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On Leap Gourava Indices of Some Wheel Related Graphs

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Abstract

Zagreb indices were introduced more than forty years ago. Naji et al. introduced the leap Zagreb indices of a graph in 2017 which are new distance-degree-based topological indices conceived while depending on the second degree of vertices. The Gourava indices were introduced by Kulli. In this paper, we defined the first and second leap Gourava indices and computed leap Gourava indices for some wheel related graphs.

Keywords: gourava indices, leap gourava indices, wheel graph, gear graph, helm graph, flower graph, sunflower graph

Introduction

In the current mathematico-chemical and mathematical literature, a large number of vertex degree based graph invariants have been studied. Among them, the first and second Zagreb indices are by far the most extensively investigated ones. These were introduced more than forty years ago [1].

The properties of the two Zagreb indices can be seen in [2], [3], [4], [5] and [6]. Many novel variants of the Zagreb indices have been studied in recent years. Some of these are multiplicative Zagreb indices [7], [8] and [6], Zagreb coincides [9], [10], multiplicative Zagreb coincides [11] and sum Zagreb index [12], [6] etc. Recently, leap Zagreb indices of a graph have been introduced by Naji et al. [13], which are new distance-degree-based topological indices conceived while depending on the second degree of vertices (number of their second neighbors). Some basic properties of these new indices have also been established. A. M. Naji and N. D. Soner [14] presented the exact expressions for the first Zagreb index of some graph operations containing the corona product, cartesian product, composition, dis junction and symmetric difference of graphs. Shiladhar P. et al. [15] computed leap Zagreb indices of some wheel related graphs.

The Gourava indices were introduced by Kulli [16]. The first and second Gourava indices are defined as follows,

$$G_1(G) = \sum_{uv\in G} ([d_u + d_v] + [d_u . d_v])$$

and

$$G_2(G) = \sum_{uv\in G} ([d_u + d_v] \, [d_u . d_v])$$

The Gourava indices of some standard classes of graphs, armchair polyhex and zigzag-edge polyhex nanotubes were computed. Kulli also introduced the sum connectivity Gourava index of a molecular graph and computed the sum connectivity Gourava index of linear [n]-Tetracene, V-Tetracenic nanotube, H Tetracenic nanotube and Tetracenic nanotori [17]. Kulli introduced the hyper Gourava indices and hyper Gouava co-indices of a graph and determined the hyper-Gourava indices of some standard classes of graphs and certain nanotubes [18]. Kulli generalized the Gouava indices and computed exact formulae for titania nanotubes [19]. Some Gourava indices and inverse sum index of some networks were computed in [20]. Kulli also developed the concept of product connectivity Gourava index of a graph and computed the product connectivity Gourava index for some standard classes of graphs, tetracenic nanotubes and tetracenic nanotori [21].

2. Definitions and Preliminaries

2.1. Definition

The wheel graph W_n with n + 1 vertices is defined as the joining of k_1 and C_n, where k_1 is the complete graph with one vertex and C_n is the cycle graph with n vertices. The vertices corresponding to k_1 and C_n are called apex and rim vertices, respectively. A wheel graph W_n is shown in figure 1.

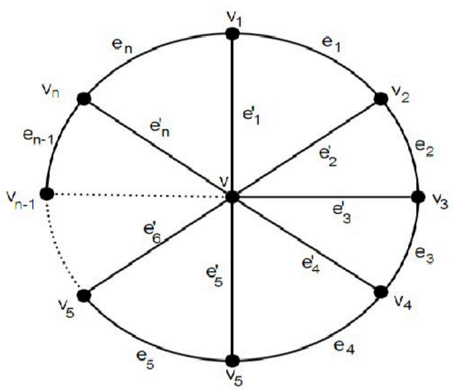

Figure 1. Wheel graph W_n

2.2. Definition

The gear graph G_n with 2n + 1 vertices (shown in figure 2) is a graph obtained from the wheel graph W_n by adding a vertex between each pair of adjacent rim vertices. The gear graph is also known as a bipartite wheel graph.

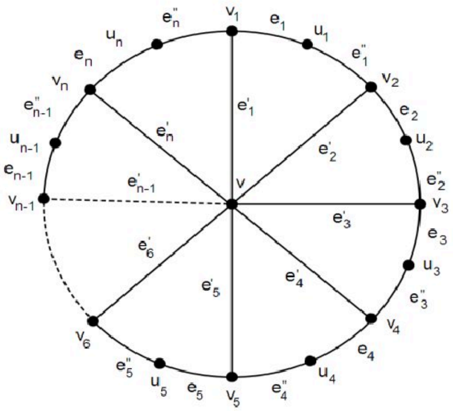

Figure 2. Gear graph G_n

2.3. Definition

The helm graph H_n is a graph with 2n + 1 vertices (shown in figure 3) obtained from the wheel graph W_n by attaching a pendant edge to each rim vertex. The helm graph contains three types of vertices, the vertex of degree n called apex, n pendant vertices and n rim vertices of degree four.

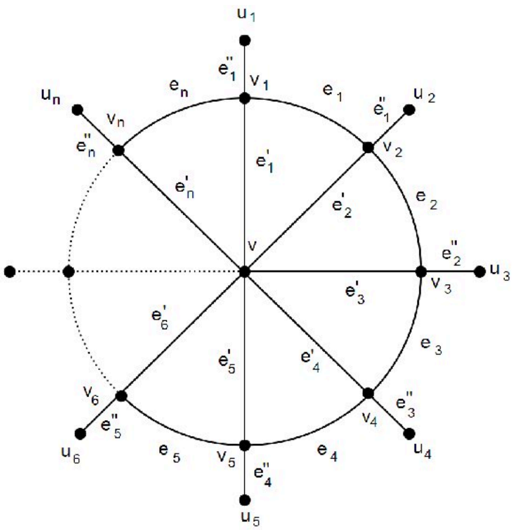

Figure 3. Helm graph H_n

2.4. Definition

The flower graph Fl_n (shown in figure 4) is a graph obtained from a helm graph H_n by joining each pendant vertex to the apex of the helm graph. There are three types of vertices, the apex of degree 2n, n vertices of degree four and n vertices of degree two.

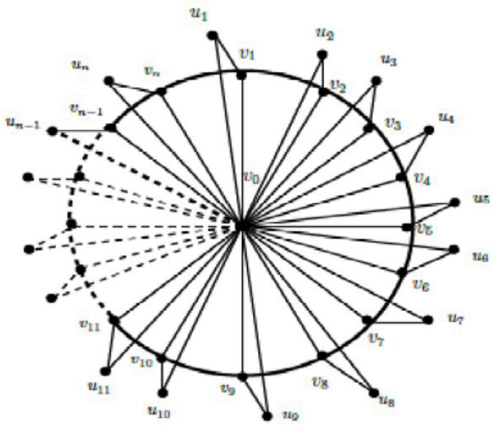

Figure 4. Flower graph Fl_n

2.5. Definition

The sunflower graph Sf_n (shown in figure 5) is a graph obtained from the flower graph Fl_n by attaching n pendant edges to the apex vertex. The sunflower graph has four types of vertices, the apex of 3n, n vertices of degree four, n vertices of degree two and n pendant vertices.

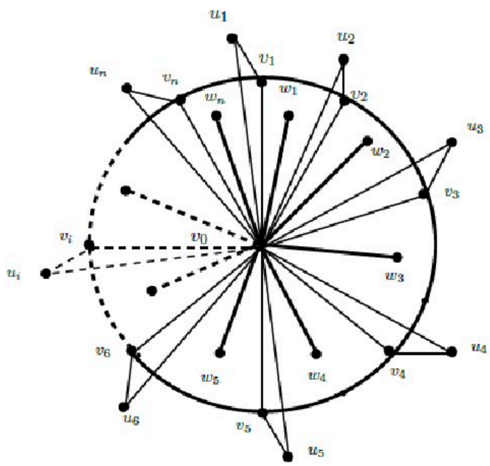

Figure 5. Sunflower graph Sf_n

2.6. Definition

The first and second leap Gourava indices are defined as follows,

$$G_1(G) = \sum_{uv\in G} ([d_2(u) + d_2(u)] + [d_2(u).d_2(u)])$$

and

$$G_2(G) = \sum_{uv\in G} ([d_2(u) + d_2(u)]\,[d_2(u).d_2(u)])$$

3. Main Results

3.1. Theorem

Let W_n be a wheel graph with $n + 1$ vertices, its leap Gourava indices are given below as

$$LG_i(G) = \begin{cases} n^2(n-3), & for\ i = 1 \\ 2n(n-3)^3, & for\ i = 2 \end{cases}$$

Proof

We compute the required result by using table 1 and formulae defined in the previous section as follows,

$$G_1(G) = \sum_{uv\in G} ([d_2(u) + d_2(u)] + [d_2(u).d_2(u)])$$

$$= n[0 + n - 3 + 0(n-3)] + n[n - 3 + n - 3 + (n-3)(n-3)]$$

$$= n(n-3) + n[2n - 6 + (n-3)^2]$$

$$= n^2(n-3)$$

and

$$G_2(G) = \sum_{uv\in G} ([d_2(u) + d_2(u)]\,[d_2(u).d_2(u)])$$

$$= n[(0 + n - 3)(0)(n-3)] + n[(n - 3 + n - 3)(n-3)(n-3)]$$

$$= n(0) + n[(2n-6)(n-3)(n-3)]$$

$$= 2n(n-3)^3$$

3.2. Theorem

Let G_n be a gear graph with $2n + 1$ vertices, its leap Gourava indices are given below as

$$LG_i(G) = \begin{cases} n^2(n^2 + 7n - 3), & for\ i = 1 \\ n(2n^3 + 3n^2 + 7n - 12), & for\ i = 2 \end{cases}$$

Proof

We compute the required result by using table 2 and formulae defined in the previous section as follows,

$$G_1(G) = \sum_{uv\in G}([d_2(u) + d_2(u)] + [d_2(u).d_2(u)])$$
$$= n[n + n - 1 + n(n-1)] + 2n[n - 1 + 3 + 3(n-1)]$$
$$= n(n^2 - n - 1) + 2n(4n - 1)$$
$$= n^2(n^2 + 7n - 3)$$

and

$$G_2(G) = \sum_{uv\in G}([d_2(u) + d_2(u)]\,[d_2(u).d_2(u)])$$
$$= n[(n + n - 1)n(n-1)] + 2n[(n - 1 + 3)3(n-1)]$$
$$= n(2n^3 + 3n^2 + 7n - 12)$$

3.3. Theorem

Let H_n be a helm graph with $2n + 1$ vertices, its leap Gourava indices are given below as

$$LG_i(G) = \begin{cases} n(2n^2 + 5n - 3), & for\ i = 1 \\ n(n-1)(4n^2 - 2n + 8), & for\ i = 2 \end{cases}$$

Proof

We compute the required result by using table 3 and formulae defined in the previous section as follows,

$$G_1(G) = \sum_{uv\in G}([d_2(u) + d_2(u)] + [d_2(u).d_2(u)])$$
$$= n[n + n - 1 + n(n-1)] + n[n - 1 + n - 1 + (n-1)(n-1)] + n[n - 1 + 3 + 3(n-1)]$$
$$= n(n^2 + n - 1) + n(n^2 - 1) + n(4n - 1)$$
$$= n(2n^2 + 5n - 3)$$

and

$$G_2(G) = \sum_{uv\in G}([d_2(u) + d_2(u)]\,[d_2(u).d_2(u)])$$

$$= n[(n+n-1)n(n-1)] + n[(n-1+n-1)(n-1)(n-1)] + n[(n-1+3)3(n-1)]$$

$$= n[n(2n-1)(n-1)] + n[2(n-1)^3] + n[3(n-1)(n+2)]$$

$$= n(n-1)(4n^2-2n+8)$$

3.4. Theorem

Let F_n be a flower graph with $2n+1$ vertices, its leap Gourava indices are given below as

$$LG_i(G) = \begin{cases} 4n(2n^2-4n+1), & for\ i=1 \\ 8n(4n^3-21n^2+37n-22), & for\ i=2 \end{cases}$$

Proof

We compute the required result by using table 4 and formulae defined in the previous section as follows,

$$G_1(G) = \sum_{uv\in G} ([d_2(u)+d_2(u)] + [d_2(u).d_2(u)])$$

$$\begin{aligned} &= n[2n-2+2n-4+(2n-2)(2n-4)] \\ &\quad + n[2n-2+0+(2n-2)(0)] \\ &\quad + n[2n-4+0+(2n-4)(0)] \\ &\quad + n[2n-4+2n-4+(2n-4)(2n-4)] \end{aligned}$$

$$\begin{aligned} &= n[4n-6+(4n^2-12n+8)] + 2n(n-1) + 2n(n-2) \\ &\quad + n[4n-8+(2n-4)^2] \end{aligned}$$

$$= 4n(2n^2-4n+1)$$

and

$$G_2(G) = \sum_{uv\in G} ([d_2(u)+d_2(u)]\,[d_2(u).d_2(u)])$$

$$\begin{aligned} &= n[(2n-2+2n-4)(2n-2)(2n-4)] \\ &\quad + n[(2n-2+0)(2n-2)(0)] \\ &\quad + n[(2n-4+0)(2n-4)(0)] \\ &\quad + n[(2n-4+2n-4)(2n-4)(2n-4)] \end{aligned}$$

$$= n[8(n-1)(n-2)(2n-3)] + n[4(n-2)(2n-4)^2]$$

$$= 8n(4n^3-21n^2+37n-22)$$

3.5. Theorem

Let Sf_n be a sunflower graph with $3n+1$ vertices, its leap Gourava indices are given below as

$$LG_i(G) = \begin{cases} 3n(6n^2 - 7n - 1), & for\ i = 1 \\ 2n(3n-4)(18n^2 - 39n + 22), & for\ i = 2 \end{cases}$$

Proof

We compute the required result by using table 5 and formulae defined in the previous section as follows,

$$G_1(G) = \sum_{uv \in G} ([d_2(u) + d_2(u)] + [d_2(u).d_2(u)])$$

$$\begin{aligned} &= n[3n-1+0+(3n-1)(0)] + n[3n-21+0+(3n-2)(0)] \\ &+ n[3n-4+0+(3n-4)(0)] \\ &+ n[3n-2+3n-4+(3n-2)(3n-4)] \\ &+ n[(3n-4+3n-4)(3n-4)(3n-4)] \\ &= n(3n-1) + n(3n-2) + n(3n-4) \\ &+ n[6n-6+9n^2-18n+8] \\ &+ n[6n-8+(3n-4)^2] \\ &= 2n(3n-4)(18n^2-39n+22) \end{aligned}$$

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