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Representation of Soft Set and Its Operations by Bipartite Graph

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Abstract

The idea of soft set was initiated by Molodstov. Soft sets have been used for decision making in dealing vague ideas. In this paper, soft sets are represented by bipartite graph. Operations on soft sets are also represented by bipartite graph.

Keywords: bipartite graph, complete bipartite graph, soft set

Introduction

Many theories have been developed to deal with uncertainty in real life problems. Molodtsov [1] pointed out the problem of the inadequacy of the fuzzy set theory and expounded the soft set theory to overcome this problem. A revolution followed when Molodstov gave the concept of soft set. Operations like union, intersection, AND-operation and OR-operation on soft set were formulated by Maji et al. [2]. Euler was the first to introduce graph theory which solved many mathematical problems in 1736. Rosenfieldin in 1975 introduced the concept of fuzzy graph. The concepts of soft graph was introduced by Rajesh K. Thumbakara and Boben George in 2014 [3]. After an year, Sumit Mohinta and T. K. Samanta [4] gave the idea of fuzzy soft graph in 2015. Further, M Irfan et al. [5] introduced a new method for graph representation based on the adjacency of vertices and soft set theory. With this representation of a graph, the application of algebraic operations in soft sets revealed many new aspects of graph theory.

In this work, a set of attributes and the universal set are represented by two partite sets and their mutual relationship is represented by joining the vertices of the two partite sets.

2. Preliminaries

2.1 Definition [3]

A graph \check{G} consists of two finite non-empty sets $V(\check{G})$ and $E(\check{G})$ called set of vertices and edges respectively, where edges are obtained by connecting vertices.

2.2 Definition [3]

A graph whose edge set is null or empty is called null graph.

2.3 Definition [3]

If two partite sets $U = \{\aleph_1, \aleph_2, \aleph_3\}$ and $W = \{\mathfrak{B}_1, \mathfrak{B}_2\}$ of graph \check{G} are such that every edge is obtained by joining the vertices of U and W , then \check{G} is called bipartite graph.

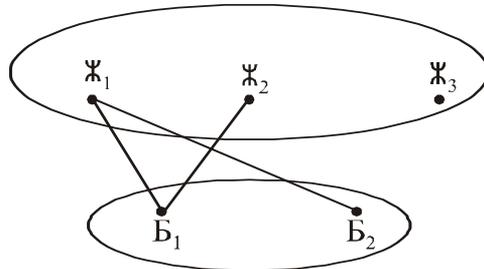


Figure 1. Bipartite graph

2.4 Definition [3]

A bipartite graph is called a complete bipartite if two partite sets U and W are such that each vertex of U is joined with all vertices of W . For example, $U = \{\mathfrak{B}_1, \mathfrak{B}_2, \mathfrak{B}_3, \mathfrak{B}_4, \mathfrak{B}_5\}$ and $W = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7\}$, then the complete bipartite graph is symbolically written by $K_{5,7}$.

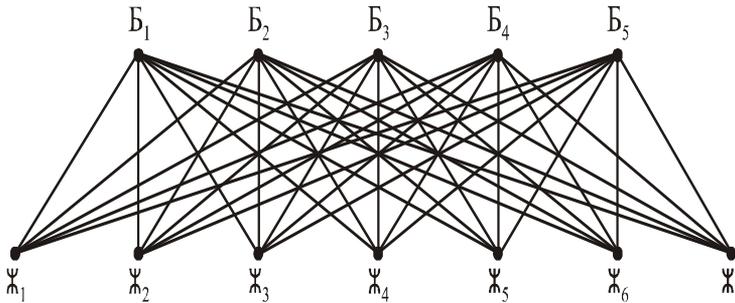


Figure 2. Complete bipartite, $K_{5,7}$

2.5 Definition [3]

Graphs \check{G}_1 and \check{G}_2 are equal if $V(\check{G}_1) = V(\check{G}_2)$ and $E(\check{G}_1) = E(\check{G}_2)$.

2.6 Definition [3]

Graph \check{G}_2 is a sub-graph of \check{G}_1 , if $V(\check{G}_2) \subseteq V(\check{G}_1)$ and $E(\check{G}_2) \subseteq E(\check{G}_1)$ and is symbolically written as $\check{G}_2 \subseteq \check{G}_1$.

2.7 Definition [3]

Complement \check{G}^c of a graph \check{G} is a graph with the same vertex set but with a different edge set, that is, if $uv \in E(\check{G}^c) \Leftrightarrow uv \notin E(\check{G})$, then $V(\check{G}^c) = V(\check{G})$ or $|V(\check{G})| = n = |V(\check{G}^c)|$; if $|E(\check{G})| = m$, then $|E(\check{G}^c)| = \binom{n}{2} - m$.

2.8 Definition [3]

Union of two graphs \check{G}_1 and \check{G}_2 is symbolically written as $\check{G}_1 \cup \check{G}_2$, which is a graph such that

$$V(\check{G}_1 \cup \check{G}_2) = V(\check{G}_1) \cup V(\check{G}_2) \text{ and } E(\check{G}_1 \cup \check{G}_2) = E(\check{G}_1) \cup E(\check{G}_2).$$

2.9 Definition [3]

Intersection of two graphs \check{G}_1 and \check{G}_2 is symbolically written as a $\check{G}_1 \cap \check{G}_2$, which is a graph such that

$$(V(\check{G}_1 \cap \check{G}_2) = V(\check{G}_1) \cap V(\check{G}_2) \text{ and } E(\check{G}_1 \cap \check{G}_2) = E(\check{G}_1) \cap E(\check{G}_2).$$

2.10 Definition [1]

Let F be a set valued function given by $F: A \rightarrow P(X)$ where $A \subseteq E$ and $P(X)$, then soft set over X is symbolically written as (F, A) or F_A and is given as $F_A = \{F(B) \in P(X); B \in A, F(B) = \emptyset \text{ if } B \notin A\}$.

2.11 Definition [2]

F_A is a soft subset of G_B and is symbolically represented by $F_A \subseteq G_B$. If (1) $A \subseteq B$ and (2) $\forall B \in A, F(B) \subseteq G(B)$, then G_B is a super SS of F_A and is symbolically represented by $G_B \supseteq F_A$.

2.12 Definition [2]

F_A is a soft equal set to G_B and symbolically represented by $F_A = G_B$ if $F_A \subseteq G_B$ and $G_B \subseteq F_A$.

2.13 Definition [2]

A SS F_A is called relative null SS w.r.t. A and symbolically represented by $\tilde{\Phi}_A$, if $F(B) = \emptyset, \forall B \in A$.

2.14 Definition [2]

A SS F_A is called relative whole SS w.r.t. A and symbolically represented by \tilde{X}_A , if $F(B) = X, \forall B \in A$.

2.15 Definition [2]

A SS F_A is called absolute SS w.r.t. E and symbolically represented by \tilde{X}_E , if $F(B) = X, \forall B \in E$.

2.16 Definition [2]

The relative complement F_A^r or $(F, A)^r$ of a SS F_A is defined as $F_A^r = (F^r, A)$, where $F^r(t) : A \rightarrow P(X)$ is the mapping given by $F^r(B) = X - F(B), \forall B \in A$.

2.17 Definition [2]

Union of two SSs F_A and G_B is a SS, symbolically represented by $F_A \cup G_B$ and is given by

$$F_A \cup G_B = \begin{cases} F(B) & B \in A - B \\ G(B) & B \in B - A \quad \forall B \in A \cup B \\ F(B) \cup G(B) & B \in A \cap B \end{cases}$$

2.18 Definition [2]

Restricted union of two SSs F_A and G_B over X is a SS, symbolically represented by $F_A \cup_R G_B$ and is given by $F(B) \cup G(B) \forall B \in A \cap B$.

2.19 Definition [2]

Intersection of SSs F_A and G_B is a SS, symbolically represented by $F_A \cap G_B$ and is given by $F(B) \cap G(B) \forall B \in A \cap B$.

2.20 Definition [2]

Extended intersection of SSs F_A and G_B is a SS, symbolically represented by $F_A \cap_\varepsilon G_B$ and is given by

$$F_A \cap_\varepsilon G_B = \begin{cases} F(B) & B \in A - B \\ G(B) & B \in B - A \quad \forall B \in A \cup B \\ F(B) \cap G(B) & B \in A \cap B \end{cases}$$

2.21 Definition [2]

Restricted difference of SSs F_A and G_B , symbolically represented by $F_A -_R G_B$ and is given by

$$F(B) - G(B), \forall B \in A \cap B.$$

2.22 Definition [2]

OR-operation of SSs F_A and G_B is a SSH_C, symbolically represented by

$F_A \vee G_B$ and is given by $(H, A \times B)$ where $H(a, b) = F(a) \cup G(b), \forall (a, b) \in A \times B$.

2.23 Definition [2]

AND-operation of two SSs F_A and G_B is a SSH_C , symbolically represented by $F_A \wedge G_B$ and is given by $(H, A \times B)$, where $H(a, b) = F(a) \cap G(b), \forall (a, b) \in A \times B$.

3. Representation of Soft Set by Bipartite Graph

3.1 Soft Set

Every SS can be graphically represented with the help of bipartite graph A by taking partite sets; the first set of attribute E with $A \sqsubset E$ and the second universe of discourse X and their elements are represented by vertices of these partite sets A and X .

Example 1

Let $X = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7\}$, $E = \{B_1, B_2, B_3, B_4, B_5\}$ and $A = \{B_1, B_2, B_3\} \subset E$

$F_A = \{(B_1, \{\aleph_1, \aleph_4, \aleph_5\}), (B_2, \{\aleph_2, \aleph_3, \aleph_6\}), (B_3, \{\aleph_3, \aleph_7\})\}$

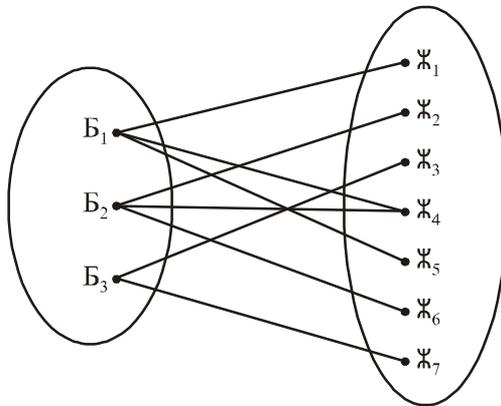


Figure 3. Soft set represented by bipartite graph

3.2 Soft Sub-Set

If G_B is the soft sub-set of a SS F_A , then the bipartite graph of G_B is also the sub-graph of F_A .

3.3 Absolute SS by Bipartite Graph

Absolute SS can be drawn by complete bipartite graph.

Example 2

Let $X = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5\}$ and $E = \{B_1, B_2, B_3, B_4\}$, then absolute SS represents complete bipartite graph $K_{4,5}$.

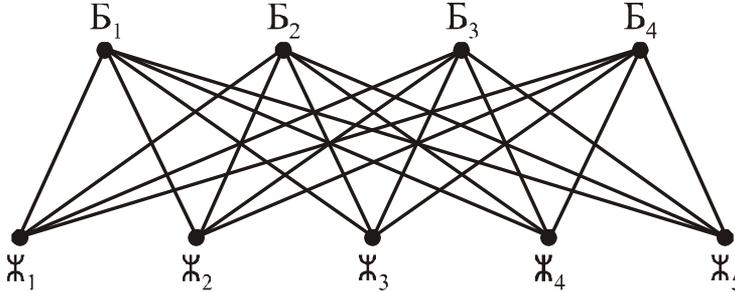


Figure 4. Absolute soft set represented by bipartite graph

3.4 Null SS

Null SS can be represented by null graph.

3.5 Relative Complement of a SS

Complement graph of a soft set F_A is the relative complement graph.

Example 3

Let $X = \{\aleph_1, \aleph_2, \aleph_3, \aleph_4, \aleph_5, \aleph_6, \aleph_7\}$, $E = \{B_1, B_2, B_3, B_4\}$, $A = \{B_1, B_2, B_3\}$ and if

$$F_A = \{(B_1, \{\aleph_2, \aleph_3\}), (B_2, \{\aleph_1, \aleph_2\}), (B_3, \{\aleph_1, \aleph_2, \aleph_3\})\}$$

then

$$F_A^r = \{(B_1, \{\aleph_1, \aleph_4, \aleph_5, \aleph_6\}), (B_2, \{\aleph_3, \aleph_4, \aleph_5, \aleph_6\}), (B_3, \{\aleph_4, \aleph_5, \aleph_6\}), (B_4, X)\}$$

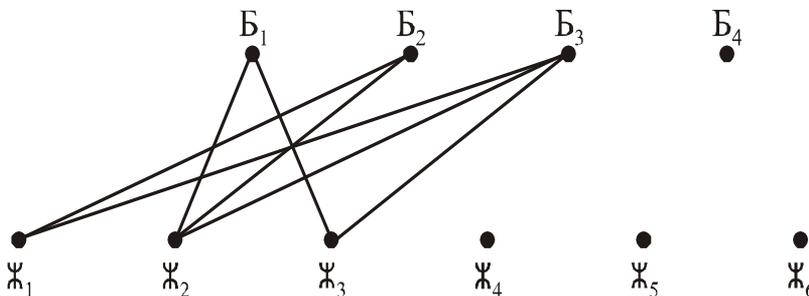


Figure 5. Soft set represented by bipartite graph

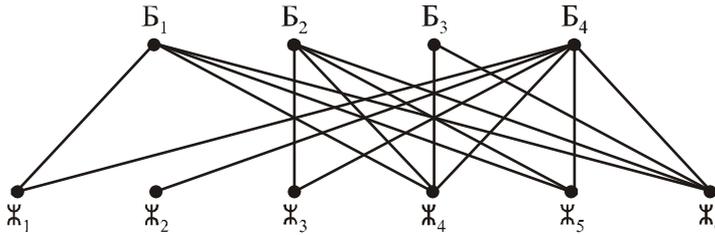


Figure 6. Relative complement of soft set represented by bipartite graph

4. Representation of Operations on Soft Set by Bipartite Graph

To illustrate operations on SSs by bipartite graph, let $X = \{K_1, K_2, K_3, K_4, K_5, K_6, K_7\}$, $E = \{B_1, B_2, B_3, B_4, B_5\}$ $A = \{B_1, B_2, B_3, \} \subset E$ and $B = \{B_3, B_4, B_5\} \subset E$

$A = \{B_1, B_2, B_3\} \subset E$ and $B = \{B_3, B_4, B_5\} \subset E$, then

$$F_A = \{(B_1, \{K_2, K_4, K_5\}), (B_2, \{K_2, K_3, K_6\}), (B_3, \{K_3, K_7\})\}$$

and $G_B = \{(B_3, \{K_1, K_3, K_4\}), (B_4, \{K_4, K_7\}), (B_5, \{K_2, K_6\})\}$.

4.1 Union of Two SSs

$$F_A \cup G_B =$$

$$\{(B_1, \{K_2, K_4, K_5\}), (B_2, \{K_2, K_3, K_6\}), (B_3, \{K_1, K_3, K_4, K_7\}), (B_4, \{K_4, K_7\}), (B_5, \{K_2, K_6\})\}$$

can be represented by bipartite graph as shown below.

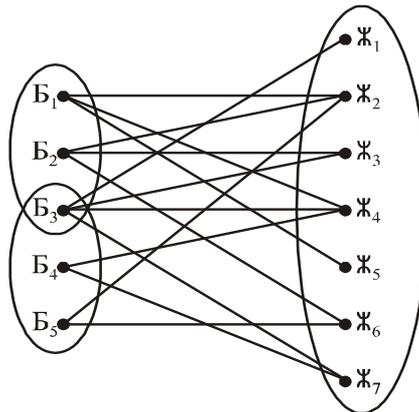


Figure 7. Union of soft sets represented by bipartite graph

4.2 Restricted Union

$$F_A \cup_R G_B = \{(B_3, \{K_1, K_3, K_4, K_7\})\}$$

graph as shown below.

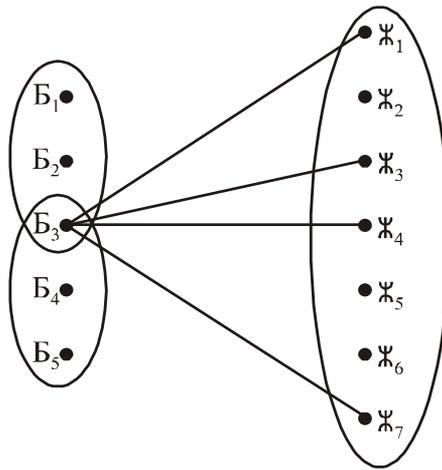


Figure 8. Restricted union of soft sets represented by bipartite graph

4.3 Intersection

$F_A \widetilde{\cap} G_B = \{(B_3, \{K_3\})\}$ can be represented by bipartite graph as shown below.

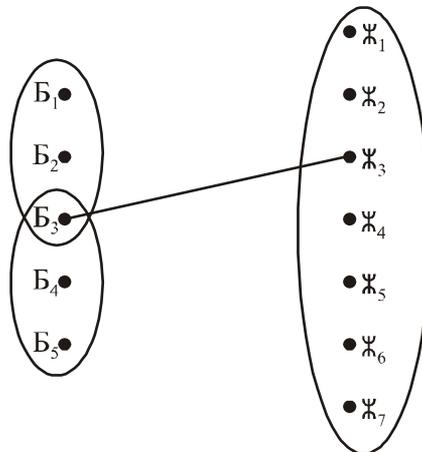


Figure 9. Intersection of soft sets represented by bipartite graph

4.4 Extended Intersection

$$F_A \cap_{\varepsilon} G_B = \{(B_1, \{K_2, K_4, K_5\}), (B_2, \{K_2, K_3, K_6\}), (B_3, \{K_3\}), (B_4, \{K_4, K_7\}), (B_5, \{K_2, K_6\})\}$$

can be represented by bipartite graph as shown below.

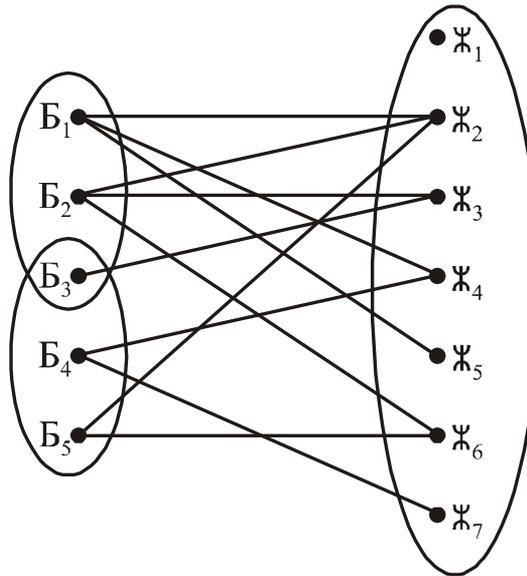


Figure 10. Extended intersection of soft sets represented by bipartite graph

4.5 Restricted Difference of SSs

$F_{A-R} G_B = \{(B_3, \{\Psi_7\})\}$ and $G_{B-R} F_A = \{(B_3, \{\Psi_1, \Psi_4\})\}$ can be represented by bipartite graph as shown below.

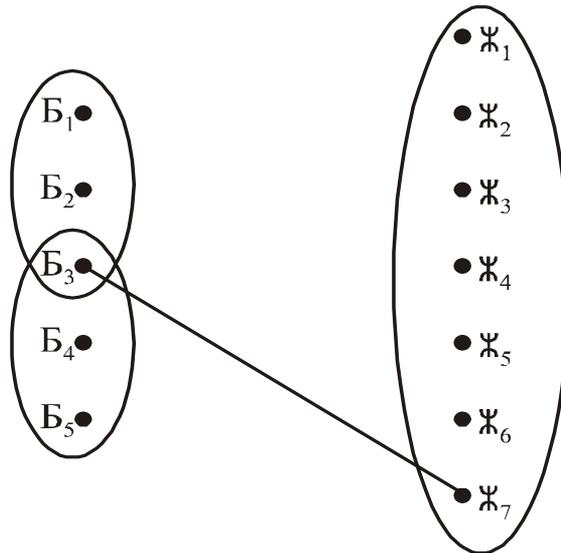


Figure 11. $F_{A-R} G_B$

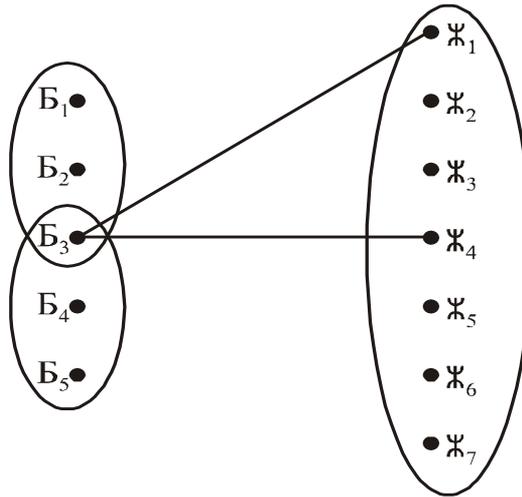


Figure 12. $G_B - R F_A$

4.6 Restricted Symmetric Difference of Two SSs

$F_A \tilde{\Delta} G_B = \{(B_3, \{K_1, K_4, K_7\})\}$ can be represented by bipartite graph as shown below.

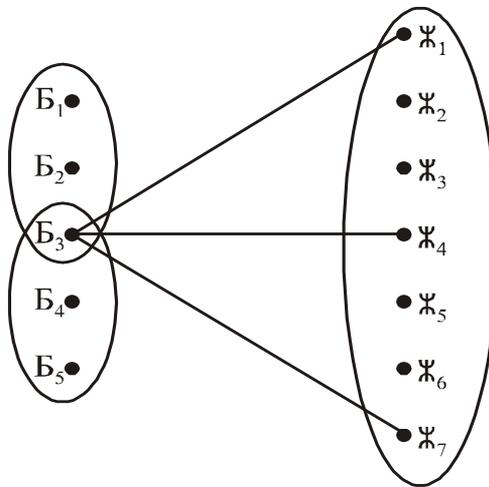


Figure 13. Restricted symmetric difference of two SSs

4.7 AND-Operation

$$\begin{aligned}
 & F_A \wedge G_B \\
 &= \{((B_1, B_3), \{K_4\}), ((B_1, B_4), \{K_4\}), ((B_1, B_5), \{K_2\}), ((B_2, B_3), \{K_3\}), \\
 & ((B_2, B_4), \{K_2, K_6\}), ((B_2, B_5), \{K_2, K_6\}), ((B_3, B_3), \{K_3\}), (B_3, B_4), \{K_7\})\}
 \end{aligned}$$

can be represented by an ordered pair (B_i, B_j) , by boxes and $F_A(B_i) \cap G_B(B_j)$ is represented by edges joining combined line of two boxes as shown by the following graph.

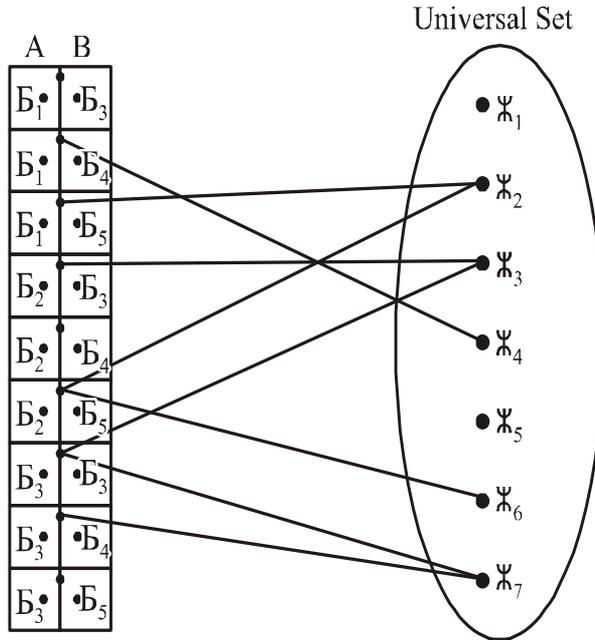


Figure 14. OR-operation

4.8 OR-Operation

OR-Operation of SSs F_A and G_B which is

$$\begin{aligned}
 &F_A \vee G_B \\
 &= \{((B_1, B_3), \{\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5\}), ((B_1, B_4), \{\Xi_2, \Xi_4, \Xi_5, \Xi_7\}), ((B_1, B_5), \\
 &\{\Xi_2, \Xi_4, \Xi_5, \Xi_6\}), \\
 &((B_2, B_3), \{\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_6\}), ((B_2, B_4), \{\Xi_2, \Xi_3, \Xi_4, \Xi_6, \Xi_7\}), ((B_2, B_5), \\
 &\{\Xi_2, \Xi_3, \Xi_6\}), \\
 &((B_3, B_3), \{\Xi_1, \Xi_3, \Xi_4, \Xi_7\}), (B_3, B_4), \{\Xi_3, \Xi_4, \Xi_7\}), ((B_3, B_5), \{\Xi_2, \Xi_3, \Xi_6, \Xi_7\})\}
 \end{aligned}$$

can be represented by an ordered pair (B_i, B_j) , by boxes and $F_A(B_i) \cap G_B(B_j)$ is represented by edges joining the combined line of two boxes and as far as values of $F_A(B_i) - (B_j)$ or $F_A(B_i) \cap G_B(B_j)$ are concerned, such values are shown by the following graph.

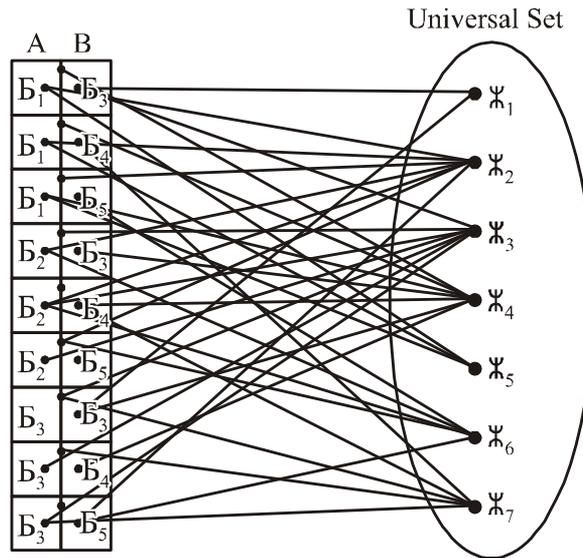


Figure 15. OR-operation

5. Conclusion

Soft set and operations on soft set can be represented by bipartite graph. In this paper, soft sets are represented by bipartite graph and some operations (union, restricted union, intersection, extended intersection, restricted difference, restricted symmetric difference, AND-Operations and OR-Operations) on soft sets are also represented by bipartite graph.

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