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Numerical Solutions for Third Order Ordinary Differential Equation by Differential Transform Method & Elzaki Transform Method

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Abstract

Diffesrential Transformation provides semi-analytical arithmetical solution. This approach is proficient at reducing calculation and works readily. In this paper, we use Elzaki Transform Method (ETM) and Differential Transformation Method (DTM) to get a numerical result of the Third Order Ordinary Differential Equation. We compare the results to see which method converges quickly with the true solution. We also offer numerical results with errors to show the efficiency of the methods.

Keywords: elzaki transform method (ETM), differential transform method (DTM), exact solutions

Introduction

The differential equations have played a vital role in every phase of applied mathematics for a long time. With the beginning of digital computing, their importance has multiplied significantly. Elzaki Transform was driven by Tarig Elzaki to empower the system of tackling conventional and differential conditions in the time space. Another fundamental change is said to be that the Elzaki Transform outlined the capacity of exponential request expect works in the set A characterized as follows,

$$A = \{f(t): \exists N \ , k1\ , k2 > 0,\ |f(t)| < Ne^{\frac{|t|}{kj}}\ , if\ \ t \in (-1)^j X\ [0, \infty)\} \qquad (1)$$

In the given capacity in set A, the steady N might be a limited number, that is, $k_{1,}k_2$. It ought to be limited and unbounded and expressed by the integral equations given below.

$$E[f(t)] = T(v) = v\int_0^\infty f(t)\, e^{\frac{-t}{v}} dt\ ,\ \ t \geq 0,\ k_1 \leq v \leq \ k_2 \qquad (2)$$

The variable in the given transform is used to factor the variable (t) in the argument of the function f. This transform has an important link with the Laplace transform. We depict several different belongings of the current transform and Sumudu transform; a few properties of these transforms are expanded. The adequate conditions for the event of Elzaki change is that $f(t)$ for $t > 0$ is piecewise nonstop and of exponential request, then Elzaki change might possibly happen. Consequently, one ought to have the capacity to match it, all things considered, in settling the issue. Defined as $Re(s) > 0$, Laplacian change is known as follows,

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)dt \quad (3)$$

The definition of Elzaki & Laplacian transform depict a double connection described as follows,

$$T(u) = uF\left(\frac{1}{s}\right)$$

$$F(s) = sT\left(\frac{1}{s}\right) \quad (4)$$

This equation is referred to as LED which is the short form of Laplace Elzaki Duality.

Zhou X. [1] was the very first person who proposed the Differential Transform Method (DTM) for solving non-linear and linear primary valued problems in electrical circuit analysis. Two-dimensional Differential Transform Method to solve partial differential equation [2] has been applied by Jan, Chen and Liu and they have also used it to take care of two boundary esteem issues [3]. The Differential Transform Method has been applied by Yu and Chen for the streamlining of the rectangular blades with variable warm parameters [4, 5]. It is an iterative method to get Taylor series results. The analytical result is constructed by this method in a polynomial form. This technique is simplest and easiest to command and works in a very short time even in relation to non-linear or parameter variable structures. However, it is unlike the Taylor arrangement strategy which needs the calculation of the high request subordinates. Reliable solutions have been shown by the implementation of DTM [3, 4, 6, 7] among others to get the solutions of ODE, Blasius equation, PDE, non-linear fractional differential equations and delay DE (Differential Equations). Firstly, all the transformed problems are attained by using Differential Transformation according to the above technique. Secondly, the results of transform problems are attained and eventually, inverse differential transformation is used that gives the results of actual problems.

$$\begin{array}{ccc} \textit{Actual problem} & \rightarrow & \textit{Solution of actual problem} \\ \downarrow & & \downarrow \end{array}$$

$$\begin{array}{ccc} Differential\ Transformatiom & & inverse\ differential\ Transformation \\ \downarrow & & \downarrow \\ Transformed\ problem & \rightarrow & solution\ of\ transformed\ problem \end{array}$$

The Differential Transform of a function is simply the nth co-efficient of the Taylor expansion of the function and inverse differential transform constitute the Taylor series. Given below is an expansion of an arbitrary function $f(t)$ in Taylor series about any point $t = 0$

$$f(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{d^k}{dt^k} f(t)\right]_{t=0} \tag{5}$$

The differential Transformation of $f(t)$ is given below as

$$F(k) = \frac{1}{k!} \left[\frac{d^k}{dt^k} f(t)\right]_{t=0} \tag{6}$$

The inverse differential change for $F(k)$ is specified through an infinite series and is defined

as $$f(t) = \sum_{k=0}^{\infty} t^k F(k)$$

The new essential change was proposed by Tarig Elzaki [8] to empower illuminating common and halfway differential condition in time area [9]. Some scientific apparatuses can be utilized to unravel differential condition like Fourier, Laplace and Sumudu change [10-14]. [15-19] which illuminate the differential condition and the acquired outcomes that were contrasted with the outcomes acquired by utilizing standard ADM. In this exposition, we used the NIT technique to acquire precise diagnostic and estimated answers for condition of fragmentary request. M. Saqlian [20] solved second order ODE in his paper.

2. Elzaki Transform Method

2.1. Problem

Let us assume a Differential Equation given below as

$$u'''(t) = e^t \tag{7}$$

The initial conditions are given below

$$u(0) = 3\ , \qquad u'(0) = 1\ , \qquad u''(0) = 5 \tag{8}$$

Applying Elzaki Transform on eq. (7) & (8). We get

$$E[u'''(t)] = E[e^t]$$

$$\frac{1}{v^3} E[u(t)] - \frac{1}{v} u(0) - u'(0) - v u''(0) = \frac{v^2}{1-v}$$

$$E[u(t)] = 2v^2 + 4v^4 + \frac{v^2}{1-v} \quad (9)$$

By taking inverse Elzaki Transform we get

$$u(t) = 2 + 2t^2 + e^t \quad (10)$$

Table 1. The Values of $u(t)$ using Elzaki Transform

t	$u(t)$
0	3.000000000
0.1	3.125170918
0.2	3.301402756
0.3	3.529858808
0.4	3.811824698
0.5	4.148721271
0.6	4.542118800
0.7	4.993757070
0.8	5.505540928
0.9	6.079603111
1	6.718281828

Table 1 shows the solution of $u(t)$ using Elzaki Transform in which independent variable is substituted by the difference of 0.1.
Differential Transform Method

2.2. Problem

Consider the Ordinary Differential Equation

$$u'''(t) - e^t = 0 \quad (11)$$

with given initial conditions

$u(0) = 3$ $\quad U[0] = 3$ $\quad U_0 = 3$ (12)

$u'(0) = 1$ $\quad U[1] = 1$ $\quad U_1 = 1$ (13)

$u''(0) = 5$ $\quad U[2] = 5$ $\quad U_2 = 5$ (14)

For k from 0 to 23

$$U[k+3] = \frac{k!}{(k+3)!}\left(\frac{1}{k!}\right)$$

For $k = 0$

$$U[0+3] = \frac{0!}{(0+3)!}\left(\frac{1}{0!}\right)$$

$$U_3 = \frac{1}{6} \quad (15)$$

$$U_{23} = \frac{1}{25852016738884976640000} \quad (16)$$

For k from 0 to 23 we get;

$u = u + U[k] * t^k$

$$u = 3$$
$$u = 3 + t$$
$$u = 5t^2 + t + 3$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10}$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11}$$
$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15} + \frac{1}{20922789888000}t^{16}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15} + \frac{1}{20922789888000}t^{16} + \frac{1}{355687428096000}t^{17}u$$

$$= 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15} + \frac{1}{20922789888000}t^{16} + \frac{1}{355687428096000}t^{17} + \frac{1}{6402373705728000}t^{18}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15} + \frac{1}{20922789888000}t^{16} + \frac{1}{355687428096000}t^{17} + \frac{1}{6402373705728000}t^{18} + \frac{1}{121645100408832000}t^{19}$$

$$u = 5t^2 + t + 3 + \frac{1}{6}t^3 + \frac{1}{24}t^4 + \frac{1}{120}t^5 + \frac{1}{720}t^6 + \frac{1}{5040}t^7 + \frac{1}{40320}t^8 + \frac{1}{362880}t^9 + \frac{1}{3628800}t^{10} + \frac{1}{39916800}t^{11} + \frac{1}{479001600}t^{12} + \frac{1}{6227020800}t^{13} + \frac{1}{87178291200}t^{14} + \frac{1}{1307674368000}t^{15} + \frac{1}{20922789888000}t^{16} + \frac{1}{355687428096000}t^{17} + \frac{1}{6402373705728000}t^{18} + \frac{1}{121645100408832000}t^{19} + \frac{1}{2432902008176640000}t^{20} \quad (17)$$

Table 2. The Value of $u(t)$ by Differential Transformation Method

t	$u(t)$
0	3
0.1	3.150170918
0.2	3.401402759
0.3	3.754858807
0.4	4.211824698
0.5	4.773721270
0.6	5.442118801
0.7	6.218752707
0.8	7.105540929
0.9	8.104603111
1	9.218281828

Table 2 and Table 3 indicate the solution of $u(t)$ using DTM in which independent variable is substituted by the difference of 0.1. The error in the solutions of *u(t)* by DTM and ETM and the graphical comparison of ETM and DTM is shown in figure 1.

Table 3. Error in the Solutions of *u(t)* by DTM and ETM

Comparison Table for *u(t)*			
T	**DTM**	**ETM**	**ERROR**
0	3	3	000000000
0.1	3.150170918	3.125170918	0.025000000
0.2	3.401402759	3.301402756	0.100000003
0.3	3.754858807	3.529858808	0.224999999
0.4	4.211824698	3.811824698	0.400000000
0.5	4.77372127	4.148721271	0.624999999
0.6	5.442118801	4.5421188	0.900000001
0.7	6.218752707	4.99375707	1.224995637
0.8	7.105540929	5.505540928	1.600000001
0.9	8.104603111	6.079603111	2.025000000
1	9.218281828	6.718281828	2.500000000

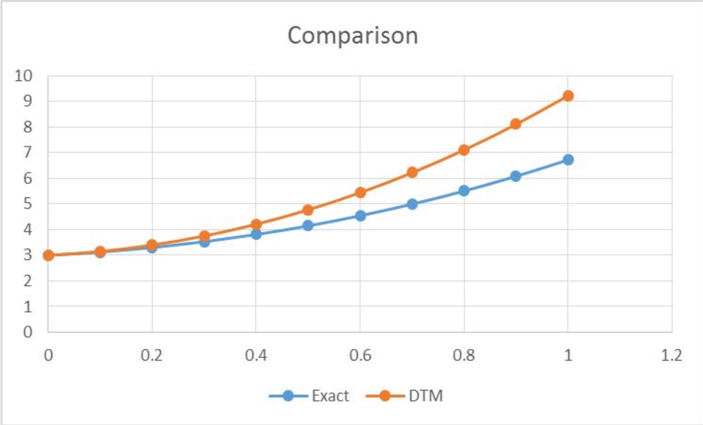

Figure 1. Graphical comparison of ETM and DTM

3. Conclusion

In this paper, differential equations are solved by using two different transformations; one is Elzaki Transformation and the other is Differential Transformation and the results obtained by these two transformations are compared. Results obtained by Differential Transformation Method are series solution; on the other hand Elzaki Transformation provides exact results. Results are compared in tabular form and by using graphs. It is found that the results obtained by Differential Transformation Method converge rapidly and are very near to the exact solution. Hence, Differential Transformation Method is a reliable, effective tool for the solution of differential equations.

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