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Author (s): Muhammad Javaid Afzal[1], and Farah Javaid[2]

Affiliation (s): [1]Government Islamia Graduate College Civil Lines, Lahore, Pakistan
[2]Government APWA Graduate College (W), Lahore, Pakistan

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Non-Kerr Law Medium: Optical Solitons and their Perturbation

Muhammad Javaid Afzal[1*], and Farah Javaid[2]

[1]Government Islamia Graduate College Civil Lines Lahore, Pakistan
[2]Government APWA Graduate College (W) Lahore, Pakistan

ABSTRACT

This investigation focuses on the perturbation analysis of optical solitons within a medium in accordance with the non-Kerr law. In the field of partial differential equations, generalized nonlinear Schrodinger equation (GNLSE) is an integrable nonlinear equation. In the non-Kerr law non-linear medium, GNLSE is utilized for doing an analytical analysis of soliton perturbations as well as the soliton itself. In case of non-Kerr law instance, the application of quasi-stationarity results in a soliton that is very close to being approximated. Several edge scenarios of nonlinearity that vary from the Kerr law remain the primary focus of the current study. Although it was found that a disturbance of the nonlinear damping kind remains present, equations can be solved to find solutions. Consequently, GNLSE cannot be integrated because of the presence of higher-order dispersion.

Keywords: GNLSE (generalized nonlinear Schrodinger equation), Ker Law, non-Ker Law, optical solitons, perturbation

1. INTRODUCTION

Optics uses ‘soliton’ to describe an optical field that remains unchanged during propagation due to nonlinear and linear influences. Several experiments have been conducted with optical fiber and light pulses. The remarkable stability of solitons allows for long-distance communication since solitons go farther without the use of amplifiers and boost the transmission capacity by a factor of two [1].

In 1973 [2], AT&T Bell labs employee Akira Hasegawa was the first to postulate that solitons may arise in optical fiber by a combination of anomalous dispersion and self-phase modulation. Robin Bullough [3] provided a mathematical demonstration of the reality of optical solitons in the same year. To improve the practical application of optical communications, the idea of a soliton-based transmitting system emerged.

*Corresponding Author: javaidphy@gmail.com

Optical solitons refer to the propagation of soliton pulses. They produce an optical field that allows them to keep their form as they travel, partly due to a combination of nonlinear and linear phenomena in the medium [4]. The two varieties of optical solitons include spatial and temporal solitons.

The diffraction phenomena during the transmission of solitonic waves over an optical fiber counteracts their linear effects. If the medium's refractive index can be altered by an electromagnetic field, a soliton pulse would be the method of choice [5]. Fiber acts like other fibers with the same grade index. During the spread of the soliton pulse, electromagnetic field maintains its restricted shape.

The presence of spatial soliton can be understood by imagining a simple convex lens. As seen in Figure l, convex lenses can be employed to focus on the visual field.

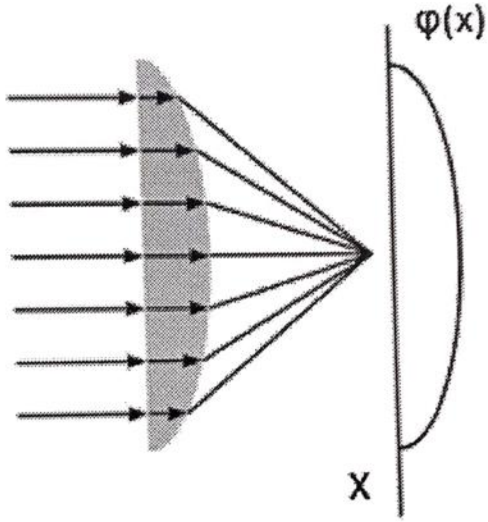

Figure 1. Convex Lens Focusing on Light Pulses

The lens's focusing action causes the phase shift to be non-uniform. Fig. 1 depicts the phase shift as a function of distance, written as $_{\varphi}$ (x). Thus, phase shift is controlled by the product of the field's phase constant (k_on) and the wave's width L(x).

Therefore, we may rewrite it as follows:

$$_{\varphi}(x) = k_0 nL(x) \tag{1}$$

So,

L(x) = breadth of the lens k_0n = total phase constant

Changing the lens's diameter to create such a phase shift is not appropriate for studying focusing phenomena. When lens width is held constant, the same focusing effect occurs when the index of refraction, that is, n(x) is altered in magnitude. Such is the operation of a fiber with a grade index. When these two forces cancel each other out, the propagating optical field remains unaffected. This primary principle underpins the existence of spatial solitons. Self-phase modulation (SPM) actually begins with the Kerr effect [6]. The index of refraction changes as a result of SPM. The strength of the soliton is essential.

$$\varphi(x) = k_0 n L(x) \tag{2}$$

Here,

$n = n_0 + n_2 I(x)$.

Therefore,

$\varphi(x) = k_0 L(n_0 + n_2 I(x))$.

Figure 1 can be reproduced by generating phase behavior befitting the wave's shape. The field then demonstrates the effect of self-focusing. Consequently, we can deduce that n, that is, the refractive index, must be positive. If the opposite is true, then the effect would be the opposite. Mathematical explanations exist for the waveguide of propagating optical solitons. Its existence proves that it can be followed by waves of other frequencies. If the medium is linear, then waves of varying frequency cannot interact with one another. Temporal solitons allow for the transmission of light pulses with no distortion to their original shape when the electric field is spatially limited [7]. Since dispersion can counteract nonlinear effects, there would be no change to their shape. Optical fiber transmission bit rate is severely constrained by GVD. On the other hand, soliton impulses are generated by GVD. The bandwidths of these are not zero. The propagation medium of solitons depends on their frequencies. Group delay dispersion (GDD) parameter D characterizes the impact itself. It's what is used to figure out the pulse's precise width [8].

$\Delta\tau \approx DL\Delta\lambda$

Here,

L = fiber length

Therefore,

$\Delta\lambda$ = bandwidth in terms of wavelength

Modern communication systems are designed to counteract the dispersal effect by coordinating with other fibers that have the same GDD parameter D' but different signs in different regions of the fiber. Thus, solitons continually enlarge and contract as they move. Temporal solitons totally do away with these issues.

The optical Kerr phenomena can be explained by a propagating electric field in a medium. Power in the x-y plane is regulated by the waveguide structure of fiber optics. When the e-field moves in a z direction with a phase constant of β_0, it can be written as follows:

$$\bar{E}\,(r, t) = A_m a(t, z) f(x,y) e^{i(\beta_0 z - \omega_0 t)} \tag{3}$$

A_m = Max. Amplitude of the e-field (4)

in the time domain, a (t, z) = the wave's envelope. Amplitude depends on z because impulse can change shape during propagation.

An electric field with a specific profile induces a soliton. It must be scaled to a power related to impulse length. In this study, equations are perturbed slightly to induce such impulses. Then, numerical methods are used to solve them. The stability of one-dimensional solitons was confirmed. These were 1+1 dimensional solitons or (1+1) since such solitons are limited to only one axis (x or t), instead of two and spread in the opposite direction, here z = [9]. A solitonic pulse is produced by adjusting the power or shape. It then modifies itself to reach the typical sech shape at the right power. Unfortunately, obtaining it requires enduring a power outage which complicates matters. It generates a second, travelling solitonic field that overlaps with the necessary field. The stability of one-dimensional solitons is excellent. If N is large enough, a first-order optical soliton can be manufactured; otherwise, we'll have to settle for a higher-order one. The high-power peaks that result could be harmful to the fiber. Limiting the field along the y-axis with a dielectric slab and along the x-axis with the soliton cause a (1+1) D spatial soliton to be generated. Unstable solitons exist in two and a half spatial dimensions and in one additional dimension. This shows that even a slight change in the medium's condition might cause the soliton pulse to diffract or collapse, causing damage. By using saturated, non-limiting material, where the Kerr relation n (I) = n + n2I is applicable unless it reaches the maximum value, it is possible to construct stable (2+1) D spatial solitons. This indicates that steady solitonic pulses can be generated in a three-dimensional environment [10].

This change to John Kerr's experimental setup makes the magnetic optical phenomenon seemingly hard to understand. Consider two types of bodily levels, simultaneously. These levels are designed to be played in pairs, so they must be paired later.

a) Quantum mechanics describes how a magnetic material reacts when exposed to an electromagnetic field from the outside.

b) Classical optics describes how a beam of light travels through space after being reflected.

The nexus between these actions and frequency-dependent organization of optical conductivity б(ω) onto permittivity ϵ (ω) is as follows:

$$f: б\,(\omega) \rightarrow \epsilon(\omega) \tag{5}$$

Important considerations in the fields of quantum mechanics and classical optics are required. Moreover, John Kerr indicators are the only ones that have been observed in different experimental examinations.

Optical solitons have been a focus of a significant number of studies. The fact that solitons cannot be integrated is crucial. Particularly, this can be observed via perturbation parameters. The problem grows more severe for nonlinearities that violate the Kerr law. The nonlinearity of the power-law also needs to be taken into account. NLSE is the guiding equation for power-law nonlinearity [11]. We investigate this equation with a Hamiltonian perturbation.

Nonlinear Schrodinger equation (NLSE) for solitonic pulse propagation in fibre optics is written in dimensionless form as [12]:

$$iq_t + aq_{xx} + b|q|^{2m}q = i\alpha q_x - i\gamma q_{xxx} + i\lambda(|q|^{2m}q)_x + iv(|q|^{2m})_x q \tag{6}$$

where,

'a' denotes the coefficients of GVD factors,

q denotes the dependent variable used for the profile of the wave, b denotes the nonlinearity coefficient, and

x and t are independent variables representing space and time accordingly.

Optical solitons were first predicted theoretically by Hasegawa [2]. These solitons are represented by locally shaped waves. They are one of the leading candidates for information transmission and processing. They may be capable of carrying out the ultra-high speed optical communication [13].

The generation of solitons depends on a fine-tuned equilibrium between nonlinearity-induced self-phase modulation (SPM) and linear group velocity dispersion (GVD). NLSE is used to characterize the behavior of solitons in motion. NLSE is fully integrable and remains the most appropriate model for a perfect Kerr medium [14].

However, solitonic pulse degrades due to the loss in fibre optics caused by the limited attenuation coefficient of communication grade optical fibre. This necessitates the use of optical amplifiers. In extremely high (frequency) bit rate transmission, the nonlinearity of Kerr law cannot depict the dynamics of solitons. Higher-order nonlinearities may become significant at average intensities in some media. Nonlinearities caused by a non-Kerr law medium become active under these conditions [5]. Changes occur in the physical characteristics of optical soliton communication. When studying ultra-high communication in a medium with coefficients, higher order nonlinearities must be accounted for [15]. Therefore, we must act in the following ways:

a) Add a new term to NLSE.

b) Since NLSE is not integrable in this setting, perturbation techniques are required.

2. MATHEMATICAL ANALYSIS AND DISCUSSION

Using the non-linearity of the non-Kerr law, we can write down the non-linear Schrodinger equation (NLSE) in a dimensionless form as [16]:

$$iq_t + \frac{1}{2} q_{xx} + F(|q|^2)q = 0 \tag{7}$$

where t is time without dimensions and x is fibre optics' dimensionless distance. The aforementioned PDE is non-integrable nonlinear (NLPDE). Algebraically speaking, F in eq. (5.1) is a function of real-value.

$$F(|q|^2)q \in U^{\infty}{}_{m,n=1} C^k ((-n,n) \, X \, (-m,m); R^2) \tag{8}$$

With the addition of the perturbation term, NLSE [17] becomes

$$iq_t + \frac{1}{2} q_{xx} + F(|q|^2)q = i\epsilon R[q,q] \tag{9}$$

The spatial differential operator is denoted by the parameter 'R'. The perturbation parameter is a measure of the spectrum's relative breadth. The latter word is derived from the concept of quasi-monochromaticity [18]. Finally, perturbation effects on the adiabatic dynamics of the soliton parameter are given below.

$$\frac{dE}{dt} = \epsilon \int_{-\infty}^{\infty} (q^* R + qR^*) dx \tag{10}$$

$$\frac{dM}{dt} = i\,\epsilon \int_{-\infty}^{\infty} (q_x^* R - q_x R^*) dx$$

$$\frac{d\kappa}{dt} = \frac{\epsilon \quad B}{I_{0,2,0,0,0,0,0,0,0} A^2} [i \int_{-\infty}^{\infty} (q_x^* R - q_x R^*) dx + \kappa \int_{-\infty}^{\infty} (q^* R + qR^*) dx]$$

$$\frac{d}{dt} = \kappa + \frac{\epsilon \quad B}{A^2 I_{0,2,0,0,0,0,0,0,0}} \int_{-\infty}^{\infty} (q^* R + qR^*) dx$$

Nonlinearity's four (04) most essential corner cases are now considered. We now detail the dynamics of the soliton's adiabatic parameters.

The fact that solitons in fiber optics experience a nonlinear response motivates the study of Kerr's law of nonlinearity. These responses are extremely lacking. The non-harmonicity of bonded electrons, however, has far-reaching effects that manifest in a variety of ways over long distances. Therefore, the induced polarization's Fourier amplitude is also nonlinear in the e-field which also involves electric field amplitude of higher-order-factors. Kerr's law states that F(s) here must equal s. This results in a change in NLSE to

$$iq_t + \frac{1}{2} q_{xx} + (|q|^2)q = 0 \tag{11}$$

IST (inverse transformation technique) can be used to integrate this fully. The soliton's shape is, therefore, demonstrated to be

$$q(x,t) = \frac{A}{\cos h[B(x - \bar{x}(t))]} e^{i(-kx + \omega t + a_0)}$$

where,

k = -v

and

$\omega = \frac{B^2 - \kappa^2}{2}$

$A = B$

There is a plethora of motion integrals involved in the Kerr law argument. Accordingly, the first three integrals of motion similar to NLSE are as follows:

$$E = \int_{-\infty}^{\infty} |q|^2 dx = 2A \quad (12)$$

$$M = \frac{i}{2} \int_{-\infty}^{\infty} (q q_x^* - q^* q_x) dx = -2kA$$

$$H = \frac{1}{2} \int_{-\infty}^{\infty} (|q_x|^2 - |q|^4) dx = \frac{2}{3} A(3\kappa^2 A^2).$$

NLSE, according to the Kerr law, is presented below in the presence of perturbational factors [19],

$$iq_t + \frac{1}{2} q_{xx} + |q|^2 q = i\epsilon R \quad (13)$$

This leads us to the following expression for the dynamics of the soliton's adiabatic parameters under the non-linear Kerr law:

$$\frac{dA}{dt} = \frac{dB}{dt} = \frac{\epsilon}{2} \int_{-\infty}^{\infty} (q^* R + q R^*) dx$$

$$\frac{d\kappa}{dt} = \frac{\epsilon}{2A}[i \int_{-\infty}^{\infty} (q^*_x R - q_x R^*)\, dx - \kappa \int_{-\infty}^{\infty} ((q^* R + q R^*) dx]$$

Therefore, the perturbed NLSE with nonlinearity in Kerr law is specified by the perturbational factors specified by eq. (5.24).

$$iq_t + \frac{1}{2} q_{xx} + |q|^2 q = =i\in[\delta |q|^{2m} q + a q_x + \beta q_{xx} - \gamma q_{xxx} + \lambda(|q|^2 q)_x + \theta(|q_x|^2)_x q +$$
$$\theta |q_x|^2 q - i\,\xi\,(q^2 q_x^*)x \;\; -i\eta q^2{}_x q^* - \; i\,\xi (q^2)_{xx} - i\mu(|q_x|^2)_x q - i\chi q_{xxxx} - i\psi q_{xxxxxxx} + (\sigma_1 q + \sigma_2 q_x) \int_{-\infty}^{\infty} |q|^2 ds.$$

Adiabatic parameter dynamics of solitons are as follows:

$$\frac{dE}{dt} = \frac{2\epsilon}{15}[15\delta A^{2m+1} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\,(m+1)}{\Gamma\,(m+\frac{3}{2})} - 10\;\beta A(A^2 + 3\kappa^2) - 4\rho A^3(A^2 + 5\kappa^2) + 10A^2\,(3\sigma_1 - \sigma_2 A)$$

$\frac{dM}{dt} = \frac{2\epsilon k}{15}[15\delta A^{2m+1}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\,(m+1)}{\Gamma\,(m+\frac{3}{2})} - 10\ \beta(A^2 + 3\kappa^2) - 4\rho A^2(A^2 + 5k^2) + 10A^2\,(3\sigma_1 - \sigma_2 A)$

$\frac{dA}{dt} = \frac{dB}{dt} = \frac{\epsilon}{15}[15\delta A^{2m+1}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\,(m+1)}{\Gamma\,(m+\frac{3}{2})} - 10\ \beta A(A^2 + 3\kappa^2) - 4\rho A^3(A^2 + 5\kappa^2) + 10A^2\,(3\sigma_1 - \sigma_2 A)$

$\frac{d\kappa}{dt} = -\frac{\epsilon A^2}{15}[10\kappa\,(2\,\beta - \sigma_2) + 8\mu A^2]$

Kerr law soliton speed dispersion is as under.

Power-law nonlinearity is observed in many different media and semiconductors are no exception [20]. Nonlinear plasma also fixes the issue of weak turbulence by revealing the nonlinearity law. Specifically, in this case F(s) = s-P. In this way, NLSE is

$$iq_t + \frac{1}{2}q_{xx} + |q|^{2p}q = 0 \quad (14)$$

To avoid soliton pulse collapse and self-focusing singularity having $0 < p < 2$, the solitonic solution to eq. (14) is as follows:

$$q(x,t) = \frac{A}{cosh^{\frac{2}{P}}[B(x-\bar{x}(t))]}e^{i(-kx+\omega t+a_0)}$$

where,

$k = -\,v$

$\omega = \frac{B^2}{2p^2} - \frac{k^2}{2}$

$B = A^p\,(\frac{2p^2}{1+p})^{1/2}$

The three motion integrals in this case are as follows:

$$E = \int_{-\infty}^{\infty}|q|^2dx$$

$$E = A^{2-p}\,(\frac{1+p}{2p^2})^{\frac{1}{2}}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\,(\frac{1}{p})}{\Gamma(\frac{1}{p}+\frac{1}{2})}$$

$$E = B^{\frac{(2-p)}{p}}(\frac{1+p}{2p^2})^{\frac{1}{p}}\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\,(\frac{1}{p})}{\Gamma(\frac{1}{p}+\frac{1}{2})}$$

$$M = \frac{i}{2}\int_{-\infty}^{\infty}(q^{*}qx \ - \ qq_{x}^{*})dx$$

$$M = \kappa A^{2-p}\left(\frac{1+p}{2p^{2}}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)}{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}$$

$$M = 2kB^{\frac{(2-p)}{p}} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)}{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}$$

and

$$H = \int_{-\infty}^{\infty}\left[\frac{1}{2}|qx|^{2} - \frac{1}{p+1}|q|^{2p+2}\right]dx$$

$$H = \frac{B^{\frac{2}{p}}}{2p^{2}}\left(\frac{1+p}{2p^{2}}\right)^{\frac{1}{p}}\left[\frac{(B^{2}+k^{2}p^{2})\Gamma\ \frac{1}{2}\ \Gamma\ \frac{1}{p}}{B \qquad \Gamma\left(\frac{1}{p}+\frac{1}{2}\right)} - 2B\ \frac{\Gamma\left(\frac{1}{2}\right)\ \Gamma\left(\frac{p+1}{p}\right)}{\Gamma\left(\frac{p+1}{p}+\frac{1}{2}\right)}\right]$$

$$H = \frac{A^{2}}{2p^{2}}\left[\left\{A^{p}\left(\frac{2p^{2}}{1+p}\right)^{\frac{2}{p}} + \frac{\kappa^{2}\ p^{2}}{A^{p}}\left(\frac{1+p}{2p^{2}}\right)^{\frac{1}{2}}\right\} \times \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)}{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)} - 2A^{p}\left(\frac{2p^{2}}{1+p}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{1}{2}\right)\ \Gamma\left(\frac{p+1}{p}\right)}{\Gamma\left(\frac{p+1}{p}+\frac{1}{2}\right)}\right.$$

NLSE with power-law nonlinearity showing the presence of perturbational factors as [21] is as follows:

$$iq_{t} + \frac{1}{2}q_{xx} + |q|^{2p}q = i\epsilon R[q,q^{*}] \tag{15}$$

In the presence of perturbing factors, the dynamics of the adiabatic parameters are as follows:

$$\frac{dA}{dt} = \frac{\epsilon}{2-p}A^{p-1}\left(\frac{2p^{2}}{1+p}\right)^{\frac{(p-1)}{2p}} \times \frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)}\int_{-\infty}^{\infty}(q^{*}R + qR^{*})dx$$

$$\frac{dB}{dt} = \epsilon\ \frac{p}{2-p}B^{(2p-2)p}\left(\frac{2p^{2}}{1+p}\right)^{\frac{1}{p}} \times \frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)}\int_{-\infty}^{\infty}(q^{*}R + qR^{*})dx,$$

$$\frac{d\kappa}{dt} = \in B^{\frac{(p-2)}{p}}\left(\frac{2p^{2}}{1+p}\right)^{\frac{1}{p}} \frac{\Gamma\left(\frac{1}{p}+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{p}\right)} \times i\int_{-\infty}^{\infty}(q^{*}_{x}R + q_{x}R^{*})dx - \kappa\int_{-\infty}^{\infty}(q^{*}R + qR^{*})\, dx.$$

The power-law nonlinear case of NLSE with perturbational factors is written as under:

$$iq_{t} + \frac{1}{2}q_{xx} + |q|^{2p}q$$

$=i\in[\delta|q|^{2m}q + aq_x + \beta ßq_{xx} - \gamma q_{xxx} + \lambda(|q|^2q)x + \theta(|q|^2)_x q + \rho(|qx|^2q) - i\,\xi\,(q^2q_x^*)x$ $-i\eta q^2{}_x q^*$- $i\,\xi(q^2)_{xx} - i\mu(|q|^2)_x q - i\chi q_{xxx} - i\psi q_{xxxxxxx} + (\sigma_1 q + \sigma_2 q_x)\int_{-\infty}^{\infty}|q|^2dx]$.

It is also possible to compute the power-law soliton's adiabatic parameter dynamics subject to perturbation. It was found that the generalized nonlinear Schrodinger equation (GNLSE) is not integrable due to higher order dispersional factors.

2.1. Conclusion

The findings can be utilized to explore the dynamics of pulse evolution in ultrahigh bit rate transmission in order to investigate this issue for a wide range of beginning pulse conditions. These findings are significant because they demonstrate that optical solitons can be accurately characterized by the behavior of a particular sort, even when perturbational elements are present. This is a phenomenon that has been observed in the past.

The purpose of these hypothetical projections is to provide a practical illustration of power-law based long distance, fiber optic communication at a rate of 40 gigabits per second over oceanic and continental distances. Consequently, the possibility that this innovation can be utilized in the field of communication technology may have a significant influence on the intensification of information.

CONFLICT OF INTEREST

The authors of the manuscript have no financial or non-financial conflict of interest in the subject matter or materials discussed in this manuscript.

DATA AVALIABILITY STATEMENT

Data availability is not applicable as no new data was created.

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