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Author (s):	Khurram Shabbir[1], Ahmad Raza[1], and Liliana Guran[2]
Affiliation (s):	[1]Government College University, Lahore, Pakistan [2]Babes-Bolyai University, Cluj-Napoca, Romania
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Computation of *Tades* of Some Families of Graphs

Khurram Shabbir[1*], Ahmad Raza[1], Liliana Guran[2]

[1]Department of Mathematics, Government College University, Lahore, Pakistan
[2]Babes-Bolyai University, Cluj-Napoca, Romania

ABSTRACT

This study presents novel and efficient techniques for computing the Total Absolute Difference Edge Irregularity Strength (TADES) of several well-known graph families. Specifically, the focus remains on book graphs with pentagonal pages, the hairy cycle graph, and three-regular graphs. For these increasingly complex graph structures, innovative algorithmic methods are introduced that significantly reduce the computational complexity of determining TADES, thus contributing valuable insights and advancements to the study of graph theory.

Keywords: TADES; Hairy cycle; Pentagonal Pages; Regular Graphs.

1.INTRODUCTION

Graph theory originated with Euler's work on the seven bridges of Königsberg problem, establishing a mathematical foundation that has evolved significantly since then. Today, it encompasses various advanced topics, including network theory, algorithmic complexity, and even quantum graph theory, which explores the implications of quantum computing for graph structures. At its core, graph theory involves the study of graphs, which are mathematical structures consisting of vertices connected by edges. This abstraction allows for the representation of pairwise relationships among objects, making graphs applicable in numerous fields, including computer science. Graphs can model everything from social networks to transportation systems, providing a framework to analyze connectivity and interactions.

This research work covers only finite, undirected graphs without numerous edges or loops. In [1], Chartrand et al. presented edge k-labeling of a graph, where for $u \neq v$ and $w(u) \neq w(v)$ for all vertices u, $v \in V(G)$. The irregularity strength of a graph G is the lowest k for which G has an irregular assignment using the labels of the maximum k.

*Corresponding Author: dr.khurramshabbir@gcu.edu.pk

This form of labeling is referred to as erratic assignments.

Baca et al. ([2]) first studied a graph's edge complete irregularity strength, which is same as the total labeling irregularity strength. According to Jendrol and Ivanco [3], the following hypothesis applies to any tree T and graph G that constitute the highest degree K_5. $\Delta(G)$

$tes(T) = max\left\{\left\lceil\frac{E(G)+2}{3}\right\rceil, \left\lceil\frac{\Delta(G)+1}{2}\right\rceil\right\}$.

The categorical product of 3-leaf and cycle by M. K. Siddiqui in [4], complete graphs and complete bipartite graphs in [5], and any tree have all been shown to satisfy Ivančo and Jendrol's hypothesis.

Motivated by the overall strength of edge irregularity and the concept of graceful labeling, Ramalakshmi and Kathiresan [6] suggested a method of decreasing edge weights by maximizing the absolute difference between the graphs' edge irregularity strength. The weight of an edge $e = uv$ under a total labeling ℵ in a graph G is $wt(e) = |\aleph(e) - \aleph(u) - \aleph(v)|$.

A graph G, labeled with ℵ. $\aleph: V \cup E \rightarrow \{1, 2, 3, ..., k$, represents an edge irregular complete

absolute difference k-labeling of G. If there is $wt(e) \neq wt(f)$ for two different edges, then $e = uv$ and $f = u_1v_1$ of G. The lowest k for which the total absolute difference of G is irregular at the edge *tades (G)* is k-labeling. Further, [6] provided proof for the following conclusions:

1. Every tree T with m vertices and a maximum degree of Δ(G)

tades(T) = max$\left\{\frac{m}{2}, \frac{\Delta(G)+1}{2}\right\}$.

2. $tes(G) \leq tades(G)$ for any graph G.

Theorem 1.1. [6] *Assume that the graph G = (V, E) has a vertex set V and edge set E. Both are non-empty. Consequently,*

$\frac{|E|}{2} \leq tades(G) \leq |E| + 1$.

Utilizing this outcome, A. Lourdusamy and F.J. Beaula in [7], [8] calculated the TADES of graphs linked to snakes, wheels, lotuses inside circles, double fans, and path-related graphs. In this research, we examine the hairy cycle graph, book graph with n-pentagonal pages, and 3-regular graph's TADES.

2. HAIRY CYCLE GRAPH

In [9], we find that a hairy cycle graph is a cycle with three pendent edges attached, such as a bared cycle. In this section, we determine the hairy cycle graph's TADES, denoted by Cp ⊙ 3k1. Its irregularity strength depends upon the value of n. For example, in Figure 1 we take the value of n as 7. Then, the TADES of C7 ⊙ 3k1 is 14.

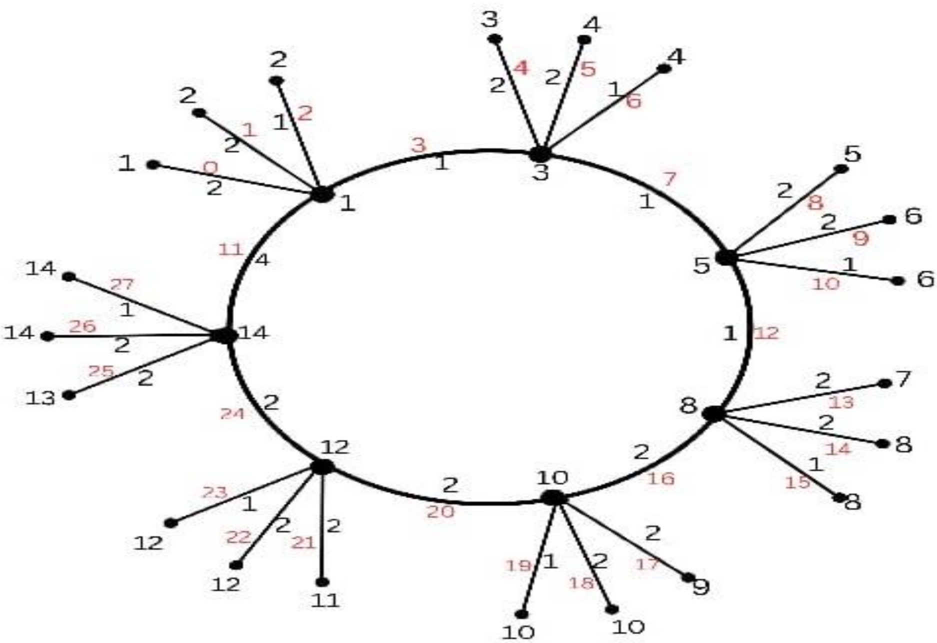

Figure 1. C7 ⊙ 3k1

Theorem 2.1. *For the comb graph* $C_p \odot 3k_1$,

tades $(C_p \odot 3k_1) = 2p$.

Proof. Say that the graph's vertex set equals

V(G) ={ $\kappa_i,\ \kappa_i{}',\kappa_i{}'',\ \kappa_i{}'''; 1 \le i \le p$},

and edge is$E(G) = \begin{cases} \kappa_i\kappa_{i+1};\ 1\ \le\ i\ \le\ p-1 \\ \kappa_i\ \kappa_i{}';\ \ 1\ \le\ i\ \le\ p \\ \kappa_i\ \kappa_i{}'';\ 1\ \le\ i\ \le\ p \\ \kappa_i\ \kappa_i{}''';\ 1\ \le\ i\ \le\ p\ and\ \kappa_p\kappa_1 \end{cases}$

By Theorem 1.1, we have

$tades\ (C_p \odot 3k_1) \geq 2p.$ (1)

As we move forward with the opposite disparity, we create

$\aleph : V \cup E \rightarrow \{1, 2, 3, ..., 2p\}$

as follows $\aleph(\kappa_l) = \begin{cases} 2(l-1)+1; & if\ 1 \leq l \leq \left\lfloor \frac{p}{2} \right\rfloor \\ 2l; & if \left\lceil \frac{p+1}{2} \right\rceil \leq l \leq p \end{cases}$

$\aleph(\kappa_l') = 2(l-1) + 1\ \boldsymbol{if}\ 1 \leq l \leq p$

$\aleph(\kappa_l'') = \aleph(\kappa_l''') = 2l \quad if\ 1 \leq l \leq p.$

Now, edge labeling is $\aleph(\kappa_l) = \begin{cases} 1; & if\ 1 \leq l \leq \left\lfloor \frac{p}{2} \right\rfloor \\ 2; & if\ \left\lceil \frac{p+1}{2} \right\rceil \leq l \leq p \end{cases}$

$\aleph(\kappa_p\kappa_1) = \begin{cases} 2; & for\ even\ p \\ 4; & for\ odd\ p \end{cases}$

$\aleph(\kappa_l\ \kappa_l') = \aleph(\kappa_l\ \kappa_l'') = 2\ \ if\ 1 \leq l \leq p.$

$\aleph(\kappa_l\ \kappa_l''') = 1\ if\ \ 1 \leq l \leq p.$

Now, the weight of the edges is

$$wt(\kappa_l\kappa_{l+1}) = \begin{cases} |\aleph(\kappa_l\ \kappa_{l+1}) - \aleph(\kappa_l) - \aleph(\kappa_{l+1})|; & \text{if } 1 \leq l \leq \left\lfloor \frac{p}{2} \right\rfloor \\ |\aleph(\kappa_l\kappa_{l+1}) - \aleph(\kappa_l) - \aleph(\kappa_{l+1})|; & if\ \left\lceil \frac{p+1}{2} \right\rceil \leq l \leq \text{p} \end{cases}$$

$$= \begin{cases} 4l - 1; & \text{if } 1 \leq l \leq \left\lfloor \frac{p}{2} \right\rfloor \\ 4l; & if\ \left\lceil \frac{p+1}{2} \right\rceil \leq l \leq \text{p} - 1 \end{cases}$$

$$wt(\kappa_p\kappa_1) = \begin{cases} \left|\aleph(\kappa_p\kappa_1) - \aleph(\kappa_p) - \aleph(\kappa_1)\right|; & for\ even\ \text{p} \\ \left|\aleph(\kappa_p\ \kappa_1) - \aleph(\kappa_p) - \aleph(\kappa_1)\right|; & for\ odd\ \text{p} \end{cases}$$

$$= \begin{cases} 2\text{p} - 1; for\ even\ \text{p} \\ 2p - 3; for\ odd\ \text{p} \end{cases}$$

$$wt(\kappa_l \kappa_l') = \begin{cases} 4l - 4; & if\ 1 \le l \le \left\lfloor \frac{p}{2} \right\rfloor \\ 4l - 3; & if\ \left\lceil \frac{p+1}{2} \right\rceil \le l \le \mathrm{p} - 1 \end{cases}$$

$$wt(\kappa_l \kappa_l'') = \begin{cases} 4l - 3; & if\ 1 \le l \le \left\lfloor \frac{p}{2} \right\rfloor \\ 4l - 2; & if\ \left\lceil \frac{p+1}{2} \right\rceil \le l \le \mathrm{p} - 1 \end{cases}$$

$$wt(\kappa_l \kappa_l''') = \begin{cases} 4l - 2; & if\ 1 \le l \le \left\lfloor \frac{p}{2} \right\rfloor \\ 4l - 1; & if\ \left\lceil \frac{p+1}{2} \right\rceil \le l \le \mathrm{p} - 1 \end{cases}$$

Clearly, from Figure (2) we have

$$tades\ (C_p \odot 3k_1) \le 2p \qquad (2)$$

From (1) and (2), we have *tades* $(C_p \odot 3k_1) = 2p$.

3. BOOK GRAPH

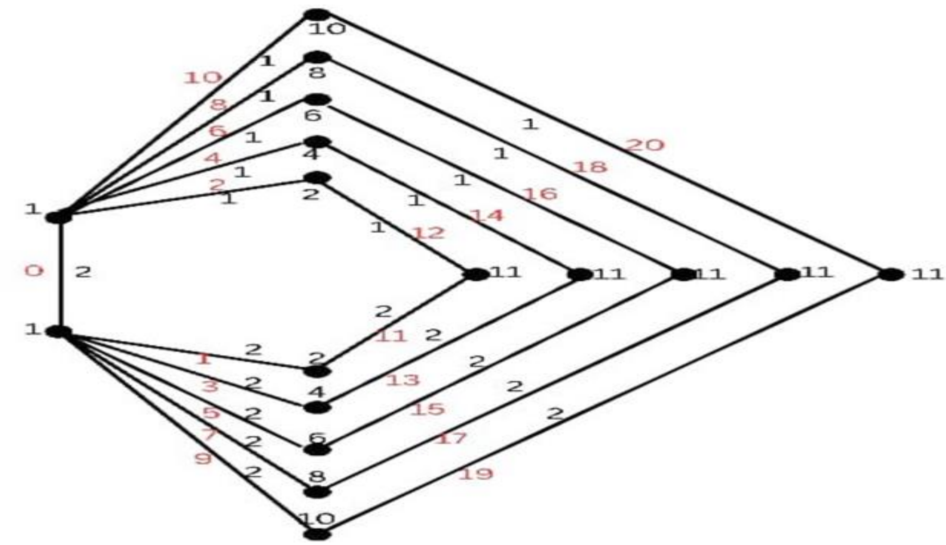

Figure 2. Book Graph

This section calculates the strength of edge irregularity for a book graph with *n*-pentagonal pages as the total absolute difference. A book graph is made up of two fixed vertices connected to each page in the book, see [10].

For example, in Figure 2 we compute the TADES of the book graph if the value of n is 5.

Theorem 3.1. *For a book with q-pentagonal pages represented by a graph G,*

tades (G) $=\left\lceil\frac{4q+1}{2}\right\rceil$.

Proof. Let us consider the graph (G) whose vertex set and edge set are

$V(G) = \{\upsilon, w, \upsilon_i, v_i, w_i: 1 \le i \le q\}$

$E(G) = \{\upsilon v\}, \{\upsilon\upsilon_i, \upsilon_i w_i, v_i w_i, vv_i\} : 1 \le i \le q\}$

By Theorem 1.1, we have

$$tades\ (G) \ge \left\lceil\frac{4q+1}{2}\right\rceil \quad (3)$$

As we move forward with the opposite disparity, we create

$$ð : V \cup E \rightarrow \left\{1,2,3,\ldots,\left\lceil\frac{q-1}{2}\right\rceil\right\}$$

as follows:

$ð(\upsilon) = ð(v) = 1;$

$ð(\upsilon_i) = ð(v_i) = 2i;\ for\ 1 \le i \le q$

$ð(w_i) = \left\lceil\frac{4q+1}{2}\right\rceil;\ for\ 1 \le i \le q$

and

$ð(vu)=2;$

$ð(\upsilon u_i) = ð(\upsilon_i w_i) = 1; \quad for\ 1 \le i \le q;$

$ð(vv_i) = ð(v_i w_i) = 2; \quad for\ 1 \le i \le q.$

We will determine how much weight each edge is

$wt(\upsilon v) = |ð(\upsilon v) - ð(\upsilon) - ð(v)|$

$= 0$

$wt(\upsilon u_i) = |ð(\upsilon u_i) - ð(\upsilon) - ð(\upsilon_i)|$

$= 2i$

$wt(vv_i) = |\eth(vv_i) - \eth(v) - \eth(v_i)|$

$= 2i - 1$

$wt(\upsilon_i w_i) = |\eth(\upsilon_i w_i) - \eth(\upsilon_i) - \eth(w_i)|$

$= 2(q + i)$

$wt(v_i w_i) = |\eth(v_i w_i) - \eth(v_i) - \eth(w_i)|$

$= 2(q + i) - 1.$

From the above calculation, we conclude that

$$tades\ (G) \leq \left\lceil \frac{4q+1}{2} \right\rceil. \quad (4)$$

Every edge weight is unique, therefore, we can derive the following from (3) and (4).

$$tades\ (G) = \left\lceil \frac{4q+1}{2} \right\rceil.$$

4.3-REGULAR GRAPH

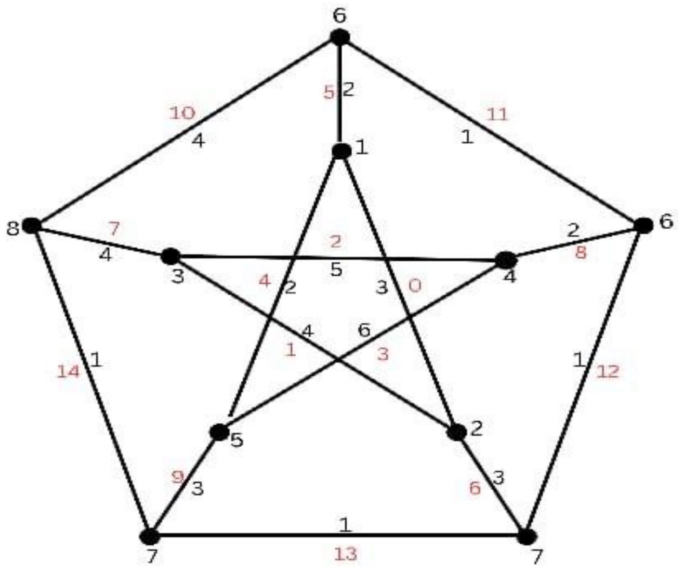

Figure 3. P (5, 2)

In this instance, a 3-regular graph is one in which each vertex has degree 3. In Figure 3 we compute the TADES of 3-regular graph if the value of n is 5. The TADES of this type of graph purely depends upon the value of *n*.

Theorem 4.1. *For a graph, say a P (q, 2) graph,*

tades(*P(q,2)*) $=\left\lceil\frac{3q}{2}\right\rceil$.

Proof. Let $P\,(q,2)$ be a graph, the set of nodes and the edge set of the graph are

$$V\big(P(q,2)\big)=\{v_z,{v_z}':1\le z\le q\}$$

$$E\big(P(q,2)\big)=\{v_zv_{z+1}:1\le z\le q-1\}\cup\{v_z{v_z}':1\le z\le q\}\\ \cup\{v_nv_1,v_{\left\lceil\frac{q}{2}\right\rceil}{v_1}'\}\cup\{{v_z}'v_{\left\lceil\frac{q}{2}\right\rceil+z},v_{\left\lceil\frac{q}{2}\right\rceil+z}{v_{z+1}}':1\le z\\ \le\left\lceil\frac{q}{2}\right\rceil-1\}.$$

By Theorem 1.1, we have

$$tades\ (\mathrm{P(q,2)})\ge\left\lceil\frac{3q}{2}\right\rceil. \qquad (5)$$

We move forward with the opposite disparity and we create Y: V ∪ E {1, 2, 3,…, $\left\lceil\frac{3q}{2}\right\rceil$}, as follows:

$$Y(v_z)=z;\quad 1\le z\le q$$

$$Y({v_z}')=\begin{cases}q+z; & for\ 1\le z\le\left\lceil\frac{q}{2}\right\rceil\\ q+z-\left\lceil\frac{q}{2}\right\rceil; & for\ \left\lceil\frac{q}{2}\right\rceil+1\le z\le \mathrm{q}\end{cases}$$

$$Y(v_zv_{z+1})=z+2;\quad 1\le z\le q-1$$

$$Y\big(v_qv_1\big)=2$$

$$Y(v_z{v_z}')=\begin{cases}z+1; & for\ 1\le z\le\left\lceil\frac{q}{2}\right\rceil\\ 2+\left(z-\left(\left\lceil\frac{q}{2}\right\rceil+1\right)\right); & for\ \left\lceil\frac{q}{2}\right\rceil+1\le z\le q\end{cases}$$

$$Y\left({v_z}'v'_{\left\lceil\frac{q}{2}\right\rceil+z}\right)=1;\quad 1\le z\le\left\lceil\frac{q}{2}\right\rceil-1$$

$$Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil+z}v'_{z+1}\right)=1;\quad 1\le z\le\left\lceil\frac{q}{2}\right\rceil-1$$

$Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil}v'_1\right) = \left\lceil\frac{q}{2}\right\rceil + 1.$

We now determine how much weight each edge is

$wt(v_z v_{z+1}) = |Y(v_z v_{z+1}) - Y(v_z) - Y(v_{z+1})|$

$= |z + 2 - (z) - (z + 1)|$

$= z - 1$

$wt(v_q v_1) = |Y(v_q v_1) - Y(v_q) - Y(v_1)|$

$= |2 - q - 1|$

$= q - 1;$

$$wt(v_z v_z') = \begin{cases} |Y(v_z v_z') - Y(v_z) - Y(v_z')|;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil \\ |Y(v_z v_z') - Y(v_z) - Y(v_z')|;\ for\ \left\lceil\frac{q}{2}\right\rceil + 1 \le z \le \mathrm{q} \end{cases}$$

$$= \begin{cases} |z + 1 - (z) - (q + z)|;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil \\ |2 + z - (2 + (z - \left(\left\lceil\frac{q}{2}\right\rceil + 1\right) - (z) - (q + z - \left\lceil\frac{q}{2}\right\rceil)|;\ for\ \left\lceil\frac{q}{2}\right\rceil + 1 \le z \le \mathrm{q} \end{cases}$$

$= q + z - 1;\quad for\ 1 \le z \le q;$

$$wt\left(v_z'v'_{\left\lceil\frac{q}{2}\right\rceil+z}\right) = \left|Y\left(v_z'v'_{\left\lceil\frac{q}{2}\right\rceil+z}\right) - Y(v_z') - Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil+z}\right)\right|;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1$$

$$= \left|1 - (q + z) - \left(q + \left\lceil\frac{q}{2}\right\rceil + z - \left\lceil\frac{q}{2}\right\rceil\right)\right|;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1$$

$$= 2(q + z) - 1;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1;$$

$$wt\left(v'_{\left\lceil\frac{q}{2}\right\rceil+z}v'_{z+1}\right) = \left|Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil+z}v'_{z+1}\right) - Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil+z}\right) - Y(v'_{z+1})\right|;\ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1$$

$$= \left|1 - \left(q + \left\lceil\frac{q}{2}\right\rceil + z - \left\lceil\frac{q}{2}\right\rceil\right) - (q + z + 1)\right|; \ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1$$

$$= 2(q + z); \ for\ 1 \le z \le \left\lceil\frac{q}{2}\right\rceil - 1;$$

$$wt\left(v'_{\left\lceil\frac{q}{2}\right\rceil}v'_1\right) = \left|Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil}v'_1\right) - Y\left(v'_{\left\lceil\frac{q}{2}\right\rceil}\right) - Y(v'_1)\right|$$

$$= \left|\left(\left\lceil\frac{q}{2}\right\rceil + 1\right) - \left(q + \left\lceil\frac{q}{2}\right\rceil\right) - (q + 1)\right| = 2q.$$

Clearly, from the calculation, we conclude that

$$tades(P(q,2)) \le \left\lceil\frac{3q}{2}\right\rceil. \quad (6)$$

Every edge weight is unique; therefore, we can derive the following from (5) and (6)

$$tades(P(q,2)) = \left\lceil\frac{3q}{2}\right\rceil.$$

5. CONCLUSION

In this paper, we examined the computation of Total Absolute Difference Edge Irregularity Strength (TADES) for several graph families in detail. The results on the TADES of the book graph, 3-regular graph, and hairy cycle graph have implications for improving network architecture, optimizing image processing, and performing data analysis in several application domains.

In the 3-regular graph P(n,2), the total edge irregularity strength (TES) is n+1 (see [11]), while the total absolute difference edge irregularity strength (TADES) is 3n/2. This comparison indicates that the irregularity strength of the 3-regular graph increases, even though the edge weights decrease. Similarly, the TADES of a graph exceeds the TES of the same graph but the edge weights decrease.

As technology continues to advance, the applications of graph theory in computer science are expected to expand, influencing areas such as artificial intelligence, smart cities, and autonomous systems. The ongoing research in this field highlights the dynamic interplay between mathematical theory and practical applications, driving innovation and connectivity in the digital landscape.

To summarize, the connection between mathematical graphs and computer science is profound and multifaceted, impacting algorithm design, network analysis, database management, and beyond (see [1], [12], [13]). Moreover, new research directions in graph theory, including solving some conjectures, are given in [14-17]. The continued exploration of graph theory promises to yield further insights and advancements in technology.

CONFLICT OF INTEREST

The authors of the manuscript have no financial or non-financial conflict of interest in the subject matter or materials discussed in this manuscript.

DATA AVALIABILITY STATEMENT

The data associated with this study will be provided by the corresponding author upon request.

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