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TOPSIS Technique of MCDM under Cubic Intuitionistic Fuzzy Soft Set Environment

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ABSTRACT

A powerful tool for dealing with ambiguity, attributive values, fuzziness, and inconsistency is the cubic intuitionistic fuzzy soft set. In this paper, multi-criteria group decision-making (MCGDM) technique called extended TOPSIS based on distance and similarity measures is presented. Furthermore, a real-world problem is resolved by applying CIFS-set to support the theoretical visualization. Arguably, the proposed technique works well and has practical uses.

Keywords: cubic set, intuitionistic fuzzy set, intuitionistic fuzzy soft set, multi-criteria group decision-making (MCGDM), TOPSIS technique

1. INTRODUCTION

Taking the best decision requires taking into account a range of factors due to increasing complexities in business, engineering, scientific, and technological environments. The most effective way to select the ideal alternative among all possible options is decision-making. Since generalized variants are frequently used in decision-making, nearly every other problem has a considerable range of requirements. Such requirements frequently come into conflict with each other and no solution may ever be able to fully satisfy all of them. To address such challenges, decision-makers need to overcome the MCGDM problem. For this purpose, many mathematical theories have been expounded such as the fuzzy set theory, intuitionistic set theory, and interval-valued intuitionistic set theory. In 1965, fuzzy set theory was introduced by Zadeh [1]. This theory deals with membership value over closed interval [0,1] and overcomes the vague and ambiguous environment. However, due to the increased complexities in the environment, a decision-maker faces difficulties to express their idea in the

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form of a single membership value. To deal with such a situation, the intuitionistic fuzzy set (IFS) [2] theory was introduced. This theory not only deals with the membership value but also with the non-membership value and hence gives a more precise result which helps in decision-making. In this context, many researchers have presented different theories based on distance and similarities measures [3–5].

The technique known as the order of preference by similarity to ideal solution (TOPSIS) was introduced by Lai et al. [6] and remains a widely known method. The purpose of this method is to identify the longest path from the negative ideal alternative (NIA) as well as the shortest path from the positive ideal alternative (PIA).

Since this technique has been introduced, various researchers have used the TOPSIS method to resolve decision-making problems in an ambiguous environment. Interval-valued intuitionistic fuzzy set and its fundamental properties was proposed by [7]. Jahanshahloo et al. [8] presented the TOPSIS method for DM under a fuzzy environment, Chu *et al.* [9] suggested the TOPSIS method for robot selection, and Zulqarnain *et al.* [10] generalized the fuzzy TOPSIS for MCGDM. Saqlain et al. [11] discussed the use of the TOPSIS method for the selection of a smartphone. Shen et al. [12] extended the application of this technique under intuitionistic fuzzy environment, Chang et al. [13] discussed the distance approaches using the TOPSIS method, Li et al. [14] suggested the Pythagorean fuzzy TOPSIS based on similarity measures, and Gupta et al. [15] discussed the extended TOPSIS under interval-valued intuitionistic fuzzy environment.

All these theories involve uncertainty to some extent as they do not deal with the membership and non-membership values simultaneously along with fuzziness. To deal with such kind of situations, Garg [16] proposed the TOPSIS method for MCGDM under a cubic environment which deals with both the intuitionistic value and fuzziness, simultaneously. Pramanik et al. [17] defined the TOPSIS method for neutrosophic cubic information, Jun et al. [18] suggested the cubic fuzzy set (CFS), while Garg [19] discussed the cubic intuitionistic fuzzy set (CIFS) and its fundamental properties. The soft set theory with its fundamentals was proposed by Maji et al. [20]. The theory of soft set was merged with cubic set and is known as cubic intuitionistic fuzzy soft set (CIFSS). It was proposed by Saqlain et al. [21].

Cubic intuitionistic fuzzy set (CIFS) is considered highly effective for decision-making problems as it involves interval-valued intuitionistic fuzzy number (IVIFS) and intuitionistic fuzzy number (IFS) over the interval. Still, this theory does not deal with multi-attributive values. Considering the soft set, this paper attempts to define the multi-attributive decision-making problem under the cubic intuitionistic fuzzy soft environment, where each element is defined by the CIF-number (CIFN). A methodology which utilizes the extended TOPSIS method is also proposed. Furthermore, some distance and similarity measure formulas have been defined which evaluate the positive ideal alternative (PIA) and negative ideal alternative (NIA).

2. PRELIMINARY SECTION

In this section, some basic concepts of intuitionistic fuzzy set (IF-set), interval-valued intuitionistic fuzzy set (IVIF-set), soft set, intuitionistic fuzzy soft set (IFS-set), CF-set, CIF-set, and CIFS-set are defined.

Definition 2.1 [2]. An intuitionistic fuzzy set (IF-set) ƙ over a crisp set Ʋ is defined by a function $\mathfrak{y}_I$ represented by the mapping

$$\mathfrak{y}_I = Ʋ \rightarrow [0,1] \times [0,1],$$

where $\mathfrak{y}_I$ is the membership function which deals with truthiness and falsity and Ʋ be a universal discourse. Thus, an intuitionistic fuzzy set (IF-set) over set Ʋ can be represented as

$$ƙ = \{\langle ʒ, (ƫ_ƙ(ʒ), ʄ_ƙ(ʒ))\rangle | ʒ \epsilon Ʋ\} \; such \; that \; 0 \leq ƫ_ƙ(ʒ) + ʄ_ƙ(ʒ) \leq 1; \; \forall ʒ \epsilon Ʋ.$$

Definition 2.2 [20]. Let Ʋ be a universal discourse. An interval-valued intuitionistic fuzzy set (IVIF-set) ƙ over a crisp set Ʋ is defined by a function $\mathfrak{y}_I$ represented by the mapping

$$\mathfrak{y}_I = Ʋ \rightarrow P([0,1] \times [0,1]),$$

where $\mathfrak{y}_I$ is the membership function which represents the degree of belongingness and the non-belongingness value. Thus, an interval-valued intuitionistic fuzzy set (IVIF-set) over set Ʋ can be represented as

$$ƙ = \{\langle ʒ, ([ƫ_ƙ(ʒ)^-, ƫ_ƙ(ʒ)^+], [ʄ_ƙ(ʒ)^-, ʄ_ƙ(ʒ)^+])\rangle | ʒ \epsilon Ʋ\} \; such \; that \; 0 \leq ƫ_ƙ(ʒ) + ʄ_ƙ(ʒ) \leq 1; \; \forall ʒ \epsilon Ʋ.$$

Definition 2.3 [19]. Consider Ʋ to be a universal set and $ƈ = \{ƈ_1, ƈ_2, ƈ_3, \ldots, ƈ_n\}$ be the set of attributive values. Then, a soft set ₰ over a crisp set Ʋ is defined by a function $\varphi_₰$ represented by the mapping

$\varphi_{\mathcal{S}}: \mathcal{C} \rightarrow \mathrm{P}(\mathrm{U})$,

where $\varphi_{\mathcal{S}}$ is an approximate function and its value $\varphi_{\mathcal{S}}(\mathcal{C}_i)$ can be represented as

$$\mathcal{S} = \left\{\left(\mathcal{C}_i, \varphi_{\mathcal{S}}(\mathcal{C}_i)\right) : \mathcal{C}_i \in \mathcal{C}, \varphi_{\mathcal{S}}(\mathcal{C}_i) \in \mathrm{P}(\mathrm{U})\right\}.$$

Definition 2.4 [17]. Let U be the set of universal discourse and $\mathfrak{z}$ be a cubic fuzzy set (CF-set) over U and defined as

$$\mathfrak{z} = \{\langle \mathcal{C}, \mathfrak{u}_{\mathfrak{k}}(\mathfrak{z}), \mathrm{v}\rangle | \mathfrak{z} \epsilon \mathrm{U}\},$$

where $\mathfrak{u}_{\mathfrak{k}}(\mathfrak{z}) = [\mathfrak{u}_{\mathfrak{k}}(\mathfrak{z})^{-}, \mathfrak{u}_{\mathfrak{k}}(v)^{+}]$ and v represents the interval-valued fuzzy set and FN in $\mathfrak{z} \epsilon \mathrm{U}$, respectively.

This pair is denoted as $\mathfrak{z} = (\mathfrak{u}_{\mathfrak{k}}, \mathrm{v})$, which is known as a cubic fuzzy set.

Definition 2.5 [18]. The term cubic intuitionistic fuzzy set (CIF-set) $\mathfrak{z}$ defined over U is defined as

$$\mathfrak{z} = \{\langle \mathcal{C}, \mathfrak{u}_{\mathfrak{k}}(\mathfrak{z}), \mathrm{v}\rangle | \mathfrak{z} \epsilon \mathrm{U}\},$$

where $\mathfrak{u}_{\mathfrak{k}}(\mathfrak{z}) = \{\langle \mathcal{C}, \langle [\mathfrak{t}_{\mathfrak{k}}(\mathfrak{z})^{-}, \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z})^{+}], [\mathfrak{f}_{\mathfrak{k}}(\mathfrak{z})^{-}, \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z})^{+}]\rangle\}$, and $\mathrm{v} = [\mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}), \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z})]$ represent the interval-valued IFS-set and intuitionistic set (IS) respectively such that $0 \leq \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}) + \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}) \leq 1.$ Also, $0 \leq \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}), \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}) \leq 1.$

This pair is denoted as $\mathfrak{z} = (\mathfrak{u}, \mathrm{v})$ and termed as a cubic intuitionistic fuzzy set.

Definition 2.6 [18]. The term cubic intuitionistic fuzzy soft set (CIFS-set) $\mathfrak{z}$ defined over U is given as

$$\mathfrak{z} = \{\langle \mathcal{C}_i, \mathfrak{u}(\mathfrak{z}_i), \mathrm{v}_i\rangle | \mathfrak{z}_i \epsilon \mathrm{U}\},$$

where $\mathfrak{u}(\mathfrak{z}_i) = \{\langle \mathcal{C}_i, \langle [\mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}_i)^{-}, \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}_i)^{+}], [\mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}_i)^{-}, \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}_i)^{+}]\rangle\}$, and $\mathrm{v}_i = [\mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}_i), \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}_i)]$ represents the interval-valued IFS-set and intuitionistic set (IS) respectively such that $0 \leq \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}_i) + \mathfrak{f}_{\tilde{\mathrm{A}}}(\mathfrak{z}_i) \leq 1.$ Also, $0 \leq \mathfrak{t}_{\mathfrak{k}}(\mathfrak{z}_i), \mathfrak{f}_{\mathfrak{k}}(\mathfrak{z}_i) \leq 1.$

This pair is denoted as $\mathfrak{z} = (\mathfrak{u}_i, \mathrm{v}_i)$ and termed as cubic intuitionistic fuzzy soft set.

Definition2.7 [18]. Consider

$$\mathfrak{B} = \left\{\begin{matrix} \langle \mathcal{C}_i, \langle [\mathfrak{t}_{\mathfrak{B}}(\mathfrak{z}_i)^{-}, \mathfrak{t}_{\mathfrak{B}}(\mathfrak{z}_i)^{+}], [\mathfrak{f}_{\mathfrak{B}}(\mathfrak{z}_i)^{-}, \mathfrak{f}_{\mathfrak{B}}(\mathfrak{z}_i)^{+}]\rangle, \\ \langle [\mathfrak{t}_{\mathfrak{B}}(\mathfrak{z}_i), \mathfrak{f}_{\mathfrak{B}}(\mathfrak{z}_i)]\rangle \end{matrix}\right\},$$

$\mathrm{y} = \left\{\left\langle \begin{matrix} c_j, \langle[\mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^-, \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^+], [\mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^-, \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^+], \\ \langle \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i), \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)\rangle \end{matrix} \right\rangle\right\}$ be the two CIFS-sets. Then, for $q \geq 1$, the distance measures are defined as

- Distance measures

$$\mathrm{d}_q(\mathrm{ß}, \mathrm{y}) = \Big(\langle \frac{1}{6}\sum_{i=1}^{n}\{|\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i)^- - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^-|^q + |\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i)^+ - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^+|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i)^- - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^-|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i)^+ - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^+|^q + |\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i) - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i) - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)|^q\}\rangle\Big)^{\frac{1}{q}}.$$

- Normalized distance measures

$$\mathrm{d}_q'(\mathrm{ß}, \mathrm{y}) = \Big(\langle \frac{1}{6n}\sum_{i=1}^{n}\{|\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i)^- - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^-|^q + |\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i)^+ - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)^+|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i)^- - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^-|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i)^+ - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)^+|^q + |\mathrm{t}_{\mathrm{ß}}(\mathfrak{z}_i) - \mathrm{t}_{\mathrm{y}}(\mathfrak{z}_i)|^q + |\mathrm{f}_{\mathrm{ß}}(\mathfrak{z}_i) - \mathrm{f}_{\mathrm{y}}(\mathfrak{z}_i)|^q\}\rangle\Big)^{\frac{1}{q}},$$

where n represents the mean of attributive values.

3. EXTENDED TOPSIS TECHNIQUE

In this section, based on the suggested distance measure, a TOPSIS (technique for order of preference by similarity to ideal solution) technique for tackling MAGDM problems in the form of CIFS sets is proposed.

3.1 Description of the Problem

A recently opened restaurant is hiring a new head chef to take charge of the kitchen. For hiring a head chief, they have published the advertisement in the newspaper and different applicants have applied in response. Assume that there is a set of m applicants (alternative) $\mathfrak{z}_i = \{\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3, \ldots, \mathfrak{z}_m\}$ chosen for the interview. The restaurant has gathered decision-makers $\{T^1, T^2, T^3, T^4\}$ and assigned them the duty to identify the best head chef for the restaurant. The selection committee has decided to assess the applicants $\mathfrak{z} = \{\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3, \ldots, \mathfrak{z}_m\}$ based on n different criteria $\mathrm{c} = \{\mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3, \ldots, \mathrm{c}_n\}$. All applicants participated in group talk for this purpose and a panel then developed results for each applicant in the form of CIFS-set criteria $\mathrm{X}_{ij} = (\mu_{ij}, \mathrm{v}_{ij})$, where

$\mu_{ij}(\mathfrak{z}_i) = \langle[\mathrm{t}_{ij}(\mathfrak{z}_i)^-, \mathrm{t}_{ij}(\mathfrak{z}_i)^+], [\mathrm{f}_{ij}(\mathfrak{z}_i)^-, \mathrm{f}_{ij}(\mathfrak{z}_i)^+]\rangle$ and $\mathrm{v}_{ij} = [\mathrm{t}_{ij}(\mathfrak{z}_i), \mathrm{f}_{ij}(\mathfrak{z}_i)]$ represent the interval-valued IFS-set and intuitionistic set

(IS), respectively. Here, the components $[ŧ_{ij}(ʒ_i)^-, ŧ_{ij}(ʒ_i)^+]$ and $ŧ_{ij}(ʒ_i)$ show that satisfaction degree corresponds to a given alternative H_i which satisfies the criterion C_j, whereas the components $[ʄ_{ij}(ʒ_i)^-, ʄ_{ij}(ʒ_i)^+]$ and $ʄ_{ij}(ʒ_i)$ represent the degree of dissatisfaction with the given alternative $ʒ_i$ regarding the criterion $ɗ_j$. Thus, overall results are recorded in the form of a CIFS-set environment and defined in matrix form which is represented as $D = (\mathrm{Ꭓ})_{m\times n}$.

3.1.1. Computing CIFS-PIA and CIFS-NIA. The CIFS-positive ideal alternative (CIFS-PIA) and CIFS-negative ideal alternative (CIFS-NIA) regarding the alternative $ʒ_i (i = 1,2,\dots,m)$ may be selected as 1 and 0, respectively. It shows the ranking values of each alternative. Thus, the ranking values of CIFS-PIA and CIFS-NIA can be represented as

$$\mathrm{Ꭓ}^+ = (\langle[1,1],[0,0]\rangle, \langle 0,1\rangle)_{1\times n} \; and \; \mathrm{Ꭓ}^- = (\langle[0,0],[1,1]\rangle, \langle 1,0\rangle)_{1\times n}.$$

So, $\mathrm{Ꭓ}^+$ and $\mathrm{Ꭓ}^-$ complement each other.

If the decision-maker wants to define the other reference points as

$$\mathrm{Ꭓ}^+{}_j = \left(\langle[t_j^{L+}, t_j^{U+}], [f_j^{L+}, f_j^{U+}]\rangle, \langle p_j^+, q_j^+\rangle\right), \tag{3.1}$$

and

$$\mathrm{Ꭓ}^-{}_j = \left(\langle[t_j^{L-}, t_j^{U-}], [f_j^{L-}, f_j^{U-}]\rangle, \langle p_j^-, q_j^-\rangle\right), \tag{3.2}$$

where

$$t_j^{L+} = \max_j(ŧ_{ij}(ʒ_i)^-), t_j^{U+} = \max_j(ŧ_{ij}(ʒ_i)^+), f_j^{L+} = \min_j(ʄ_{ij}(ʒ_i)^-), f_j^{U+} = \min_j(ʄ_{ij}(ʒ_i)^+),$$

$$p_j^+ = \min_j(ŧ_{ij}(ʒ_i)),\; q_j^+ = \max_j(ʄ_{ij}(ʒ_i))\; \forall i$$

and $t_j^{L-} = \min_j(ŧ_{ij}(ʒ_i)^-), t_j^{U-} = \min_j(ŧ_{ij}(ʒ_i)^+), f_j^{L-} = \max_j(ʄ_{ij}(ʒ_i)^-),\; f_j^{U-} = \max_j(ʄ_{ij}(ʒ_i)^+),\; p_j^- = \max_j\left(ŧ_{ij}(ʒ_i)\right),\; q_j^- = \min_j\left(ʄ_{ij}(ʒ_i)\right) \forall i.$

3.1.2. Computing Distance Measures Between Alternatives. Consider the criteria in terms of the weight vector $w_i = (w_1, w_2, \dots, w_n)$ along with CIFS-PIA($\mathrm{Ꭓ}^+$) and CIFS-NIA($\mathrm{Ꭓ}^-$). We evaluate the weighted distances between the alternatives $ʒ_i$ and its $\mathrm{Ꭓ}^+$ as well as $\mathrm{Ꭓ}^-$ defined as

$$\mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^+) = \Big[\frac{1}{6}\sum_{j=1}^{n} w_j\Big\{\big|t_j^{L+} - \mathfrak{t}_{ij}(\mathfrak{z}_i)^-\big|^q + \big|t_j^{U+} - \mathfrak{t}_{ij}(\mathfrak{z}_i)^+\big|^q + \big|\mathfrak{f}_{ij}(\mathfrak{z}_i)^- - f_j^{L+}\big|^q + \big|\mathfrak{f}_{ij}(\mathfrak{z}_i)^+ - f_j^{U+}\big|^q + \big|\mathfrak{t}_{ij}(\mathfrak{z}_i) - p_j^+\big|^q + \big|q_j^+ - \mathfrak{f}_{ij}(\mathfrak{z}_i)\big|^q\Big\}\Big]^{1/q}, \tag{3.3}$$

and

$$\mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^-) = \Big[\frac{1}{6}\sum_{j=1}^{n} w_j\Big\{\big|\mathfrak{t}_{ij}(\mathfrak{z}_i)^- - t_j^{L-}\big|^q + \big|\mathfrak{t}_{ij}(\mathfrak{z}_i)^+ - t_j^{U-}\big|^q + \big|f_j^{L-} - \mathfrak{f}_{ij}(\mathfrak{z}_i)^-\big|^q + \big|f_j^{U-} - \mathfrak{f}_{ij}(\mathfrak{z}_i)^+\big|^q + \big|p_j^- - \mathfrak{t}_{ij}(\mathfrak{z}_i)\big|^q + \big|\mathfrak{f}_{ij}(\mathfrak{z}_i) - q_j^-\big|^q\Big\}\Big]^{1/q}, \tag{3.4}$$

where $q \geq 1 \in$ R".

The relative closeness coefficient of alternative $\mathfrak{z}_i (i = 1,2, \ldots, m)$ with respect to CIFS-PIA$(\mathfrak{X}^+)$ (which is based on weighted distances) is defined as

$$\mathbb{C}_i = \frac{\mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^-)}{\mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^-) + \mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^+)};\ \mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^+) \neq 0\ . \tag{3.5}$$

Further,

$$0 \leq \mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^-) \leq \mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^-) + \mathrm{d}''_q(\mathfrak{z}_i, \mathfrak{X}^+)\ and\ 0 \leq \mathbb{C}_i \leq 1.$$

3.1.3. Group Decision-making TOPSIS Approach. Using the above analysis, an approach is presented to solve the group decision-making problem under the CIFS-set environment. For this purpose, consider there are $'T'$ decision-makers $\{T^1, T^2, \ldots, T^k\}$ which evaluate the set of 'm' alternatives $\mathfrak{z}_i (i = 1,2, \ldots, m)$ under the set of 'n' different criteria $\mathrm{d} = \{\mathrm{d}_1, \mathrm{d}_2, \mathrm{d}_3, \ldots, \mathrm{d}_n\}$.

These decision-makers give the results in the form of CIFS-set $\left(\mathfrak{X}_{ij}\right)^k = \left((\mu_{ij})^k, (\mathrm{v}_{ij})^k\right)$,

where $(\mu_{ij}(\mathfrak{z}_i))^k$
$= \langle[(\mathfrak{t}_{ij}(\mathfrak{z}_i)^-)^k, (\mathfrak{t}_{ij}(\mathfrak{z}_i)^+)^k], [(\mathfrak{f}_{ij}(\mathfrak{z}_i)^-)^k, (\mathfrak{f}_{ij}(\mathfrak{z}_i)^+)^k]\rangle$ and $(\mathrm{v}_{ij})^k$
$= [\left(\mathfrak{t}_{ij}(\mathfrak{z}_i)\right)^k, (\mathfrak{f}_{ij}(\mathfrak{z}_i)])^k,$ where $K = 1,2, \ldots, k$.

Further, suppose that the criteria of the weight vector $(w_i)^k = ((w_1)^k, (w_2)^k, \ldots, (w_n)^k)^T$ such that each $(w_i)^k > 0$ and $\sum_{i=1}^{n}(w_i)^k = 1$.

Moreover, to overcome the diverse judgments by different decision-makers, their decision is prioritized according to weight vector $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_K)$, such that $\gamma_k > 0$ *and* $\sum_{k=1}^{K} \gamma_k = 1$.

The stepwise algorithm of TOPSIS is presented below.

Step 1: Arrange the ranking values of the alternative given by each decision-maker in the form of a matrix

$$T^{(k)} = \begin{array}{c} \\ ʒ_1 \\ ʒ_2 \\ \vdots \\ ʒ_m \end{array} \begin{bmatrix} ɗ_1 & ɗ_2 & \dots & ɗ_n \\ Ꭓ_{11}{}^{(k)} & Ꭓ_{12}{}^{(k)} & \dots & Ꭓ_{1n}{}^{(k)} \\ Ꭓ_{21}{}^{(k)} & Ꭓ_{22}{}^{(k)} & \dots & Ꭓ_{2n}{}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ Ꭓ_{m1}{}^{(k)} & Ꭓ_{m2}{}^{(k)} & \dots & Ꭓ_{mn}{}^{(k)} \end{bmatrix}$$

Step 2: For each decision-maker $T^k = \{T^1, T^2, \dots, T^K\}$, compute CIFS-PIA and CIFS-NIA corresponding to the applicants $ʒ_i (i = 1,2, \dots, m)$ using the **Eq.** (**3.4**) and (**3.5**) respectively and defined as

$$(Ꭓ^+{}_j)^k = \left(\left\langle\left[(t_j^{L+})^k, (t_j^{U+})^k\right], \left[(f_j^{L+})^k, (f_j^{U+})^k\right]\right\rangle, \left\langle(p_j^+)^k, (q_j^+)^k\right\rangle\right) \qquad (3.6)$$

and

$$(Ꭓ^-{}_j)^k = \left(\left\langle\left[(t_j^{L-})^k, (t_j^{U-})^k\right], \left[(f_j^{L-})^k, (f_j^{U-})^k\right]\right\rangle, \left\langle(p_j^-)^k, (q_j^-)^k\right\rangle\right), \qquad (3.7)$$

where

$$\begin{aligned}(t_j^{L+})^k = \max_j\left\{(ƭ_{ij}(ʒ_i)^-)^{(k)}\right\}, (t_j^{U+})^k = \max_j\left\{(ƭ_{ij}(ʒ_i)^+)^{(k)}\right\}, (f_j^{L+})^k \\ = \min_j\left\{(ʄ_{ij}(ʒ_i)^-)^{(k)}\right\}, (f_j^{U+})^k = \min_j\left\{(ʄ_{ij}(ʒ_i)^+)^{(k)}\right\}, \\ (p_j^+)^k = \min_j\left\{(ƭ_{ij}(ʒ_i))^{(k)}\right\}, (q_j^+)^k = \max_j\left\{(ʄ_{ij}(ʒ_i))^{(k)}\right\} \forall i\end{aligned}$$

and

$$\begin{aligned}(t_j^{L-})^k = \min_j\left\{(ƭ_{ij}(ʒ_i)^-)^{(k)}\right\}, (t_j^{U-})^k = \min_j\left\{(ƭ_{ij}(ʒ_i)^+)^{(k)}\right\}, (f_j^{L-})^k \\ = \max_j\left\{(ʄ_{ij}(ʒ_i)^-)^{(k)}\right\}, (f_j^{U-})^k = \max_j\left\{(ʄ_{ij}(ʒ_i)^+)^{(k)}\right\}, \\ (p_j^-)^k = \max_j\left\{\left(ƭ_{ij}(ʒ_i)\right)^{(k)}\right\}, (q_j^-)^k \\ = \min_j\left\{\left(ʄ_{ij}(ʒ_i)\right)^{(k)}\right\} \forall i.\end{aligned}$$

Step 3: For each decision-maker, compute the separation measures between the alternatives $\mathfrak{z}_i$ from its CIFS-PIA$(\mathfrak{X}^+)$ and CIFS-NIA$(\mathfrak{X}^-)$ which are denoted by $\mathrm{d}_q''((\mathfrak{z}_i)^k, (\mathfrak{X}^+)^k)$ and $\mathrm{d}_q''((\mathfrak{z}_i)^k, (\mathfrak{X}^-)^k)$, respectively.

Step 4: For each decision-maker, the relative closeness coefficient is determined as

$$\mathbb{C}_i{}^{(k)} = \frac{\mathrm{d}_q''((\mathfrak{z}_i)^k,(\mathfrak{X}^-)^k)}{\mathrm{d}_q''((\mathfrak{z}_i)^k,(\mathfrak{X}^+)^k) + \mathrm{d}_q''((\mathfrak{z}_i)^k,(\mathfrak{X}^-)^k)}; \quad k = 1,2,\ldots,K \tag{3.8}$$

where $\mathrm{d}_q''((\mathfrak{z}_i)^k, (\mathfrak{X}^+)^k) \neq 0$.

Step 5: Rank the alternatives based on the descending values of $\mathbb{C}_i$s

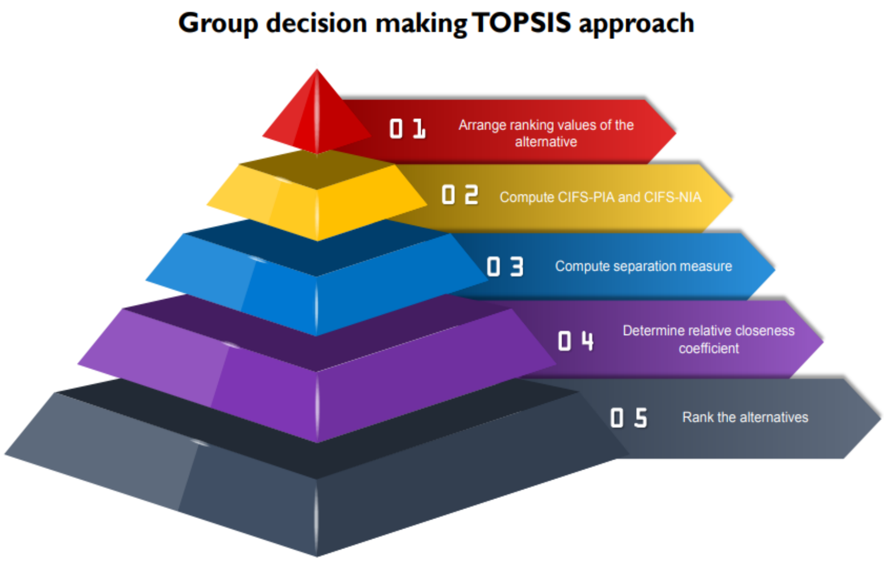

Figure 1. TOPSIS Algorithm

Example: To demonstrate the above-mentioned approach depicted in Figure 1, an example is discussed below.

4. RESULTS

4.1 Case Study

A recently opened restaurant is hiring a new head chef to take charge of the kitchen. For hiring a head chef, they have published the advertisement in the newspaper and different applicants have applied in response. A total of four individuals $\mathfrak{z}_i; i = 1,2,3,4$ have been chosen for the interview. The restaurant has gathered decision-makers $\{T^1, T^2, T^3, T^4\}$ and given them the responsibility to identify the best head chef for the restaurant. The selection committee has decided to assess the applicants $\mathfrak{z}_i; i = 1,2,3,4$ based on four criteria $\mathfrak{d} = \{\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4\}$ defined as $\mathfrak{d}_1: cooking\ experience\ \mathfrak{d}_2: time\ management, \mathfrak{d}_3: presentable, \mathfrak{d}_4: team\ handling.$ For the assessment, they conducted group discussion with all the applicants and the results are formulated by a panel in the form of an IVIFS set. From lots of applicants appearing for group discussion, only four applicants are shortlisted for the interview. At this stage, the results are recorded in the form of an IFS set. Then, the following steps of the proposed approach are executed to find the best head chef for the kitchen.

4.1.1. Step 1: Ranking Values of Alternatives Corresponding to Each Decision-maker

Table 1. Ranking Values of Alternatives Corresponding to $T^{(1)}$ Decision-maker

Decision maker	Applicants and Weighs	ɗ$_1$	ɗ$_2$	ɗ$_3$	ɗ$_4$
$T^{(1)}$	ʒ$_1$	(⟨[0.30,0.50], [0.20,0.40]⟩ ,⟨0.35,0.29⟩)	(⟨[0.25,0.35], [0.10,0.30]⟩, ⟨0.19,0.27⟩)	(⟨[0.32,0.40], [0.38,0.49]⟩, ⟨0.35,0.49⟩)	(⟨[0.40,0.44], [0.50,0.52]⟩, ⟨0.55,0.35⟩)
	ʒ$_2$	(⟨[0.15,0.30], [0.35,0.40]⟩ ,⟨0.17,0.40⟩)	(⟨[0.15,0.18], [0.20,0.30]⟩, ⟨0.29,0.63⟩)	(⟨[0.30,0.40], [0.35,0.42]⟩, ⟨0.40,0.40⟩)	(⟨[0.55,0.59], [0.18,0.33]⟩, ⟨0.16,0.20⟩)
	ʒ$_3$	(⟨[0.39,0.48], [0.10,0.18]⟩, ⟨0.20,0.15⟩)	(⟨[0.23,0.32], [0.47,0.50]⟩, ⟨0.30,0.60⟩)	(⟨[0.40,0.45], [0.20,0.25]⟩, ⟨0.29,0.19⟩)	(⟨[0.37,0.42], [0.40,0.50]⟩, ⟨0.30,0.70⟩)
	ʒ$_4$	(⟨[0.42,0.55], [0.30,0.45]⟩, ⟨0.15,0.30⟩)	(⟨[0.44,0.50], [0.19,0.25]⟩ ,⟨0.42,0.20⟩)	(⟨[0.16,0.32], [0.18,0.22]⟩, ⟨0.17,0.34⟩)	(⟨[0.20,0.55], [0.38,0.49]⟩, ⟨0.80,0.17⟩)
	weights	0.40	0.20	0.25	0.15

Table 2. Ranking Values of Alternatives Corresponding to $\boldsymbol{T^{(2)}}$ Decision-maker

Decision maker	Applicants and Weighs	ɗ$_1$	ɗ$_2$	ɗ$_3$	ɗ$_4$
$T^{(2)}$	ʒ$_1$	(⟨[0.35,0.45], [0.48,0.54]⟩, ⟨0.60,0.27⟩)	(⟨[0.40,0.60], [0.15,0.20]⟩ ,⟨0.35,0.19⟩)	(⟨[0.16,0.35], [0.24,0.32]⟩, ⟨0.32,0.20⟩)	(⟨[0.11,0.20], [0.16,0.35]⟩, ⟨0.40,0.17⟩)
	ʒ$_2$	(⟨[0.15,0.30], [0.35,0.40]⟩ ,⟨0.20,0.70⟩)	(⟨[0.60,0.70], [0.19,0.25]⟩, ⟨0.60.0.10⟩)	(⟨[0.30,0.45], [0.21,0.35]⟩, ⟨0.50,0.30⟩)	(⟨[0.40,0.50], [0.10,0.20]⟩, ⟨0.40,0.30⟩)
	ʒ$_3$	(⟨[0.19,0.39], [0.10,0.29]⟩, ⟨0.11,0.20⟩)	(⟨[0.20,0.35], [0.17,0.36]⟩, ⟨0.10,0.40⟩)	(⟨[0.10,0.18], [0.12,0.15]⟩, ⟨0.20,0.35⟩)	(⟨[0.19,0.27], [0.25,0.32]⟩, ⟨0.20,0.70⟩)
	ʒ$_4$	(⟨[0.10,0.29], [0.18,0.36]⟩, ⟨0.30,0.42⟩)	(⟨[0.15,0.40], [0.16,0.32]⟩, ⟨0.30,0.63⟩)	(⟨[0.47,0.50], [0.32,0.42]⟩, ⟨0.15,0.29⟩)	(⟨[0.25,0.29], [0.40,0.44]⟩, ⟨0.66,0.18⟩)
	weights	0.35	0.30	0.15	0.20

Table 3. Ranking Values of Alternatives Corresponding to $\boldsymbol{T^{(3)}}$ Decision-maker

Decision maker	Applicants & Weighs	d_1	d_2	d_3	d_4
$T^{(1)}$	$\mathfrak{z}_1$	(⟨[0.30,0.50], [0.20,0.40]⟩ ,⟨0.35,0.29⟩)	(⟨[0.25,0.35], [0.10,0.30]⟩, ⟨0.19,0.27⟩)	(⟨[0.32,0.40], [0.38,0.49]⟩, ⟨0.35,0.49⟩)	(⟨[0.40,0.44], [0.50,0.52]⟩, ⟨0.55,0.35⟩)
	$\mathfrak{z}_2$	(⟨[0.15,0.30], [0.35,0.40]⟩ ,⟨0.17,0.40⟩)	(⟨[0.15,0.18], [0.20,0.30]⟩, ⟨0.29,0.63⟩)	(⟨[0.30,0.40], [0.35,0.42]⟩, ⟨0.40,0.40⟩)	(⟨[0.55,0.59], [0.18,0.33]⟩, ⟨0.16,0.20⟩)
	$\mathfrak{z}_3$	(⟨[0.39,0.48], [0.10,0.18]⟩, ⟨0.20,0.15⟩)	(⟨[0.23,0.32], [0.47,0.50]⟩, ⟨0.30,0.60⟩)	(⟨[0.40,0.45], [0.20,0.25]⟩, ⟨0.29,0.19⟩)	(⟨[0.37,0.42], [0.40,0.50]⟩, ⟨0.30,0.70⟩)
	$\mathfrak{z}_4$	(⟨[0.42,0.55], [0.30,0.45]⟩, ⟨0.15,0.30⟩)	(⟨[0.44,0.50], [0.19,0.25]⟩ ,⟨0.42,0.20⟩)	(⟨[0.16,0.32], [0.18,0.22]⟩, ⟨0.17,0.34⟩)	(⟨[0.20,0.55], [0.38,0.49]⟩, ⟨0.80,0.17⟩)
	weights	0.40	0.20	0.25	0.15

Table 4. Ranking Values of Alternatives Corresponding to $\boldsymbol{T^{(4)}}$ Decision-maker

Decision maker	Applicants & Weighs	d_1	d_2	d_3	d_4
$T^{(4)}$	$\mathfrak{z}_1$	(⟨[0.20,0.35], [0.48,0.54]⟩, ⟨0.30,0.45⟩)	(⟨[0.36,0.40], [0.11,0.50]⟩, ⟨0.45,0.20⟩)	(⟨[0.30,0.55], [0.24,0.32]⟩, ⟨0.10,0.30⟩)	(⟨[0.11,0.20], [0.16,0.29]⟩, ⟨0.35,0.19⟩)
	$\mathfrak{z}_2$	(⟨[0.18,0.30], [0.25,0.40]⟩, ⟨0.20,0.70⟩)	(⟨[0.18,0.30], [0.19,0.34]⟩, ⟨0.40,0.20⟩)	(⟨[0.15,0.35], [0.20,0.60]⟩, ⟨0.25,0.40⟩)	(⟨[0.40,0.20], [0.10,0.50]⟩, ⟨0.50,0.20⟩)
	$\mathfrak{z}_3$	(⟨[0.10,0.29], [0.15,0.35]⟩, ⟨0.11,0.30⟩)	(⟨[0.20,0.30], [0.11,0.60]⟩, ⟨0.30,0.50⟩)	(⟨[0.10,0.29], [0.40,0.50]⟩, ⟨0.30,0.40⟩)	(⟨[0.29,0.37], [0.25,0.35]⟩, ⟨0.25,0.60⟩)
	$\mathfrak{z}_4$	(⟨[0.25,0.32], [0.40,0.45]⟩, ⟨0.20,0.32⟩)	(⟨[0.16,0.20], [0.25,0.32]⟩, ⟨0.10,0.63⟩)	(⟨[0.45,0.60], [0.20,0.25]⟩, ⟨0.29,0.63⟩)	(⟨[0.21,0.39], [0.20,0.49]⟩, ⟨0.66,0.28⟩)
	weights	0.45	0.19	0.25	0.11

4.1.2. Step 2: Computing CIFS-PIA and CIFS-NIA for Each Decision-maker. Computing the CIFS-PIA and CIFS-NIA for each decision-maker is depicted below in Table 5.

Using the **Eq.** (**3.1**) and **(3.2**), CIFS-PIA and CIFS-NIA are computed respectively as follows:

$$(Ꝁ^{+}{}_{j})^{k} = \left(\langle\left[(t_j^{L+})^k, (t_j^{U+})^k\right], \left[(f_j^{L+})^k, (f_j^{U+})^k\right]\rangle, \langle(p_j^{+})^k, (q_j^{+})^k\rangle\right)$$

and

$$(Ꝁ^{-}{}_{j})^{k} = \left(\langle\left[(t_j^{L-})^k, (t_j^{U-})^k\right], \left[(f_j^{L-})^k, (f_j^{U-})^k\right]\rangle, \langle(p_j^{-})^k, (q_j^{-})^k\rangle\right),$$

where

$$(t_j^{L+})^k = \max_j\left\{(ŧ_{ij}(ʒ_i)^{-})^{(k)}\right\}, (t_j^{U+})^k = \max_j\left\{(ŧ_{ij}(ʒ_i)^{+})^{(k)}\right\}, (f_j^{L+})^k = \min_j\left\{(ƒ_{ij}(ʒ_i)^{-})^{(k)}\right\}, (f_j^{U+})^k = \min_j\left\{(ƒ_{ij}(ʒ_i)^{+})^{(k)}\right\},$$
$$(p_j^{+})^k = \min_j\left\{(ŧ_{ij}(ʒ_i))^{(k)}\right\}, (q_j^{+})^k = \max_j\left\{(ƒ_{ij}(ʒ_i))^{(k)}\right\} \forall i$$

and

$$(t_j^{L-})^k = \min_j\left\{(ŧ_{ij}(ʒ_i)^{-})^{(k)}\right\}, (t_j^{U-})^k = \min_j\left\{(ŧ_{ij}(ʒ_i)^{+})^{(k)}\right\}, (f_j^{L-})^k = \max_j\left\{(ƒ_{ij}(ʒ_i)^{-})^{(k)}\right\}, (f_j^{U-})^k = \max_j\left\{(ƒ_{ij}(ʒ_i)^{+})^{(k)}\right\},$$
$$(p_j^{-})^k = \max_j\left\{\left(ŧ_{ij}(ʒ_i)\right)^{(k)}\right\}, (q_j^{-})^k = \min_j\left\{\left(ƒ_{ij}(ʒ_i)\right)^{(k)}\right\} \forall i.$$

Table 5. Positive and Negative Ideal for Each Decision-maker

DM	PIA / NIA	$ɗ_1$	$ɗ_2$	$ɗ_3$	$ɗ_4$
$T^{(1)}$	$Ꝁ^{+}{}_{1}$	$(\langle[0.42,0.50],[0.10,0.18]\rangle, \langle0.17,0.40\rangle)$	$(\langle[0.44,0.50],[0.10,0.25]\rangle, \langle0.19,0.63\rangle)$	$(\langle[0.40,0.45],[0.18,0.22]\rangle, \langle0.17,0.49\rangle)$	$(\langle[0.55,0.59],[0.18,0.33]\rangle, \langle0.16,0.70\rangle)$
	$Ꝁ^{-}{}_{1}$	$(\langle[0.15,0.30],[0.35,0.45]\rangle, \langle0.35,0.15\rangle)$	$(\langle[0.15,0.18],[0.47,0.50]\rangle, \langle0.42,0.20\rangle)$	$(\langle[0.16,0.32],[0.38,0.49]\rangle, \langle0.40,0.19\rangle)$	$(\langle[0.20,0.42],[0.50,0.52]\rangle, \langle0.80,0.17\rangle)$

DM	PIA NIA	d_1	d_2	d_3	d_4
$T^{(2)}$	$X^{+}{}_2$	$\left(\langle[0.35,0.45],[0.10,0.29]\rangle,\langle0.11,0.70\rangle\right)$	$\left(\langle[0.60,0.70],[0.15,0.20]\rangle,\langle0.10,0.63\rangle\right)$	$\left(\langle[0.47,0.50],[0.12,0.15]\rangle,\langle0.15,0.35\rangle\right)$	$\left(\langle[0.40,0.50],[0.10,0.20]\rangle,\langle0.20,0.70\rangle\right)$
	$X^{-}{}_2$	$\left(\langle[0.10,0.29],[0.48,0.54]\rangle,\langle0.60,0.20\rangle\right)$	$\left(\langle[0.15,0.35],[0.19,0.36]\rangle,\langle0.60,0.10\rangle\right)$	$\left(\langle[0.10,0.18],[0.32,0.42]\rangle,\langle0.50,0.20\rangle\right)$	$\left(\langle[0.11,0.20],[0.40,0.44]\rangle,\langle0.66,0.17\rangle\right)$
$T^{(3)}$	$X^{+}{}_3$	$\left(\langle[0.30,0.50],[0.10,0.30]\rangle,\langle0.20,0.35\rangle\right)$	$\left(\langle[0.30,0.38],[0.20,0.30]\rangle,\langle0.11,0.40\rangle\right)$	$\left(\langle[0.32,0.60],[0.30,0.30]\rangle,\langle0.11,0.70\rangle\right)$	$\left(\langle[0.32,0.45],[0.10,0.20]\rangle,\langle0.30,0.40\rangle\right)$
	$X^{-}{}_3$	$\left(\langle[0.10,0.35],[0.40,0.45]\rangle,\langle0.45,0.10\rangle\right)$	$\left(\langle[0.10,0.15],[0.40,0.45]\rangle,\langle0.60,0.10\rangle\right)$	$\left(\langle[0.10,0.40],[0.50,0.50]\rangle,\langle0.25,0.32\rangle\right)$	$\left(\langle[0.10,0.29],[0.50,0.50]\rangle,\langle0.45,0.20\rangle\right)$
$T^{(4)}$	$X^{+}{}_4$	$\left(\langle[0.25,0.35],[0.15,0.35]\rangle,\langle0.11,0.70\rangle\right)$	$\left(\langle[0.36,0.40],[0.11,0.32]\rangle,\langle0.10,0.63\rangle\right)$	$\left(\langle[0.30,0.55],[0.20,0.25]\rangle,\langle0.10,0.63\rangle\right)$	$\left(\langle[0.40,0.39],[0.10,0.29]\rangle,\langle0.25,0.60\rangle\right)$
	$X^{-}{}_4$	$\left(\langle[0.10,0.29],[0.40,0.54]\rangle,\langle0.30,0.30\rangle\right)$	$\left(\langle[0.16,0.20],[0.25,0.60]\rangle,\langle0.45,0.20\rangle\right)$	$\left(\langle[0.10,0.29],[0.40,0.60]\rangle,\langle0.30,0.30\rangle\right)$	$\left(\langle[0.11,0.20],[0.25,0.50]\rangle,\langle0.66,0.19\rangle\right)$

4.1.3. Step 3: Evaluation of Separation Measures. Without the loss of generality, assuming q=2 separation measures are calculated between the applicants from their CIFS-PIA and CIFS-NIA corresponding to each decision-maker and denoted by $\mathrm{d}_{\boldsymbol{q}}''\left((\mathfrak{z}_{\boldsymbol{i}})^{\boldsymbol{k}},(X^{+})^{\boldsymbol{k}}\right)$ **and** $\mathrm{d}_{\boldsymbol{q}}''\left((\mathfrak{z}_{\boldsymbol{i}})^{\boldsymbol{k}},(X^{-})^{\boldsymbol{k}}\right)$ respectively, as depicted in Table 6.

Using **Eqs.** (**3.6**) and (**3.7**)

$$\mathrm{d}_q''(\mathfrak{z}_i,X^{+}) = \left[\frac{1}{6}\sum_{j=1}^{n} w_j\left\{\left|t_j^{L+}-\mathfrak{t}_{ij}(\mathfrak{z}_i)^{-}\right|^q+\left|t_j^{U+}-\mathfrak{t}_{ij}(\mathfrak{z}_i)^{+}\right|^q+\left|\mathfrak{f}_{ij}(\mathfrak{z}_i)^{-}-f_j^{L+}\right|^q+\left|\mathfrak{f}_{ij}(\mathfrak{z}_i)^{+}-f_j^{U+}\right|^q+\left|\mathfrak{t}_{ij}(\mathfrak{z}_i)-p_j^{+}\right|^q+\left|q_j^{+}-\mathfrak{f}_{ij}(\mathfrak{z}_i)\right|^q\right\}\right]^{1/q}$$

and

$$\mathrm{d}_q''(\mathfrak{z}_i,X^{-}) = \left[\frac{1}{6}\sum_{j=1}^{n} w_j\left\{\left|\mathfrak{t}_{ij}(\mathfrak{z}_i)^{-}-t_j^{L-}\right|^q+\left|\mathfrak{t}_{ij}(\mathfrak{z}_i)^{+}-t_j^{U-}\right|^q+\left|f_j^{L-}-\mathfrak{f}_{ij}(\mathfrak{z}_i)^{-}\right|^q+\left|f_j^{U-}-\mathfrak{f}_{ij}(\mathfrak{z}_i)^{+}\right|^q+\left|p_j^{-}-\mathfrak{t}_{ij}(\mathfrak{z}_i)\right|^q+\left|\mathfrak{f}_{ij}(\mathfrak{z}_i)-q_j^{-}\right|^q\right\}\right]^{1/q}.$$

Solving these separation measures corresponding to each decision-maker, we get

Table 6. Evaluation of Separation Measures

Applicants	$T^{(1)}$		$T^{(2)}$		$T^{(3)}$		$T^{(4)}$	
	$T_i^{(1+)}$	$T_i^{(1-)}$	$T_i^{(2+)}$	$T_i^{(2-)}$	$T_i^{(3+)}$	$T_i^{(3-)}$	$T_i^{(4+)}$	$T_i^{(4-)}$
$ʒ_1$	0.3553	0.3102	0.4961	0.3927	0.2703	0.2501	0.3651	0.2461
$ʒ_2$	0.3927	0.4116	0.4087	0.4672	0.2180	0.2151	0.2583	0.2076
$ʒ_3$	0.3784	0.3478	0.2645	0.4783	0.2158	0.2426	0.2764	0.2123
$ʒ_4$	0.4067	0.3600	0.3538	0.3330	0.2058	0.2037	0.1947	0.1582

4.1.4. Step 4: Computing the Relative Closeness Coefficient. Using Eq. (3.8), the relative closeness coefficient and the ranking order of the applicants corresponding to each decision-maker is computed as

$$\mathbb{C}_{\boldsymbol{i}}{}^{(k)} = \frac{\mathrm{d}_q''\left((ʒ_i)^k,(Ꭓ^-)^k\right)}{\mathrm{d}_q''\left((ʒ_i)^k,(Ꭓ^+)^k\right)+ \mathrm{d}_q''\left((ʒ_i)^k,(Ꭓ^-)^k\right)}; \;\; k = 1,2,\dots,K$$

where $\mathrm{d}_q''\left((ʒ_i)^k,(Ꭓ^+)^k\right) \neq 0$.

The outcome is manifested in Table 7.

Table 7. Relative Closeness

Applicants	$\boldsymbol{T}^{(\mathbf{1})}$	$\boldsymbol{T}^{(\mathbf{2})}$	$\boldsymbol{T}^{(\mathbf{3})}$	$\boldsymbol{T}^{(\mathbf{4})}$
	$\mathbb{C}_{\boldsymbol{i}}{}^{(1)}$	$\mathbb{C}_{\boldsymbol{i}}{}^{(2)}$	$\mathbb{C}_{\boldsymbol{i}}{}^{(3)}$	$\mathbb{C}_{\boldsymbol{i}}{}^{(4)}$
$ʒ_1$	0.4661	0.4418	0.4806	0.4027
$ʒ_2$	0.5117	0.5334	0.4967	0.4456
$ʒ_3$	0.4789	0.6439	0.5292	0.4344
$ʒ_4$	0.4695	0.4849	0.4974	0.4483

4.1.5. Step 5: Ranking of Alternatives. Using the relative closeness coefficient, the ranking of alternatives corresponding to each decision-maker is manifested in Table 8.

Table 8. Ranking of Alternatives

Applicants	$T^{(1)}$		$T^{(2)}$		$T^{(3)}$		$T^{(4)}$	
	$\mathbb{C}_{\boldsymbol{i}}{}^{(1)}$	Ranking	$\mathbb{C}_{\boldsymbol{i}}{}^{(2)}$	Ranking	$\mathbb{C}_{\boldsymbol{i}}{}^{(3)}$	Ranking	$\mathbb{C}_{\boldsymbol{i}}{}^{(4)}$	Ranking
$ʒ_1$	0.4661	4	0.4418	4	0.4806	4	0.4027	4
$ʒ_2$	0.5117	1	0.5334	2	0.4967	3	0.4456	2

Applicants	$T^{(1)}$		$T^{(2)}$		$T^{(3)}$		$T^{(4)}$	
	$\mathbb{C}_i^{(1)}$	Ranking	$\mathbb{C}_i^{(2)}$	Ranking	$\mathbb{C}_i^{(3)}$	Ranking	$\mathbb{C}_i^{(4)}$	Ranking
$\mathfrak{z}_3$	0.4789	2	0.6439	1	0.5292	1	0.4344	3
$\mathfrak{z}_4$	0.4695	3	0.4849	3	0.4974	2	0.4483	1

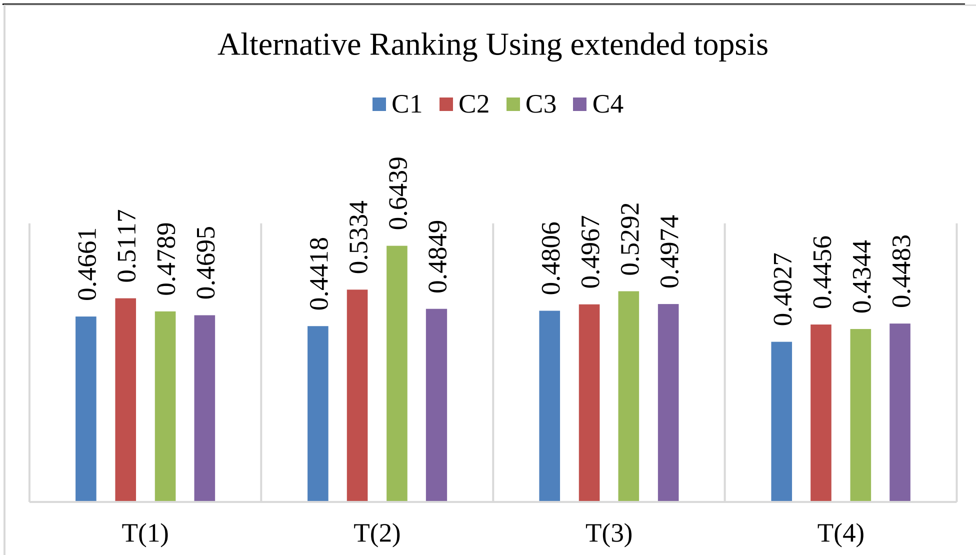

Figure 2. Alternative Ranking using Extended TOPSIS Method

Table 9. Final Scores of Alternatives

	$T^{(1)}$	$T^{(2)}$	$T^{(3)}$	$T^{(4)}$
$\mathbb{C}_i^{(1)}$	0.4661	0.4418	0.4806	0.4027
$\mathbb{C}_i^{(2)}$	0.5117	0.5334	0.4967	0.4456
$\mathbb{C}_i^{(3)}$	0.4789	0.6439	0.5292	0.4344
$\mathbb{C}_i^{(4)}$	0.4695	0.4849	0.4974	0.4483

5. DISCUSSION

The problem of the selection of the head chef at a famous restaurant has been solved by applying the extended TOPSIS method based on the suggested distance and similarities measure. Based on the proposed method, four decision-makers were selected to select the best head chef to take charge of the kitchen. Each decision-maker assigned the rating values that corresponded to each attribute and alternative, as shown in tables 1-4. Using CIF-PIA and CIF-NIA, the separation measure between the alternatives was computed, as shown in tables 5-6. Then, the last relative closeness coefficient was calculated, as shown in Table 7. Each value of the relative closeness coefficient was arranged in descending order and all alternatives were ranked corresponding to each decision-maker, as shown in Table 8 and Figure 2. According to $\mathsf{T}^{(1)}$ decision-maker, $\mathfrak{z}_2$ remains the PIA for the selection of head chef. According to $\mathsf{T}^{(2)}$ decision-maker, $\mathfrak{z}_3$ is the PIA for the selection of head chef. According to $\mathsf{T}^{(3)}$ decision-maker, $\mathfrak{z}_3$ is the PIA for the selection of head chef and according to $\mathsf{T}^{(4)}$ decision-maker, $\mathfrak{z}_4$ is the PIA for the selection of head chef. However, if only one head chef needs to be selected, then the highest ranking PIA should be arranged in descending order and the best head chef should be chosen to take charge of the kitchen.

5.1.Conclusion

In this project, the issue of MCGDM under the CIFS-set environment has been discussed. An adaptation of the TOPSIS method has been illustrated to show the effectiveness of the proposed operators. According to the findings, these decision-making techniques can represent uncertainty more effectively than the current approaches and provide us with a comprehensive understanding of real-life scenarios. The problem of the selection of head chef for the newly opened restaurant has been dealt with in this research. The proposed approach of TOPSIS yields the best head chef. The findings of this research can be applied in the future to hypersoft set, interval-valued soft set, bi-polar soft set, and other ambiguous environments.

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