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Title: **Generators to Construct an Efficient Generalized Class of Minimal Circular Neighbor Designs**

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Generators to Construct an Efficient Generalized Class of Minimal Circular Neighbor Designs

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ABSTRACT

A design that is robust to neighbor effects is one that can protect against the neighbor effects. In situations where minimal circular balanced neighbor designs (MCBNDs) cannot be constructed, minimal circular weakly balanced neighbor designs-I (MCWBNDs-I) are preferred, which is an efficient generalized class. MCWBNDs-I are neighbor designs in which $v/2$ pairs of distinct treatments appear twice as neighbors, while the remaining pairs appear once. New generators are developed in this study to obtain MCWBNDs-I in blocks of three different sizes.

Keywords: circular block, minimal design, neighbor balanced design, strongly balanced neighbor design, neighbor effect

1. INTRODUCTION

In field experiments, competition or interference between adjacent units can increase the variability of the results and may reduce efficiency. In experiments where neighboring unit treatments influence the effect of the current unit treatment, balanced neighbor designs (BNDs) are used to control neighbor effects. BNDs reduce bias caused by neighbor effect [1-3].

- If each treatment appears λ' times with all other treatments as neighbor, then it is neighbor balanced. For $\lambda' = 1$, design would be minimal.
- A neighbor design is known as generalized neighbor design (GND) in which the constancy of λ' is relaxed. GN_2-design refers to GND with only two possible values of λ'.
- Minimal weakly BNDs are GN_2-designs in which $\lambda'_1 = 1$ and $\lambda'_2 = 2$.

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- MCWBNDs come with $v/2$ unordered pairs appearing twice as neighbors and the remaining pairs appearing once are known as MCWBNDs-I.

Rees [4] applied CBNDs in serology (virus research) and presented minimal CBNDs for some odd value of v (number of treatments). Azais [5] suggested that BNDs or partial BNDs control neighbor effects. Hwang [6], Cheng [7], Iqbal [8], Akhtar [9], Shehzad [10], Ahmed [11], Akhtar [12], Shahid [13], and Shahid [14] constructed BNDs for some cases. Misra [15], Chaure and Misra [16], Mishra [17], and Kedia and Misra [18] constructed GNDs for some specific cases. Ahmed [19], Zafaryab [20], Iqbal [21], Ahmed and Akhtar [22] developed some series to generate circular partial BNDs. Noreen [23] developed some series for MCWBNDs-I and MCWBNDs-II in equal block sizes. In this study, some generators have been developed to generate sets of cyclic shifts for MCWBNDs-I in blocks of three different sizes.

2. METHOD OF CYCLIC SHIFTS

For neighbor designs, Iqbal [24] introduced the method of cyclic shifts. Its Rule I is described here only for MCWBNDs-I.

If $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$, where $j = 1, 2, \ldots, l$ and $1 \leq q_{ji} \leq v-1$. For v, even if each of 1, 2, …, v-1 appears once in S* but $v/2$ appears twice, then these sets produce MCBND, where S* contains

(i) All values in each of sets S_j, along with their sum (mod v), and

(ii) Complements of all values present in (i). Complement of 'd' is 'v-d' in Rule I.

Example 2.1. S_1 = [2,4,5,12], S_2 = [9,10,21], S_3 = [7,11] produce MCWBND-I for $v = 24$, $k_1 = 5$, $k_2 = 4$, and $k_3 = 3$.

Proof: S* = [2,4,5,12,23,9,10,21,8,7,11,18,22,20,19,12,1,15,14,3,16,17, 13,6]. Here, each of 1, 2, …, 23 appears once except 12 which appears twice. Hence, S_1 = [2,4,5,12], S_2 = [9,10,21], S_3 = [7,11] produce MCWBND-I for $v = 24$, $k_1 = 5$, $k_2 = 4$, and $k_3 = 3$.

To generate the design, use v blocks for S_1. Write 0, 1, …, v-1 in Row 1. Add the 1st value of S_1 (mod v) to Row 1 to get Row 2. Similarly, add the 2nd value of S_1 (mod v) to Row 2 to get Row 3, and so on, see Table 1.

Table 1. Blocks Obtained from $S_1 = [2,4,5,12]$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14	15	16	17
11	12	13	14	15	16	17	18	19	20	21	22
23	0	1	2	3	4	5	6	7	8	9	10
13	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
12	13	14	15	16	17	18	19	20	21	22	23
14	15	16	17	18	19	20	21	22	23	0	1
18	19	20	21	22	23	0	1	2	3	4	5
23	0	1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20	21	22

v more blocks are taken for S_2, see Table 2.

Table 2. Blocks Obtained from $S_2 = [9,10,21]$

25	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
0	1	2	3	4	5	6	7	8	9	10	11
9	10	11	12	13	14	15	16	17	18	19	20
19	20	21	22	23	0	1	2	3	4	5	6
16	17	18	19	20	21	22	23	0	1	2	3
37	**38**	**39**	**40**	**41**	**42**	**43**	**44**	**45**	**46**	**47**	**48**
12	13	14	15	16	17	18	19	20	21	22	23
21	22	23	0	1	2	3	4	5	6	7	8
7	8	9	10	11	12	13	14	15	16	17	18
4	5	6	7	8	9	10	11	12	13	14	15

v more blocks are taken for S_3, see Table 3.

Table 3. Blocks Obtained from $S_3 = [7,11]$

49	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
0	1	2	3	4	5	6	7	8	9	10	11
7	8	9	10	11	12	13	14	15	16	17	18
18	19	20	21	22	23	0	1	2	3	4	5
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**
12	13	14	15	16	17	18	19	20	21	22	23
19	20	21	22	23	0	1	2	3	4	5	
6	7	8	9	10	11	12	13	14	15	16	17

Here, pairs (0,12), (1,13), (2,14), (3,15), (4,16), (5,17), (6,18), (7,19), (8,20), (9,21), (10,22), and (11,23) appear twice as neighbors.

3. MCWBNDS-I FOR *V* (EVEN) = $2IK_1+2K_2+2K_3$ IN THREE DIFFERENT BLOCK SIZES

For *v* even, let *m* = (*m*-2)/2. The procedure is described to generate sets of shifts from constructor **A** or **B** to obtain MCWBNDs-I in three different block sizes.

- Constructor **A** = [1, 2,…, *m*, (*m*+1)] for *m* (mod 4) ≡ 2.
- Constructor **B** = [1, 2, …, (*m*-3)/4, (*m*+5)/4, (*m*+9)/4, …, *m*, (*m*+1), and 7(*m*+1)/4] for *m* (mod 4) ≡ 3.

Divide values of the selected constructor (C) into *i* groups, each of k_1 values, one of k_2 values, and one of k_3 values, such that the sum of each group is divisible by *v*. The required *i* sets of shifts for k_1 and one each for k_2 and k_3 would be obtained by discarding any one value from each group. Here, *a* (mod *b*) ≡ *c*, which means '*c*' is remainder if '*a*' is divided by '*b*'.

Example 3.1. Constructor **B** = [1, 2, 4, 5, …, 12, 21] for *v* = 24 can be divided into the following three groups.

G-I = (1,2,4,5,12), G-II = (8,9,10,21), G-III = (6,7,11)

Deleting the smallest value of each group, following are the sets of shifts to obtain MCWBND-I for *v* = 14, $k_1 = 5$, $k_2 = 4$, $k_3 = 3$, Es = 0.7410, and E*n* = 0.7772.

$S_1 = [2,4,5,12]$, $S_2 = [9,10,21]$, $S_3 = [7,11]$

4. GENERATORS TO OBTAIN MCWBNDS-I FROM CONSTRUCTOR A (*M* (MOD 4) ≡ 2)

4.1. MCWBNDs-I for $k_1 = 4l$

MCWBNDs-I can be generated for $k_1 = 4l$ and

- $k_2 = k_1-1$, $k_3 = 4$, *i* integer and $l > 1$
- $k_2 = k_1-2$, $k_3 = k_1-3$, $l > 1$ and *i* integer
- $k_2 = k_1-2$, $k_3 = 5$, $l > 2$ and *i* integer

Table 4. MCWBNDs-I obtained from Generator 4.1

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
54	12	11	4	1	A	[4,5,6,7,8,9,10,11,12,16,17],[14,15,18,19,20,21,22,23,25,26], [2,4,27]	0.87	0.90
38	8	6	5	1	A	[2,3,4,5,6,8,9],[19,11,12,13,14],[15,16,17,18]	0.84	0.86
54	12	10	5	1	A	[2,4,8,12,13,16,17,19,21,23,26],[9,10,11,14,18,20,22,25,27], [5,7,15,24]	0.85	0.90

4.2. MCWBNDs-I for $k_1 = 4l+2$

MCWBNDs-I can be generated for $k_1 = 4l+2$ and

- $k_2 = k_1-1, k_3 = k_1-2$ and i odd
- $k_2 = k_1-1$, $k_3 = 4$ and i odd
- $k_2 = k_1-2$, $k_3 = k_1-3$ and i even
- $k_2 = k_1-2$, $k_3 = 3$, $l > 1$ and i even
- $k_2 = k_1-2$ and $k_3 = 5$, $l > 1$ and i odd

Table 5. MCWBNDs-I obtained from Generator 4.2

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
30	6	5	4	1	A	[6,10,12,13,15], [3,7,8,11],[5,9,14]	0.82	0.82
46	10	9	4	1	A	[4,5,6,7,8,9,10,17,23],[12,13,14,15,16,18,19,20],[2,21,22]	0.85	0.89
38	6	4	3	1	A	[3,4,5,6,18],[10,11,12,19,15],[8,13,16],[14,17]	0.84	0.81
62	10	8	3	2	A	[4,8,9,10,14,16,17,21,23],[7,12,18,19,26,27,28,31], [3,6,15,20,24,25,30],[22,29]	0.87	0.88
46	10	8	5	1	A	[4,5,6,7,8,9,10,17,23],[14,15,16,18,19,21,22],[2,11,12,20]	0.85	0.89

4.3. MCWBNDs-I for k_1 odd

MCWBNDs-I can be generated for k_1 odd and

- $k_2 = k_1-1$, $k_3 = k_1-2$, $i(\text{mod } 4) \equiv 0$
- $k_2 = k_1-1$, $k_3 = 4$ and $i(\text{mod } 4) \equiv 3$
- $k_2 = k_1-2$, $k_3 = k_1-3$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1-2$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 1$
- $k_2 = k_1-2$, $k_3 = 5$ and $i(\text{mod } 4) \equiv 3$

Table 6. MCWBNDs-I obtained from Generator 4.3

v	k_1	k_2	k_3	*i*	C	Sets of Shifts	Es	E*n*
54	5	4	3	4	A	[5,14,15,16],[8,9,13,17],[2,6,22,23],[19,20,24,27],[11,12,21], [25,26]	0.87	0.79
78	9	8	4	3	A	[3,4,5,6,7,8,9,34],[14,15,16,17,18,20,21,22], [23,24,25,26,29,27,30,31],[12,28,33,36,37,38,39],[10,32,35]	0.89	0.88
46	7	5	4	2	A	[3,4,5,6,7,19],[11,12,13,14,15,17],[18,20,22,23],[8,16,21]	0.85	0.76
30	7	5	3	1	A	[2,3,4,5,7,8],[11,12,13,15],[10,14]	0.82	0.82
78	9	7	5	3	A	[3,4,5,6,7,8,9,34],[12,13,14,15,16,17,19,39], [22,23,24,25,26,28,27,38], [31,32,33,35,36,37],[10,18,20,29]	0.89	0.88

4.4. MCWBNDs-I for $k_1(\text{mod } 4) \equiv 1$

MCWBNDs-I can be generated for $k_1(\text{mod } 4) \equiv 1$ and

- $k_2 = k_1-1$, $k_3 = k_1-3$ and $i(\text{mod } 4) \equiv 1$
- $k_2 = k_1-1$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 0$
- $k_2 = k_1-1$, $k_3 = 5$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1-2$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1-2$, $k_3 = 4$ and $i(\text{mod } 4) \equiv 0$

Table 7. MCWBNDs-I obtained from Generator 4.4

v	k_1	k_2	k_3	*i*	C	Sets of Shifts	Es	E*n*
46	9	8	6	1	A	[2,3,4,5,6,7,8,10],[14,15,16,17,18,22,23],[11,12,19,20,21]	0.85	0.89
94	9	8	3	4	A	[3,4,5,6,8,9,10,47],[13,14,15,16,17,18,37,46], [24,25,26,27,28,41,44,45], [23,31,32,33,35,36,42,43], [11,19,21,29,30,38,39],[34,40]	0.90	0.88
86	13	12	5	2	A	[3,4,5,6,7,8,9,10,11,43,23,41], [14,15,16,17,19,20,21,22,24,25,26,27], [18,28,29,30,31,32,33,34,35,36,37],[38,39,40,42]	0.89	0.92
56	9	7	3	2	A	[3,4,5,6,8,9,26,49],[14,15,17,18,19,23,24,25], [11,12,16,20,21,22],[27,28].	0.87	0.87
94	9	7	4	4	A	[3,4,5,6,8,9,10,47],[13,14,15,16,17,18,37,46], [24,25,26,27,28,42,43,44], [29,30,31,32,33,34,35,36], [19,20,21,38,39,40],[7,41,45]	0.90	0.88

4.5. Generators to Obtain MCWBNDs-I for $k_1(\text{mod } 4) \equiv 3$

MCWBNDs-I can be generated for $k_1(\text{mod } 4) \equiv 1$ and

- $k_2 = k_1\text{-}1$, $k_3 = k_1\text{-}3$ and $i(\text{mod } 4) \equiv 3$
- $k_2 = k_1\text{-}1$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1\text{-}2$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 0$
- $k_2 = k_1\text{-}2$, $k_3 = 4$ and $i(\text{mod } 4) \equiv 2$

Table 8. MCWBNDs-I obtained from Generator 4.5Example 4.5.

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
62	7	6	4	3	A	[4,5,6,7,18,19],[11,13,14,15,30,31],[12,17,20,21,23,29], [16,22,25,24,28],[8,26,29]	0.87	0.85
46	7	6	3	2	A	[4,5,6,9,10,11],[7,12,14,15,19,23],[13,16,18,17,20],[21,22]	0.85	0.84
72	7	5	3	4	A	[10,22,23,24,26,33],[7,11,28,30,31,35],[13,17,18,25,32,34], [3,12,16,20,29,63],[8,14,19,27],[21,36]		
70	11	9	4	2	A	[2,3,4,5,7,8,9,10,11,12],[14,15,16,18,19,20,21,22,23,35], [45,29,30,31,32,33,34,28],[17,24,25]	0.88	0.90

5. GENERATORS TO OBTAIN MCWBNDS-I FROM CONSTRUCTOR B (M (MOD 4) ≡ 3)

5.1. MCWBNDs-I for $k_1 = 4l$

MCWBNDs-I can be generated for $k_1 = 4l$ and

- $k_2 = k_1\text{-}1$, $k_3 = k_1\text{-}3$ and i integer
- $k_2 = k_1\text{-}1$, $k_3 = 5$, $l > 1$ and i integer

Table 9. MCWBNDs-I obtained from Generator 5.1

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
40	8	7	5	1	B	[3,4,6,7,8,35,12],[10,11,12,13,16,17],[14,18,19,20]	0.83	0.87
56	12	11	5	1	B	[26,5,6,8,9,10,11,49,12,25,4],[2,13,14,15,18,19,20,21,22,23], [17,24,27,28]	0.87	0.91

5.2. MCWBNDs-I for $k_1 = 4l+2$

MCWBNDs-I can be generated for $k_1 = 4l+2$ and

- $k_2 = k_1$-1, $k_3 = k_1$-3 and *i* even
- $k_2 = k_1$-1, k_3 = 3, *l* > 1 and *i* even
- $k_2 = k_1$-1, k_3 = 5 and *i* odd
- $k_2 = k_1$-2, k_3 = 4 and *i* even

Table 10. MCWBNDs-I obtained from Generator 5.2

v	k_1	k_2	k_3	*i*	C	Sets of Shifts	Es	E*n*
40	6	5	3	2	B	[4,8,12,35,20],[11,13,14,16,17],[6,7,10,15],[18,19]	0.84	0.82
64	10	9	3	2	B	[3,4,5,6,7,9,10,26,56],[14,15,16,17,18,19,20,28,32], [12,21,23,24, 25,27,30,29],[22,31]	0.88	0.89
48	10	9	5	1	B	[2,3,4,5,8,9,10,42,12],[13,14,15,16,17,18,19,21], [20,22,23,24]	0.86	0.81
64	10	8	4	2	B	[3,4,5,6,7,9,10,26,56],[14,15,16,17,18,19,20,28,32], [21,23,24,25,27,31,29],[11,22,30]	0.88	0.89

5.3. MCWBNDs-I for k_1 odd

MCWBNDs-I can be generated for k_1 odd and

- $k_2 = k_1$-1, $k_3 = k_1$-3 and *i*(mod 4) ≡ 2
- $k_2 = k_1$-1, k_3 = 3 and *i*(mod 4) ≡ 1
- $k_2 = k_1$-1, k_3 = 5 and *i*(mod 4) ≡ 3
- $k_2 = k_1$-2, $k_3 = k_1$-3 and *i*(mod 4) ≡ 1
- $k_2 = k_1$-2, k_3 = 4 and *i*(mod 4) ≡ 1

Table 11. MCWBNDs-I obtained from Generator 5.3

v	k_1	k_2	k_3	*i*	C	Sets of Shifts	Es	E*n*
32	7	6	3	1	B	[3,5,6,7,13,28],[9,10,11,12,14],[15,16]	0.83	0.83
80	9	8	5	3	B	[3,4,5,6,7,8,9,36],[14,15,16,17,38,18,39,70], [22,23,24,25,26,27,32,40], [35,29,30,31,33,28,34],[11,12,19,37]	0.89	0.88
32	7	5	4	1	B	[3,5,6,7,13,28],[11,12,15,16],[8,9,14]	0.83	0.83
40	9	7	4	1	B	[7,8,9,10,11,18,19,35],[6,12,13,14,15,16], [2,17,20]	0.84	0.87
48	7	6	4	2	B	[4,5,7,17,18,42],[10,11,13,15,14,24],[12,19,20,21,22],[8,16,23]	0.86	0.84

5.4. MCWBNDs-I for k_1(mod 4) ≡ 1

MCWBNDs-I can be generated for k_1(mod 4) ≡ 1 and

- $k_2 = k_1-1$, $k_3 = k_1-2$ and $i(\text{mod } 4) \equiv 1$
- $k_2 = k_1-1$, $k_3 = 4$ and $i(\text{mod } 4) \equiv 3$
- $k_2 = k_1-2$, $k_3 = k_1-3$ and $i(\text{mod } 4) \equiv 3$
- $k_2 = k_1-2$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1-2$, $k_3 = 5$ and $i(\text{mod } 4) \equiv 0$

Table 12. MCWBNDs-I obtained from Generator 5.4

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
24	5	4	3	1	B	[2,4,5,12],[9,10,21],[7,11]	0.80	0.78
98	9	8	4	3	B	[3,4,5,7,8,9,10,48],[13,14,15,16,17,18,46,47], [24,25,26,27,34,42,43,44], [29,30,31,32,33,35,37,40], [11,19,20,28,36,38,39],[41,45,84]	0.87	0.88
80	9	7	6	3	B	[3,4,5,6,7,8,9,36],[13,14,16,17,22,38,39,70], [23,24,25,26,27,31,32,34], [19,20,28,29,30,33], [15,21,35,37,40]	0.89	0.88
56	9	7	3	2	B	[2,5,9,10,14,19,24,28],[4,6,8,13,15,18,22,23], [16,17,21,26,27,49],[20,25]		
96	9	7	5	4	B	[3,5,7,8,9,10,19,33],[13,14,17,30,37,40,47,84], [15,21,22,24,25,26,27,28], [23,29,31,32,34,35,36,48], [11,18,38,39,42,43],[41,44,45,46]	0.90	0.88

5.5. Generators to Obtain MCWBNDs-I for $k_1(\text{mod } 4) \equiv 3$

MCWBNDs-I can be generated for $k_1(\text{mod } 4) \equiv 3$ and

- $k_2 = k_1-1$, $k_3 = k_1-2$ and $i(\text{mod } 4) \equiv 3$
- $k_2 = k_1-1$, $k_3 = 4$ and $i(\text{mod } 4) \equiv 2$
- $k_2 = k_1-2$, $k_3 = 3$ and $i(\text{mod } 4) \equiv 0$
- $k_2 = k_1-2$, $k_3 = 5$ and $i(\text{mod } 4) \equiv 2$

Table 13. MCWBNDs-I obtained from Generator 5.5

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
64	7	6	5	3	B	[4,5,6,7,19,20],[10,11,13,15,22,56],[16,17,18,21,23,31], [14,24,25,26,27], [28,29,30,32]	0.88	0.85
48	7	6	4	2	B	[5,7,11,12,15,42],[9,10,13,14,18,24],[16,17,19,20,22], [3,21,23]	0.86	0.85

v	k_1	k_2	k_3	i	C	Sets of Shifts	Es	En
72	7	5	3	4	B	[4,5,6,7,23,24],[11,12,13,14,21,63],[16, 17,18,20,22,36], [2,25,26,27,28,35], [29,31,32,33],[30,34]	0.88	0.85
72	11	9	5	2	B	[2,3,4,5,6,7,8,15,30,63],[13,16,17,18,19 ,20,21,22,23,36], [14,24,25,26,27, 29,28,31],[32,33,34,35]	0.88	0.90

6. EFFICIENCY OF SEPARABILITY AND OF NEIGHBOR EFFECTS

6.1. Efficiency of Neighbor Effects

The efficiency factor for both direct and neighbor effects is the harmonic mean of eigenvalues (non-zero) of the respective information matrix [25, 26]. For a high value of En, the design would be suitable to estimate neighbor effects.

6.2 Efficiency for Separability

[27] developed the following measure of efficiency for separability (Es).

$$Es = \left[1 - 1/(v\sqrt{v-1})\right] \times 100\%$$

7. CONCLUSION

New generators have been developed to generate sets of shifts in order to obtain MCWBNDs-I in blocks of three different sizes. MCWBNDs-I obtained through these newly developed generators possess high values of Es and En. Therefore, these designs are efficient to control neighbor effects as well as to estimate the direct effect and neighbor effects independently.

CONFLICT OF INTEREST

The authors of the manuscript have no financial or non-financial conflict of interest in the subject matter or materials discussed in this manuscript.

DATA AVALIABILITY STATEMENT

Data availability is not applicable as no new data was created.

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