate will be transferred to the fluid. Thus, the boundary conditions for the problem under consideration are as follows,

$$U(0,t) = \begin{cases} 0 & for \quad t \le 0, \\ U_o & for \quad t > 0, \end{cases} \tag{5}$$

$$U(\infty, t) = 0$$

Since we have the governing equations, initial conditions and boundary conditions, therefore, the problem is well-posed. Through the application of ETM, PDE reduces into ODE, that is, it causes the reduction of two variables into a single variable. The whole set of governing equations, initial conditions and boundary conditions are arranged and this problem can be solved with the help of ETM. Equation (4) and the boundary conditions takes the following form,

$$E\left[\frac{\partial U(x,t)}{\partial t}\right] = \vartheta E\left[\frac{\partial^2 U(x,t)}{\partial x^2}\right]$$

$$\frac{\bar{U}(x,u)}{v} - vU(x,0) = \vartheta \frac{d^2\bar{U}(x,u)}{dx^2}$$

The general solution to eq. (4) is given below.

$$\bar{U}(x,u) = A\, e^{\frac{-1}{\sqrt{\vartheta u}}x} + B\, e^{\frac{1}{\sqrt{\vartheta u}}x} \tag{6}$$

The boundary conditions are as follows,

$$\bar{U}(0,u) = u^2 U_o \tag{7}$$

$$\bar{U}(\infty,u) = 0 \tag{8}$$

By using these boundary conditions to solve the arbitrary constants A and B and by inserting the values of these constants into eq. (6) we get the following solution,

$$\bar{U}(x,u) = U_o u^2\, e^{\frac{-1}{\sqrt{\vartheta u}}x} \tag{9}$$

Taking the inverse Elzaki Transform to eq. (9), the velocity profile is given below.

$$U(x,t) = E^{-1}\left[U_o u^2\, e^{\frac{-1}{\sqrt{\vartheta u}}x}\right] \tag{10}$$

$$U(x,t) = U_o erfc\left(\frac{x}{2\sqrt{\vartheta t}}\right)$$

$$= U_o\left[1 - erf\left(\frac{x}{2\sqrt{\vartheta t}}\right)\right] \tag{11}$$

Where

$$\text{erf}(\eta) = \frac{2}{\sqrt{\pi}}\int_0^\eta e^{-k^2}\, dk$$

The values of complementary error function are given in Table 1.

3.3. Suddenly Stopped Plate

In this case, initially the fluid and plate are moving with constant velocity U_o and the plate is suddenly stopped, that is, achieves zero velocity. Therefore, the boundary conditions are as follows,

$$U(0,t) = \begin{cases} U_o & for \quad t \leq 0, \\ 0 & for \quad t > 0, \end{cases} \tag{12}$$

$$U(\infty\,, t) = 0 \tag{13}$$

Taking the Elzaki Transform of eq. (4) and using the boundary and initial conditions, we get

$$\frac{d^2\bar{U}(x,u)}{dx^2} - \frac{1}{\vartheta u}\bar{U} = -\frac{u}{\vartheta}\,U_o \tag{14}$$

$$\left.\begin{aligned} \bar{U}(0,u) &= U_o u^2 \\ \bar{U}(\infty,u) &= 0 \end{aligned}\right\} \tag{15}$$

The general solution of the non-homogeneous differential eq. (14) is as follows,

$$\bar{U}(x,u) = C\, e^{\frac{-1}{\sqrt{\vartheta u}}x} + D\, e^{\frac{1}{\sqrt{\vartheta u}}x} + u^2\, U_o$$

Using boundary conditions (15), we obtain the following

$$\bar{U}(x,u) = U_o u^2\, e^{\frac{-1}{\sqrt{\vartheta u}}x} + u^2\, U_o \tag{16}$$

$$\bar{U}(x,u) = U_o u^2 \left(1 - e^{\frac{-1}{\sqrt{\vartheta u}}x}\right) \tag{17}$$

Taking inverse Elzaki Transform to obtain

$$U(x,t) = U_o erfc\left(\frac{x}{2\sqrt{\vartheta t}}\right) \tag{18}$$

The solution given by eqs. (11) and (18) are identical to those given by Laplace Transform and other similarity methods [1, 2, 3, 4, 5]. It is obvious from the eqs. (11) and (18) that in both cases the plate's effect diffuses into the fluid at a rate proportional to the square root of kinematics viscosity. It is customary to define the shear layer thickness as the point where the wall effect on fluid drops to 1 percent, that is in the first case (suddenly started plate) where $\frac{U}{U_o} = 0.01$ and in the second case (suddenly stopped plate) where $\frac{U}{U_o} = 0.99$. Both of them correspond to $\text{erf}(\eta) = 0.01\,,\ \ or\ \ \eta = 1.82$. Then, the shear layer thickness δ in these flows is approximately $\delta = 3.64\sqrt{\nu t}$. Consider the flow of a viscous fluid on an infinite plate under a constant pressure. If $x - axis$ is the direction of main flow then $U \neq 0\,, V = 0, W = 0$. The continuity equation simplifies as follows,

$$\frac{\partial U}{\partial x} = 0 \tag{19}$$

If $y - axis$ is taken normal to the plate, then we conclude that $U = U(y)$. The equations of motion are reduced to one equation which is given below.

$$\frac{\partial^2 U}{\partial x^2} = 0\,, p = constant \tag{20}$$

Taking no-slip condition

$$U = 0\,, y = 0 \tag{21}$$

And denoting the wall shear as

$$\tau_w = \mu\left(\frac{\partial U}{\partial y}\right)_{y=0} \tag{22}$$

The Elzaki Transform of eq. (20) and the use of conditions (21) and (22) gives

$$\bar{U}(u) = \frac{\tau_w}{\mu}\, u^3 \tag{23}$$

And the inverse Elzaki Transform of the above equation gives

$$U(y) = \frac{\tau_w}{\mu}\, y \tag{24}$$

The eq. (24) shows that the distribution in velocity is linear in y. This result is identical to analytical solution [9].

Table 1. Numerical Values of the Complementary Error Function.

$\boldsymbol{\eta}$	$\boldsymbol{erfc(\eta)}$	$\boldsymbol{\eta}$	$\boldsymbol{erfc(\eta)}$
0	1.0	1.1	0.11980
0.05	0.94363	1.2	0.08969
0.1	0.88754	1.3	0.06599
0.15	0.83200	1.4	0.04772
0.2	0.77730	1.5	0.03390
0.25	0.72367	1.6	0.02365
0.3	0.67137	1.7	0.01621
0.35	0.62062	1.8	0.01091
0.4	0.57161	1.9	0.00721
0.5	0.47950	2.0	0.00468
0.6	0.39615	2.5	0.000407
0.7	0.32220	3.0	0.0000221
0.8	0.25790	3.5	0.00000074
0.9	0.20309	4.0	0.00000001
1.0	0.15730	∞	0.0000000000

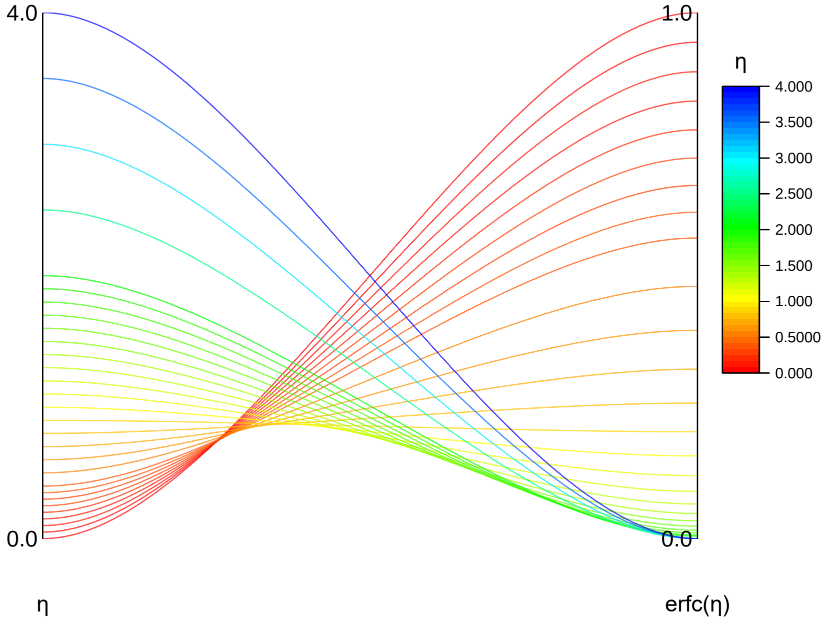

Figure 1. Graphical representation of the complementary error function defined in (Table: 1)

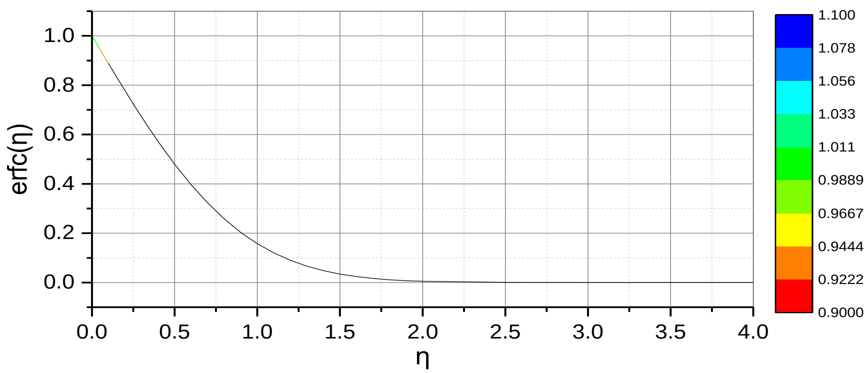

Figure 2. Graphical representation of the complementary error function of eq (11) and (18)

4. Conclusion

In the current research, NFFP are successfully solved by Elzaki Transform Method. The solutions are identical to Sumudu Transform, Laplace Transform and supported by literature. The results reveal that the computation period for Elzaki Transform is less than the other transforms discussed in the study. The consequences depict that the method is very effective and simple. Hence, Elzaki Transform can be used as a powerful mathematical tool to solve Newtonian Fluid Flow Problems.

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