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On Reverse Super Edge Magic Total Labeling of Subdivided Trees

Abdul Raheem
Department of Mathematics,
National University of Singapore, Singapore 119076

rahimciit7@gmail.com

Abstract

A reverse edge magic total labeling of a graph G is a one-to-one map $\lambda : V(G) \cup E(G) \rightarrow \{1,2, \ldots, |V(G) \cup E(G)|\} = [1, |V(G) \cup E(G)|]$ with the property that there is an integer constant k such that $\{\lambda(xy) - (\lambda(x) + \lambda(y))/xy \in V(G)\} = k$. If $(V(G)) = [1, |V(G)|]$ then the reverse edge magic labeling is called reverse super edge-magic labeling. In this paper we will formulate the reverse edge magic labeling of two subclasses of trees.

***Keywords*:** star, subdivision of star, reverse super edge magic labeling

Introduction

All graphs in this paper are finite, undirected, simple and planar. The graph G has the vertex set V(G) and the edge set E(G). A (v, e) graph G is a graph such that |V(G) |= v and | E(G) |= e. A general reference for graph-theoretic ideas can be seen in [1,2]. A labeling (or valuation) of a graph is a map that converts graph elements into numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is known as total labeling. Some labeling alternatively uses either the vertex-set or the edge-set and we shall address them as vertex-labeling or edge-labeling, respectively.

Definition 1.1. A reverse edge magic total labeling of a graph G is a one-to-one map $\lambda : V(G) \cup E(G) \rightarrow \{1,2, \ldots, |V(G) \cup E(G)|\} = [1, |V(G) \cup E(G)|]$ with the property that there is an integer constant k such that $\{\lambda(xy) - (\lambda(x) + \lambda(y))/xy \in V(G)\} = k$.

Definition 1.2. λ is called the reverse super edge magic total labeling and G is known as a reverse super edge magic total graph. Enomoto *et al.* [3] proposed the following conjecture,

Conjecture 1.1. *Every tree admits a super edge magic total labeling.*

In favor of this conjecture, many authors have considered a super edge magic total labeling for different classes of trees. For detailed studies the reader can see [4–14].

Definition 1.3. Let $n_i \geq 1, 1 \leq i \leq r$ and $r \geq 2$. A subdivided star $Sb(n_1, n_2, \dots, n_r)$ is a tree obtained by inserting $n_i - 1$ nodes to each of the ith edge of the star $K_{1,r}$. Let us define the set of nodes and edges as follows,

$V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r;\ 1 \leq l \leq n_i\}$ and $E(G) = \{cx_i^{l} | 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} | 1 \leq i \leq r;\ 1 \leq l \leq n_i - 1\}$.

However, the investigation of different results related to a reverse super edge magic total labeling of the subdivided star $Sb(n_1, n_2, \dots, n_r)$ for $n_1 \neq n_2, \dots, \neq\ n_r$ is still an open problem. In this paper, we will formulate a reverse super edge magic total labeling of the subclasses of subdivided stars denoted by $Sb(mn, mn, mn, 2mn, n_6, n_r, \dots, n_r)$ and $Sb(mn, mn. 2n, 2n + 2, 4n + 3, n_6, \dots, n_r)$ under certain conditions.

Let us consider the following Lemma which we will use frequently in the main theorems.

Lemma 1.1. A graph with vertices v and e edges is reverse super edge magic total labeling if and only if there exists a bijective function $\lambda: V(G) \rightarrow [1, v]$ such that the set consists of consecutive integers. In such a case, λ extends to a reverse super edge magic total labeling of G with magic constant $k = 2v - s - 1$, where $s = \max(S)$.

2. Main Results

In this section, we will prove the main results related to a reverse super edge magic total labeling of the more generalized subclasses of subdivided trees.

Theorem 2.1. The graph $G \cong Sb(n, n, n, n, 2n, n_6, \dots, n_r)$ admits the reverse edge magic total labeling for any odd $n \geq 3$, $r \geq 6$, $n_p = 2^{p-4}n - 2p + 11$ and $6 \leq p \leq r$

Proof: Let $v = |V(G)|$ and $e = |E(G)|$ then

$v = 6n + 1 + \sum_{t=6}^{r}[2^{t-4}n - 2t]$ and $e = v - 1$

Let us define $\lambda : V(G) \rightarrow [1, v]$ as follows,

$$\lambda(c) = (4n + 2) + \sum_{t=6}^{r}[2^{t-5}n - t + 6]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $1 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \frac{l_1+1}{2}, & if\ w = {x_1}^{l_1} \\ n+1-\frac{l_2-1}{2}, & if\ w = {x_2}^{l_2} \\ (n+2)+\frac{l_3-1}{2}, & if\ w = {x_3}^{l_3} \\ 2(n+1)+\frac{l_4-1}{2}, & if\ w = {x_4}^{l_4} \\ (3n+2)-\frac{l_5-1}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = (3n+2) + \sum_{t=6}^{i}[2^{t-5}n - t + 6] - \frac{l_i-1}{2}$ accordingly.

For even $1 \le l \le n_i$ and $\gamma = (3n+2) + \sum_{t=6}^{r}[2^{t-6}n + 1]$, where $1 \le i \le 5$ and for $6 \le i \le r$:

$$\lambda(w) = \begin{cases} (\gamma+1)\frac{l_1-2}{2}, & if\ w = {x_1}^{l_1} \\ (\gamma+n-1)-\frac{l_2-2}{2}, & if\ w = {x_2}^{l_2} \\ (\gamma+n+1)+\frac{l_3-2}{2}, & if\ w = {x_3}^{l_3} \\ (\gamma+2n-1)+\frac{l_4-2}{2}, & if\ w = {x_4}^{l_4} \\ (\gamma+3n-1)-\frac{l_5-2}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = (\gamma+3n-1) + \sum_{t=6}^{i}\ [2^{t-5}(3n) - 2t + 11] - \frac{l_i-2}{2}$ Accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma+2, \gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant $k = (3n-2) + \sum_{t=6}^{i}[2^{t-6}(3n) - 2t + 10]$.

Theorem 2.2. The graph $G \cong T(3n, 3n, 3n, 3n, 6n, n_6, \ldots, n_r)$ admits the reverse edge magic total labeling with $k = 2v - s - 1$ for any odd $n \ge 3$, $r \ge 6$, $n_p = 2^{t-6}(3n) - 2t + 11$ and $6 \le p \le r$

Proof: Let $v = |V(G)|$ and $e = |E(G)|$ then

$v = (18n+1) + \sum_{t=6}^{r}[2^{t-4}(3n) - 2t + 11]$ and $e = v - 1$

Let us define $\lambda: V(G) \to [1, v]$ as follows,

$$\lambda(c) = (12n+2) + \sum_{t=6}^{r}[2^{t-5}(3n) - t + 6]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $1 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \frac{l_1+1}{2}, & if\ w = x_1^{l_1} \\ 3n+1-\frac{l_2-1}{2}, & if\ w = x_2^{l_2} \\ (3n+2)+\frac{l_3-1}{2}, & if\ w = x_3^{l_3} \\ (6n+2)-\frac{l_4-1}{2}, & if\ w = x_4^{l_4} \\ (9n+2)-\frac{l_5-1}{2}, & if\ w = x_5^{l_5} \end{cases}$$

$\lambda(x_i^{l_i}) = (9n+2) + \sum_{t=6}^{i}[2^{t-5}(3n) - t + 6] - \frac{l_i-1}{2}$ accordingly.

For even $1 \leq l \leq n_i$ and $\gamma = (9n+2) + \sum_{t=6}^{r} 2^{t-5}(3n) - t + 6$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} (\gamma+1)\frac{l_1-2}{2}, & if\ w = x_1^{l_1} \\ (\gamma+3n-1)-\frac{l_2-2}{2}, & if\ w = x_2^{l_2} \\ (\gamma+3n+1)+\frac{l_3-2}{2}, & if\ w = x_3^{l_3} \\ (\gamma+6n-1)-\frac{l_4-2}{2}, & if\ w = x_4^{l_4} \\ (\gamma+9n-1)-\frac{l_5-2}{2}, & if\ w = x_5^{l_5} \end{cases}$$

$\lambda(x_i^{l_i}) = (\gamma+9n-1) + \sum_{t=6}^{i} [2^{t-5}(3n) - 2t + 11] - \frac{l_i-2}{2}$ accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma+2, \gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant$k = (9n-2) + \sum_{t=6}^{i}[2^{t-6}(3n) - t + 5]$.

Theorem 2.3 The graph $G \cong Sb(mn, mn, mn, 2mn, n_6, \dots, n_r)$ admits the reverse edge magic total labeling with $k = 2v - s - 1$ for any odd $n \geq 3, r \geq 6, n_p = 2^{p-4}kn - 2p + 11$ and $6 \leq p \leq r$

Proof: Let Let $v = |V(G)|$ and $e = |E(G)|$ then

$v = (6kn + 1) + \sum_{t=6}^{r}[2^{m-4}kn - 2m + 11]$ and $e = v - 1$

Let us define $\lambda: V(G) \to [1, v]$ as follows,

$$\lambda(c) = (4kn + 2) + \sum_{t=6}^{r}[2^{m-5}kn - m + 6]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $1 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \frac{l_1 + 1}{2}, & if\ w = {x_1}^{l_1} \\ mn + 1 - \frac{l_2 - 1}{2}, & if\ w = {x_2}^{l_2} \\ (mn + 2) + \frac{l_3 - 1}{2}, & if\ w = {x_3}^{l_3} \\ 2(mn + 2) - \frac{l_4 - 1}{2}, & if\ w = {x_4}^{l_4} \\ (3mn + 2) - \frac{l_5 - 1}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = (3mn + 2) + \sum_{t=6}^{i}[2^{t-5}mn - m + 6] - \frac{l_i - 1}{2}$ accordingly.

For even $1 \leq l \leq n_i$ and $\gamma = (3kn + 2) + \sum_{t=6}^{r}[2^{t-6}2kn - (m - 6)]$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} (\gamma + 1)\frac{l_1 - 2}{2}, & if\ w = {x_1}^{l_1} \\ (\gamma + mn - 1) - \frac{l_2 - 2}{2}, & if\ w = {x_2}^{l_2} \\ (\gamma + mn + 1) + \frac{l_3 - 2}{2}, & if\ w = {x_3}^{l_3} \\ (\gamma + 2mn - 1) - \frac{l_4 - 2}{2}, & if\ w = {x_4}^{l_4} \\ (\gamma + 3mn - 1) - \frac{l_5 - 2}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = (\gamma + 3mn - 1) + \sum_{t=6}^{i}\ [2^{t-5}(mn) - 2m + 11] - \frac{l_i - 2}{2}$ accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma + 2, \gamma + 1 + e]$. Therefore, by

Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant$k = (3mn - 2) + \sum_{t=6}^{i}[2^{t-6}(mn) - t + 10]$.

Theorem 2.4 The graph $G \cong Sb(mn, mn. 2n, 2n + 2, 4n + 3, n_6, \dots, n_r)$ admits the reverse edge magic total labeling with$k = 2v - s - 1$, for any odd $n \geq 3, r \geq 6, n_p = 2^{p-5}(4n + 2) + 1$ and $6 \leq p \leq r$

Proof: Let Let $v = |V(G)|$and $e = |E(G)|$ then

$v = [(2k + 8)n + 6] + \sum_{t=6}^{r}[2^{m-5}(4n + 2) + 1]$ and $e = v - 1$

Let us define $\lambda: V(G) \to [1, v]$ as follows,

$$\lambda(c) = [(2k + 4)n + 4] + \sum_{t=6}^{r}[2^{m-6}(4n + 2) + 1]$$

For odd $1 \leq l \leq n_i$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} \frac{l_1 + 1}{2}, & if\ w = {x_1}^{l_1} \\ mn + 1 - \frac{l_2 - 1}{2}, & if\ w = {x_2}^{l_2} \\ (mn + 2) + \frac{l_3 - 1}{2}, & if\ w = {x_3}^{l_3} \\ (m + 2)n + 2 - \frac{l_4 - 1}{2}, & if\ w = {x_4}^{l_4} \\ (m + 2)n + 4 - \frac{l_5 - 1}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda({x_i}^{l_i}) = [(2k + 4)n + 4] + \sum_{t=6}^{r}[2^{m-6}(4n + 2) + 1]\frac{l_i - 1}{2}$ accordingly.

For even $1 \leq l \leq n_i$ and $\gamma = (3kn + 2) + \sum_{t=6}^{r}[2^{t-6}2kn - (m - 6)]$, where $1 \leq i \leq 5$ and for $6 \leq i \leq r$:

$$\lambda(w) = \begin{cases} (\gamma + 1)\frac{l_1 - 2}{2}, & if\ w = {x_1}^{l_1} \\ (\gamma + mn - 1) - \frac{l_2 - 2}{2}, & if\ w = {x_2}^{l_2} \\ (\gamma + mn + 1) + \frac{l_3 - 2}{2}, & if\ w = {x_3}^{l_3} \\ (\gamma + (m + 2)n - 1) - \frac{l_4 - 2}{2}, & if\ w = {x_4}^{l_4} \\ (\gamma + (m + 2)n + 2) - \frac{l_5 - 2}{2}, & if\ w = {x_5}^{l_5} \end{cases}$$

$\lambda(x_i^{l_i}) = [\gamma + (m+4)n + 2] + \sum_{t=6}^{i} [2^{t-6}(4n+2)+1] - \frac{l_i-2}{2}$ accordingly.

By using the above scheme of labeling, we get the set of edge-sums consecutive integer sequence $S = [\gamma+2, \gamma+1+e]$. Therefore, by Lemma 1.1 can be extended to a reverse edge magic total labeling with magic constant $k = (mn+4) + \sum_{t=6}^{i}[2^{t-6}(4n+2)]$.

3. Conclusion

In this paper, we have proved that the following subclasses of subdivided stars admit reverse super edge magic total labeling,

- For $n \geq 3$ and $k \geq 1$ are odd, $r \geq 6$, $Sb(mn, mn, mn, 2mn, n_6, \dots, n_r)$ with $n_p = 2^{p-4} - 2p + 11$ for $6 \leq p \leq r$.
- For $n \geq 3$ and $k \geq 1$ are odd, $r \geq 6$, $Sb(mn, mn, 2n, 2n+2, 4n+3, n_6, \dots, n_r)$ with $n_p = 2^{p-5}(4n+2)+1$ for $6 \leq p \leq r$.

The problem is still open for the remaining subclasses of subdivided stars with different combinations of m and n.

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