Scientific Inquiry and Review (SIR)
Volume 3, Issue 3, August 2019
ISSN (P): 2521-2427, ISSN (E): 2521-2435
Journal DOI: https://doi.org/10.32350/sir
Issue DOI: https://doi.org/10.32350/sir.33
Homepage: https://ssc.umt.edu.pk/sir/Home.aspx

Journal QR Code:

Article	**Representation of Soft Set and Its Operations by Bipartite Graph**	Indexing Agency
Author(s)	**Faiz Farid Muhammad Saeed Muzammil Ali**	
Online Published	August 2019	
Article DOI	https://doi.org/10.32350/sir.33.03	
Article QR Code	Faiz Farid	
To cite this article	Farid F, Saeed M, Ali M. Representation of soft and its operations by Bipartite. *Sci Inquiry Rev.* 2019;3(3):30–42. Crossref	
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A publication of the
School of Science, University of Management and Technology
Lahore, Pakistan.

Representation of Soft Set and Its Operations by Bipartite Graph

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Abstract

The idea of soft set was initiated by Molodstov. Soft sets have been used for decision making in dealing vague ideas. In this paper, soft sets are represented by bipartite graph. Operations on soft sets are also represented by bipartite graph.

Keywords: bipartite graph, complete bipartite graph, soft set

Introduction

Many theories have been developed to deal with uncertainty in real life problems. Molodtsov [1] pointed out the problem of the inadequacy of the fuzzy set theory and expounded the soft set theory to overcome this problem. A revolution followed when Molodstov gave the concept of soft set. Operations like union, intersection, AND-operation and OR-operation on soft set were formulated by Maji et al. [2]. Euler was the first to introduce graph theory which solved many mathematical problems in 1736. Rosenfieldin in 1975 introduced the concept of fuzzy graph. The concepts of soft graph was introduced by Rajesh K. Thumbakara and Boben George in 2014 [3]. After an year, Sumit Mohinta and T. K. Samanta [4] gave the idea of fuzzy soft graph in 2015. Further, M Irfan et al. [5] introduced a new method for graph representation based on the adjacency of vertices and soft set theory. With this representation of a graph, the application of algebraic operations in soft sets revealed many new aspects of graph theory.

In this work, a set of attributes and the universal set are represented by two partite sets and their mutual relationship is represented by joining the vertices of the two partite sets.

2. Preliminaries

2.1 Definition [3]

A graph $\breve{G}$ consists of two finite non-empty sets $V(\breve{G})$ and $E(\breve{G})$ called set of vertices and edges respectively, where edges are obtained by connecting vertices.

2.2 Definition [3]

A graph whose edge set is null or empty is called null graph.

2.3 Definition [3]

If two partite sets U $=$ {$Ӿ_1, Ӿ_2, Ӿ_3$} and W $=$ {$Б_1, Б_2$} of graph Ğ are such that every edge is obtained by joining the vertices of U and W, then Ğ is called bipartite graph.

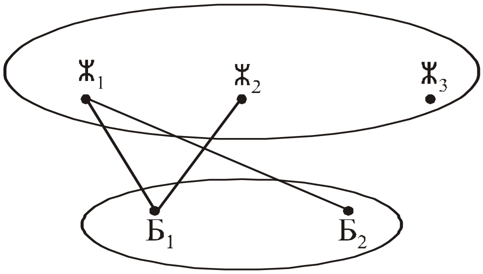

Figure 1. Bipartite graph

2.4 Definition [3]

A bipartite graph is called a complete bipartite if two partite sets U and W are such that each vertex of U is joined with all vertices of W. For example, U $=$ {$Б_1, Б_2, Б_3, Б_4, Б_5$} and W $=$ {$Ӿ_1, Ӿ_2, Ӿ_3, Ӿ_4, Ӿ_5, Ӿ_6, Ӿ_7$}, then the complete bipartite graph is symbolically written by $K_{5,7}$.

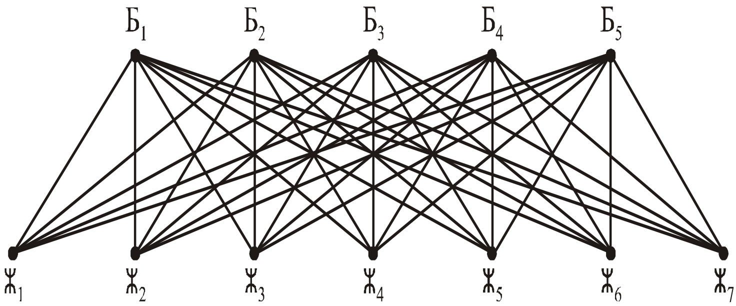

Figure 2. Complete bipartite, $K_{5,7}$

2.5 Definition [3]

Graphs $Ğ_1$ and $Ğ_2$ are equal if V ($Ğ_1$) $=$ V ($Ğ_2$) and E ($Ğ_1$) $=$ E ($Ğ_2$).

2.6 Definition [3]

Graph $Ğ_2$ is a sub-graph of $Ğ_1$, if V($Ğ_2$)$\subseteq$V($Ğ_2$) and E($Ğ_2$)$\subseteq$E($Ğ_1$) and is symbolically written as $Ğ_2 \subseteq Ğ_2$.

2.7 Definition [3]

Complement $\breve{G}^c$ of a graph $\breve{G}$ is a graph with the same vertex set but with a different edge set, that is, if uv$\in$ E($\breve{G}^c$) $\Leftrightarrow$uv$\notin$ E($\breve{G}$), then $V(\breve{G}^c) = V(\breve{G})or|V(\breve{G})| = n = |V(\breve{G}^c)|$; if $|E(\breve{G})| = m$, then $|E((\breve{G}^c)| = \binom{n}{2} - m$.

2.8 Definition [3]

Union of two graphs $\breve{G}_1$ and $\breve{G}_2$ is symbolically written as $\breve{G}_2 \cup \breve{G}_2$, which is a graph such that

$V(\breve{G}_1 U \breve{G}_2) = V(\breve{G}_1) \cup V(\breve{G}_2)$ and $E(\breve{G}_1 U \breve{G}_2) = E(\breve{G}_1) \cup E(\breve{G}_2)$.

2.9 Definition [3]

Intersection of two graphs $\breve{G}_1$ and $\breve{G}_2$is symbolically written as a $\breve{G}_1 \cap \breve{G}_2$, which is a graph such that

$(V(\breve{G}_1 \cap \breve{G}_2) = V(\breve{G}_1) \cap V(\breve{G}_2)$ and $E(\breve{G}_1 \cap \breve{G}_2) = E(\breve{G}_1) \cap E(\breve{G}_2)$.

2.10 Definition [1]

Let F be a set valued function given by $F: A \rightarrow P(X)$ where $A \subseteq E$ and P(X), then soft set over X is symbolically written as (F, A) or F_A and is given as $F_A = \{F(Б) \in P(X);\ Б \in A, F(Б) = \varnothing \text{ if } Б \notin A\}$.

2.11 Definition [2]

F_A is a soft subset of G_B and is symbolically represented by $F_A \tilde{\subseteq} G_B$. If (1) $A \subseteq B$ and (2) $\forall Б \in A, F(Б) \subseteq G(Б)$, then G_B is a super SS of F_A and is symbolically represented by $G_B \tilde{\supseteq} F_A$.

2.12 Definition [2]

F_A is a soft equal set to G_B and symbolically represented by $F_A = G_B$ if $F_A \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_A$.

2.13 Definition [2]

A SS F_A is called relative null SS w.r.t. A and symbolically represented by $\tilde{Ф}_A$, if $F(Б) = \varnothing, \forall Б \in A$.

2.14 Definition [2]

A SS F_A is called relative whole SS w.r.t. A and symbolically represented by $\tilde{X}_A$, if $F(Б) = X, \forall Б \in A$.

2.15 Definition [2]

A SS F_A is called absolute SS w.r.t. E and symbolically represented by $\widetilde{X}_E$, if $F(Б) = X, \forall Б \in E$.

2.16 Definition [2]

The relative complement F_A^r or $(F, A)^r$ of a SS F_A is defined as $F_A^r = (F^r, A)$, where $F^r(t) : A \rightarrow P(X)$ is the mapping given by $F^r(Б) = X - F(Б)$, $\forall Б \in A$.

2.17 Definition [2]

Union of two SSs F_A and G_B is a SS, symbolically represented by $F_A \tilde{\cup} G_B$ and is given by

$$F_A \tilde{\cup} G_B = \begin{cases} F(Б) & Б \in A - B \\ G(Б) & Б \in B - A \quad \forall\ Б \in A \cup B \\ F(Б) \cup G(Б) & Б \in A \cap B \end{cases}$$

2.18 Definition [2]

Restricted union of two SSs F_A and G_B over X is a SS, symbolically represented by $F_A \cup_R G_B$ and is given by $F(Б) \cup G(Б)\ \forall Б \in A \cap B$.

2.19 Definition [2]

Intersection of SSs F_A and G_B is a SS, symbolically represented by $F_A \tilde{\cap} G_B$ and is given by $F(Б) \cap G(Б)\ \forall\ Б \in A \cap B$.

2.20 Definition [2]

Extended intersection of SSs F_A and G_B is a SS, symbolically represented by $F_A \cap_\varepsilon G_B$ and is given by

$$F_A \cap_\varepsilon G_B = \begin{cases} F(Б) & Б \in A - B \\ G(Б) & Б \in B - A \quad \forall\ Б \in A \cup B \\ F(Б) \cap G(Б) & Б \in A \cap B \end{cases}$$

2.21 Definition [2]

Restricted difference of SSs F_A and G_B, symbolically represented by $F_A -_R G_B$ and is given by

$F(Б) - G(Б), \forall\ Б \in A \cap B$.

2.22 Definition [2]

OR-operation of SSs F_A and G_B is a SSH_C, symbolically represented by

$F_A \vee G_B$ and is given by $(H, A \times B)$ where $H(a,b) = F(a) \cup G(b), \forall (a,b) \in A \times B$.

2.23 Definition [2]

AND-operation of two SSs F_A and G_B is a SSH$_C$, symbolically represented by $F_A \wedge G_B$ and is given by $(H, A \times B)$, where $H(a,b) = F(a) \cap G(b)$, $\forall (a,b) \in A \times B$.

3. Representation of Soft Set by Bipartite Graph

3.1 Soft Set

Every SS can be graphically represented with the help of bipartite graph A by taking partite sets; the first set of attribute E with $A \sqsubset E$ and the second universe of discourse X and their elements are represented by vertices of these partite sets A and X.

Example 1

Let $X = \{Ӿ_1, Ӿ_2, Ӿ_3, Ӿ_4, Ӿ_5, Ӿ_6, Ӿ_7\}$, $E = \{Б_1, Б_2, Б_3, Б_4, Б_5\}$ and $A = \{Б_1, Б_2, Б_3\} \subset E$

$F_A = \{(Б_1, \{Ӿ_1, Ӿ_4, Ӿ_5\}), (Б_2, \{Ӿ_2, Ӿ_3, Ӿ_6\}), (Б_3, \{Ӿ_3, Ӿ_7\})\}$

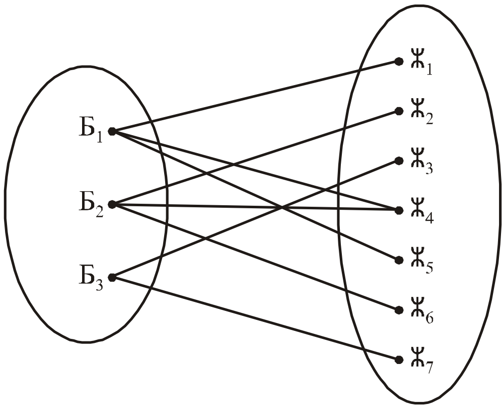

Figure 3. Soft set represented by bipartite graph

3.2 Soft Sub-Set

If G_B is the oft sub-set of a SS F_A, then the bipartite graph of G_B is also the sub-graph of F_A.

3.3 Absolute SS by Bipartite Graph

Absolute SS can be drawn by complete bipartite graph.

Example 2

Let $X = \{ⵣ_1, ⵣ_2, ⵣ_3, ⵣ_4, ⵣ_5\}$ and $E = \{Б_1, Б_2, Б_3, Б_4\}$, then absolute SS represents complete bipartite graph $K_{4,5}$.

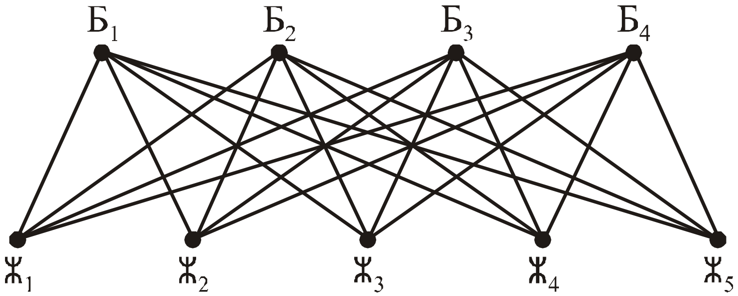

Figure 4. Absolute soft set represented by bipartite graph

3.4 Null SS

Null SS can be represented by null graph.

3.5 Relative Complement of a SS

Complement graph of a soft set F_A is the relative complement graph.

Example 3

Let $X = \{ⵣ_1, ⵣ_2, ⵣ_3, ⵣ_4, ⵣ_5, ⵣ_6, ⵣ_7\}$, $E = \{Б_1, Б_2, Б_3, Б_4\}$, $A = \{Б_1, Б_2, Б_3\}$ and if

$F_A = \{(Б_1, \{ⵣ_2, ⵣ_3\}), (Б_2, \{ⵣ_1, ⵣ_2\}), (Б_3, \{ⵣ_1, ⵣ_2, ⵣ_3\})\}$

then

$F_A^r =$
$\{(Б_1, \{ⵣ_1, ⵣ_4, ⵣ_5, ⵣ_6\}), (Б_2, \{ⵣ_3, ⵣ_4, ⵣ_5, ⵣ_6\}), (Б_3, \{ⵣ_4, ⵣ_5, ⵣ_6\}), (Б_4, X)\}$

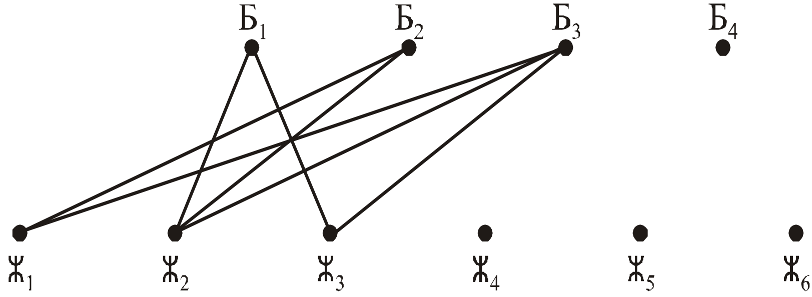

Figure 5. Soft set represented by bipartite graph

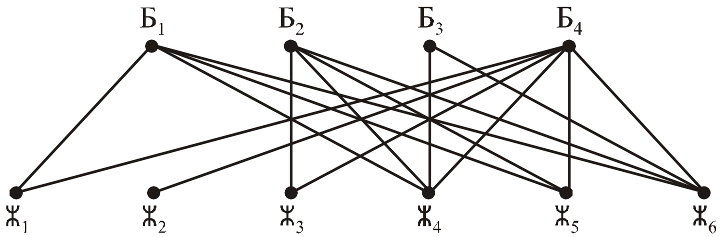

Figure 6. Relative complement of soft set represented by bipartite graph

4. Representation of Operations on Soft Set by Bipartite Graph

To illustrate operations on SSs by bipartite graph, let $X = \{Ж_1, Ж_2, Ж_3, Ж_4, Ж_5, Ж_6, Ж_7\}$, $E = \{Б_1, Б_2, Б_3, Б_4, Б_5\}$ $A = \{Б_1, Б_2, Б_3,\} \subset E$ and $B = \{Б_3, Б_4, Б_5\} \subset E$

$A = \{Б_1, Б_2, Б_3\} \subset E$ and $= \{Б_3, Б_4, Б_5\} \subset E$, then

$F_A = \{(Б_1, \{Ж_2, Ж_4, Ж_5\}), (Б_2, \{Ж_2, Ж_3, Ж_6\}), (Б_3, \{Ж_3, Ж_7\})\}$

and $G_B = \{(Б_3, \{Ж_1, Ж_3, Ж_4\}), (Б_4, \{Ж_4, Ж_7\}), (Б_5, \{Ж_2, Ж_6\})\}$.

4.1 Union of Two SSs

$$F_A \tilde{\cup} G_B = \{(Б_1, \{Ж_2, Ж_4, Ж_5\}), \begin{pmatrix} Б_2, \{Ж_2, Ж_3, Ж_6\}, (Б_3, \{Ж_1, Ж_3, Ж_4, Ж_7\}), (Б_4, \{Ж_4, Ж_7\}), \\ (Б_5, \{Ж_2, Ж_6\}) \end{pmatrix}\}$$

can be represented by bipartite graph as shown below.

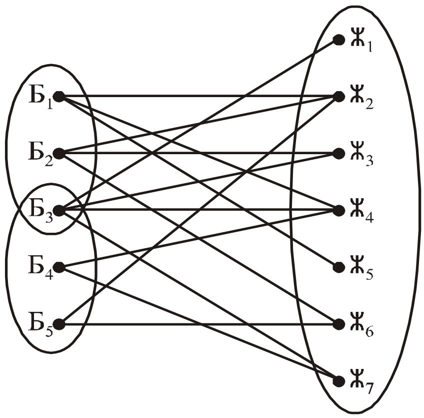

Figure 7. Union of soft sets represented by bipartite graph

4.2 Restricted Union

$F_A \cup_R G_B = \{(Б_3, \{Ж_1, Ж_3, Ж_4, Ж_7\})\}$ can be represented by bipartite graph as shown below.

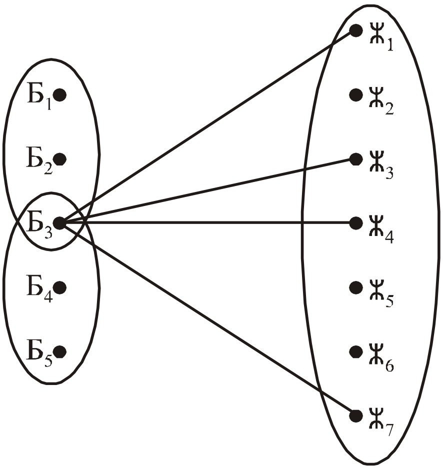

Figure 8. Restricted union of soft sets represented by bipartite graph

4.3 Intersection

$F_A \tilde{\cap} G_B = \{(Б_3, \{Ж_3\})\}$ can be represented by bipartite graph as shown below.

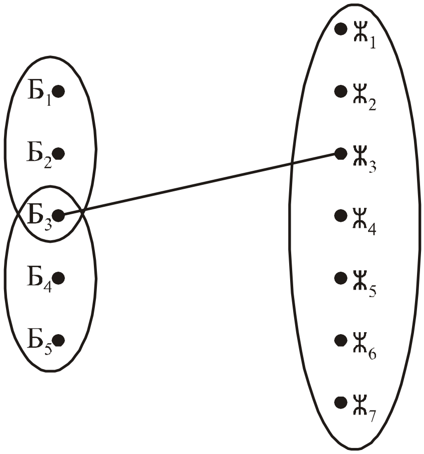

Figure 9. Intersection of soft sets represented by bipartite graph

4.4 Extended Intersection

$$F_A \cap_\varepsilon G_B = \{(Б_1, \{Ж_2, Ж_4, Ж_5\}), \begin{pmatrix} Б_2, \{Ж_2, Ж_3, Ж_6\}, (Б_3, \{Ж_3, \}), (Б_4, \{Ж_4, Ж_7\}), \\ (Б_5, \{Ж_2, Ж_6\}) \end{pmatrix}\}$$

can be represented by bipartite graph as shown below.

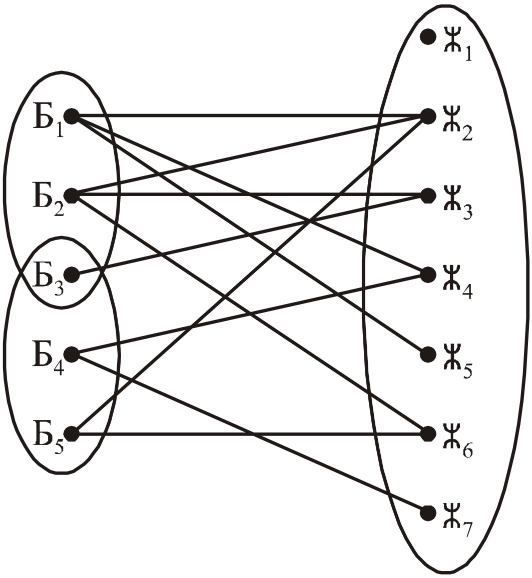

Figure 10. Extended intersection of soft sets represented by bipartite graph

4.5 Restricted Difference of SSs

$F_A -_R G_B = \{(Б_3, \{Ӿ_7\})\}$ and $G_B -_R F_A = \{(Б_3, \{Ӿ_1, Ӿ_4\})\}$ can be represented by bipartite graph as shown below.

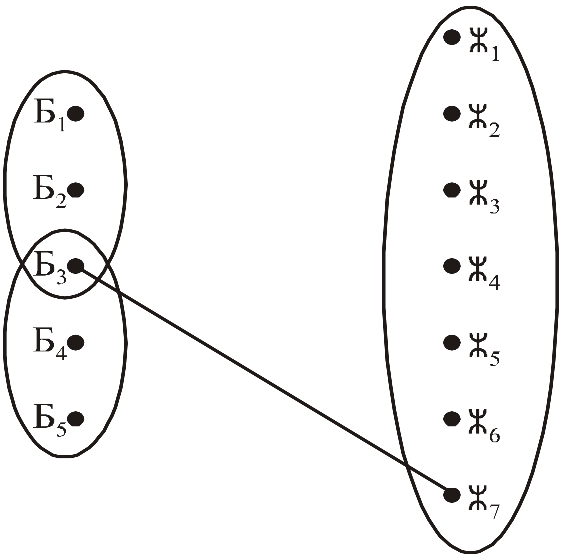

Figure 11. $F_A -_R G_B$

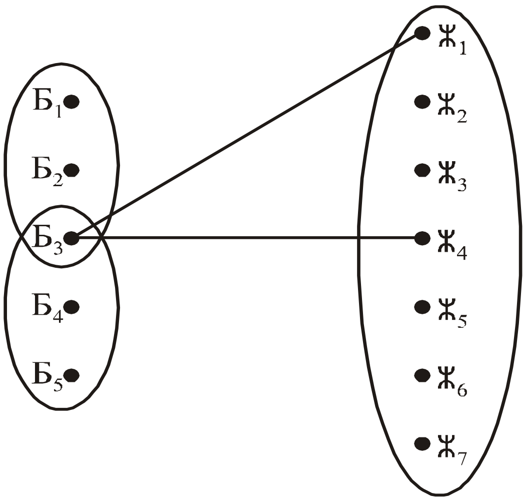

Figure 12. $G_B -_R F_A$

4.6 Restricted Symmetric Difference of Two SSs

$F_A \widetilde{\Delta} G_B = \{(\text{Б}_3, \{\text{Ж}_1, \text{Ж}_4, \text{Ж}_7\})\}$ can be represented by bipartite graph as shown below.

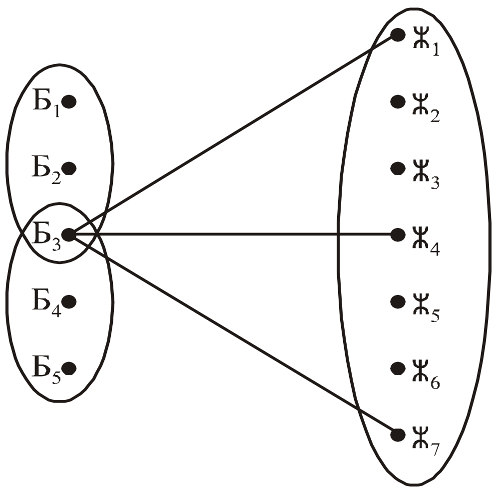

Figure 13. Restricted symmetric difference of two SSs

4.7 AND-Operation

$$F_A \wedge G_B = \{((\text{Б}_1, \text{Б}_3), \{\text{Ж}_4\}), ((\text{Б}_1, \text{Б}_4), (\text{Ж}_4\}), ((\text{Б}_1, \text{Б}_5), \{\text{Ж}_2\}), ((\text{Б}_2, \text{Б}_3), \{\text{Ж}_3\}),$$

$$((\text{Б}_2, \text{Б}_4), \{\text{Ж}_2, \text{Ж}_6\}), ((\text{Б}_2, \text{Б}_5), \{\text{Ж}_2, \text{Ж}_6\}), ((\text{Б}_3, \text{Б}_3), \{\ \text{Ж}_3, \}), (\text{Б}_3, \text{Б}_4), \{\text{Ж}_7\})\}$$

can be represented by an ordered pair $(\text{Б}_i, \text{Б}_j)$, by boxes and $F_A(\text{Б}_i) \cap G_B(\text{Б}_j)$ is represented by edges joining combined line of two boxes as shown by the following graph.

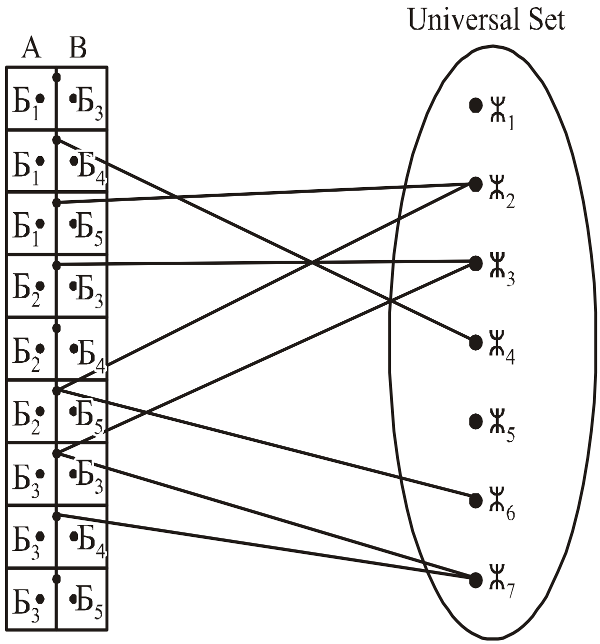

Figure 14. OR-operation

4.8 OR-Operation

OR-Operation of SSs F_A and G_B which is

$$F_A \vee G_B = \{((\text{Б}_1, \text{Б}_3), \{\text{Ж}_1, \text{Ж}_2, \text{Ж}_3, \text{Ж}_4, \text{Ж}_5\}), ((\text{Б}_1, \text{Б}_4), (\text{Ж}_2, \text{Ж}_4, \text{Ж}_5, \text{Ж}_7\}), ((\text{Б}_1, \text{Б}_5), \{\text{Ж}_2, \text{Ж}_4, \text{Ж}_5, \text{Ж}_6\}),$$

$$((\text{Б}_2, \text{Б}_3), \{\text{Ж}_1, \text{Ж}_2, \text{Ж}_3, \text{Ж}_4, \text{Ж}_6\}), ((\text{Б}_2, \text{Б}_4), \{\text{Ж}_2, \text{Ж}_3, \text{Ж}_4, \text{Ж}_6, \text{Ж}_7\}), ((\text{Б}_2, \text{Б}_5), \{\text{Ж}_2, \text{Ж}_3, \text{Ж}_6\}),$$

$$\left((\text{Б}_3, \text{Б}_3), \{\text{Ж}_1, \text{Ж}_3, \text{Ж}_4, \text{Ж}_7\}\right), (\text{Б}_3, \text{Б}_4), \{\text{Ж}_3, \text{Ж}_4, \text{Ж}_7), ((\text{Б}_3, \text{Б}_5), \{\text{Ж}_2, \text{Ж}_3, \text{Ж}_6, \text{Ж}_7\}\}$$

can be represented by an ordered pair $(\text{Б}_i, \text{Б}_j)$, by boxes and $F_A(\text{Б}_i) \cap G_B(\text{Б}_j)$ is represented by edges joining the combined line of two boxes and as far as values of $F_A(\text{Б}_i) - (\text{Б}_j)$ or $F_A(\text{Б}_i) \cap G_B(\text{Б}_j)$ are concerned, such values are shown by the following graph.

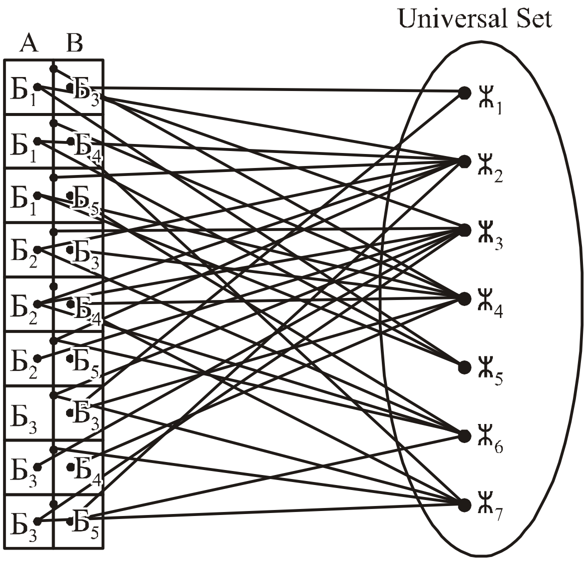

Figure 15. OR-operation

5. Conclusion

Soft set and operations on soft set can be represented by bipartite graph. In this paper, soft sets are represented by bipartite graph and some operations (union, restricted union, intersection, extended intersection, restricted difference, restricted symmetric difference, AND-Operations and OR-Operations) on soft sets are also represented by bipartite graph.

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