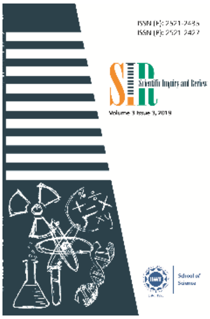

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Author(s)	**Zameer Abbas Sohaib Abdal Fayyaz Hussain Nasir Hussain Muhammad Adnan Bagh Ali Rana Muhammad Liaqat Ali Zulqarnain Saba Younas**
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MHD Boundary Layer Flow and Heat Transfer of Nanofluid over a Vertical Stretching Sheet in the Presence of a Heat Source

Zameer Abbas[1], Sohaib Abdal[2*], Fayyaz Hussain[1], Nasir Hussain[1], Muhammad Adnan[1], Bagh Ali[3], Rana Muhammad Zulqarnain[2], Liaqat Ali[4], Saba Younas[1]

[1]Department of Mathematics, National College of Business Administration & Economics, Lahore, Pakistan
[2]School of Mathematics, Northwest University, Xi'an, China
[3]Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an, China
[4]School of Energy and power, Xi'an Jiaotong University, Xi'an, China

*sohaib@stumail.nwu.edu.cn

Abstract

The nanoparticles used in nanofluid are prepared for carbides and oxides. In this paper, a nanofluid flow over a stretching sheet in the presence of viscous dissipation, heat source, and magnetic field was numerically explored with the help of the Runge-Kutta shooting technique and the effects of various parameters were analyzed using graphical representation.

Keywords: buoyancy assisting, buoyancy opposing, MHD Boundary Layer Flow, nanofluid, Runge-Kutta shooting technique

Introduction

Nanofluid contains a nanometer sized metallic component in base fluid particles. The nanoparticles used in nanofluid are typical, same as those prepared for carbides and oxides. Afridib et al. [1] considered MHD stagnation point flows over a stretching sheet. Alawi et al. [2] focused on determining and modelling the forceful thermal conductivity and viscosity of nanofluid. Ali at al. [3] enhanced the thermophysical heat transference fluid. Awaludin et al. [4] studied the effects of MHD stagnation point fluid over a stretching sheet. Baag et al. [5] investigated heat transfer and boundary layer flow of MHD on a stretching sheet. Bhatti et al. [6] described the flow of a shrinking sheet of MHD. Chaudhary et al. [7] analysed the viscous flow of a shrinking surface in a porous medium. Ganesh et al. [8] analyzed the influence of non-linear thermal radiation on boundary layer flow and convective heat transference of AI_2O_3 nanofluid.

Ghan et al. [9] discussed an incompressible and unidirectional MHD flow of fluids on an oscillating in-clines belt embedded in the porous medium. Hamid et al. [10] discussed the influence of Williamson fluid. Ishaq et al. [11] investigated two dimensional nanofluid flow over a stretching sheet. Kumar et al. [12] studied the numerically unsteady flow of melting heat transference of nanofluids over a stretched sheet. Kamal et al. [13] investigated the MHD stagnation point draw of a nanofluid at a permeable stretching sheet with chemical attitude effect. Kumar et al. [14] studied the unsteady hydromagnetic boundary layer stagnation point nanofluid flow over a non-linear stretching surface. Kumar et al. [15] discussed MHD on the stretching surface. Maripala [16] investigated the nanofluid flow in the occurrence of radiations.

Mahmood et al. [17] examined the axisymmetric fluid on a disc. Makinde et al. [18] investigated the steady MHD nanofluid boundary layer flow over a non-linear stretching sheet. Nojoomizadeh et al. [19] numerically investigated the heat transfer and laminar flow with a previous medium. Reddy and Krishna [20] described the unstable flood of the transversely magnetic field of constant strength. Shaba and Ali [21] investigated the problems of hydromagnetics boundary layer flow of a dusty fluid over a stretching sheet. Singh et al. [22] investigated MHD slip flow over a flat platter. Waqas et al. [23] described the idea of computation to investigate for buoyance and radiation effect on the MHD stagflation points of micropolar fluid. Wen et al. [24] studied the natural carbon dioxide (Co_2), the field at low pressure (below 1 MPa) in a closed system. Younas et al. [25] studied external heat and radiated heat sources. Zaib et al. [26] discussed the axisymmetric flood of homogeneous-heterogeneous reactional. Sohaib et al. [27] obtained the numerical solution for radiation on hydromagnetic stagnation point flow.

According to the best knowledge of the author, the numerical study of the effect of heat and boundary layer flow on steady convection flow and heat transfer past a vertical stretching sheet is not available. Energy and momentum equations are obtained with the help of similarity variables. The governing partial differential equations are transmuted into ordinary differential equations and numerically solved by using Runge-Kutta shooting technique. The effects of various parameters are analyzed using their graphical representation.

2. Problem Description

In this study, the vertical stretching sheet is in the direction of the x-axis and the y-axis is orthogonal to the sheet. u is the velocity component in the x-direction and v is the velocity component in the y-direction. Taking into consideration c which is a positive constant; u = $u_e(x)$= ax represents the unrestricted stream velocity, while u = $u_w(x)$= cx represents the velocity when there is stretching on the sheet. When a heat source / sink is present, H_0 is an external magnetic field that is practically perpendicular to the sheet. The principal equations of continuity, momentum, and energy are written as follows,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma {\mu_e}^2 {H_0}^2}{\rho}u+g\beta\,(T-T_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{Q}{\rho C_p}(T-T_\infty)+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where D_B is Brownian diffusion, D_T is Thermophoresis diffusion, σ is electrical conductivity, μ_e is magnetic permeability, T_∞ is the temperature of free stream, g is the acceleration due to gravity, β is the volumetric coefficient of thermal expansion, k is thermal conductivity, $\upsilon=\frac{\mu}{\rho}$ is the kinematic viscosity, and $T_w=T_\infty+bx$ is the temperature of the sheet. $C_w=C_\infty+bx$, τ is the ratio of heat capacities.

The boundary conditions pertaining to the horizontally moving boundary and convective heat transfer at the wall are formulated below.

$$v=0, u=u_w(x)=cx, -k\frac{\partial T}{\partial y}=h_f\left(T_f-T\right), C=C_w at\ y=0 \tag{5}$$

$$u=u_e(x)=ax,\ T=T_\infty, C=C_\infty\ as\ y\to\infty,$$

3. Similarity Analysis

The subsequent change and dimensionless quantities are used into equations while taking into account the boundary conditions.

We have

$$\eta=\sqrt{\frac{a}{v}}y, \psi=x\sqrt{av}f(\eta), \phi(\eta)=\frac{C-C_\infty}{C_W-C_\infty}\ ,\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty}\quad and$$

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x} \tag{6}$$

$$u = xa\, f'(\eta)\ ,\quad v = -\sqrt{av}\, f(\eta).$$

The equation of continuity satisfies equations (2-4) and transforms them as shown below.

$$f''' + ff'' - f'^2 + 1 + H_a^{\ 2}(1-f') + \lambda\theta = 0 \tag{7}$$

$$\theta'' + pr\left[f\theta' - f'\theta + \delta\theta + N_b\theta'\Phi' + N_t\theta'^2 \right] = 0 \qquad (8)$$

$$\Phi'' + Le\left(f\Phi' - f'\Phi\right) + \frac{Nt}{Nb}\theta'' = 0 \tag{9}$$

The boundary conditions (5) become

$$f = 0,\ f' = \frac{c}{a} = A,\ \theta' = B(\theta - 1), \phi = 1\ at\ \eta = 0$$

(10)

$$f' = 1,\ = 0,\ \phi = 0\, as\, \eta \to \infty.$$

$H_a = \mu_e H_o\sqrt{\frac{\sigma}{\rho a}}$ is the Hartmann number, $\lambda = \frac{Gr_x}{\mathrm{Re}_x^{\ 2}}$ is the mixed convection parameter,

$Gr_x = g\beta(T_w - T_\infty)\frac{x^3}{v^2}$ is the local Grashof number, $\mathrm{Re}_x = u_e(x)\frac{x}{v}$ and $p_r = \frac{v}{\alpha}$,

$\delta = \frac{Q}{\rho a C_p}$ represents the factor of heat generation or absorption. L_e = $\frac{v}{D_B}$ is the Lewis number parameter, N_b = $\tau D_B\frac{bx}{v}$ is the Brownainmotion parameter, and N_t = $\tau\frac{DT}{T_\infty}\frac{bx}{v}$ is the Thermophoresis parameter.

4. Results and Discussion

A steady laminar flow above a vertical stretching sheet with the existence of viscous dissipation, heat sink or source, and magnetic field was explored numerically with the help of the Runge-Kutta shooting technique. physical parameter effects L_e, N_b, N_t, Hartmann number, stretching velocity ratio, Biot number and velocity distribution along with skin friction and heat transfer coefficient.

In Table 1, numerical comparison of the values of Pr for heat transformation is obtained by using the Runge-Kutta shooting scheme. The skin friction coefficient is compared with previously studied results in Table 2. In Figure 1, the effect of Ha on velocity for opposing and assisting flow is shown ($N_b = N_t = L_e = 0$). Figure 2 shows the effects of A when stretching is in flow in the opposite direction ($N_b = N_t = L_e = 0$). In Figure 3, the effects of Bi in the presence of a heat source on the dimensionless temperature for assisting and opposing flow can be seen. A similar effect for Bi can be seen in Figure 4 which illustrates the result of the Biot number in the company of heat sink on dimensionless temperature for assisting and opposing flow at $P_r = H_a = A$ ($N_b = N_t = L_e = 0$). Figure 5 displays the effects of stretching velocity ratio and mixed convection parameter on dimensionless skin friction for both assisting and opposing flow. It is found that skin friction increases with the mixed convection parameter and H_a, even though it drops with the stretching velocity ratio for mutually opposing and assisting flow ($N_b = N_t = L_e = 0$). The effects of Biot number B_i, heat generation / absorption coefficient δ, and mixed convection parameter λ on the dimensionless heat transfer rate for both assisting and opposing flow are illustrated respectively in Figure 6 ($N_b = N_t = L_e = 0$). The effects of Brownian motion N_b in the presence of heat source on the dimensionless temperature for assisting and opposing flows are shown in Figure 7, respectively (N_t = 0.1, 0.2) (L_e = 0.1, 0.5). Figure 8 illustrates the result of Thermophoresis N_t in the company of heat sink on the dimensionless temperature for assisting and opposing flow at $P_r = H_a = A$ (N_b = 0.1, 0.2) (L_e = 0.1, 0.5). The effects of Lewis number L_e and λ for both assisting and opposing flow are illustrated respectively in Figure 9 (N_t = 0.1, 0.2) (N_b = 0.1, 0.5).

Table 1. Numerical Comparison of $\theta'(0)$ for Pr

Pr	Liaqat et al. [29]	Sohaib et al. [28]	Our Results
0.72	0.8086	0.80863	0.80866
1.00	1.0000	1.00000	1.00000
3.00	1.9236	1.92367	1.92376
10.0	3.7206	3.72066	3.72084
100	12.2946	12.29405	12.29452

Table 2. Numerical Comparison of $-f''(0)$ for M

M	Bagh et al. [31]	Liaqat et al. [30]	Our Results
0.0	1.0000080	1.0000078	1.0000196
0.2	1.0954458	1.0954462	1.0954480
0.5	1.2247446	1.2247452	1.2247476
1.0	1.4142132	1.4142142	1.4142192
1.2	1.4832393	1.4832385	1.4832416
1.5	1.5811384	1.5811392	1.5811402
2.0	1.7320504	1.7320515	1.7320522

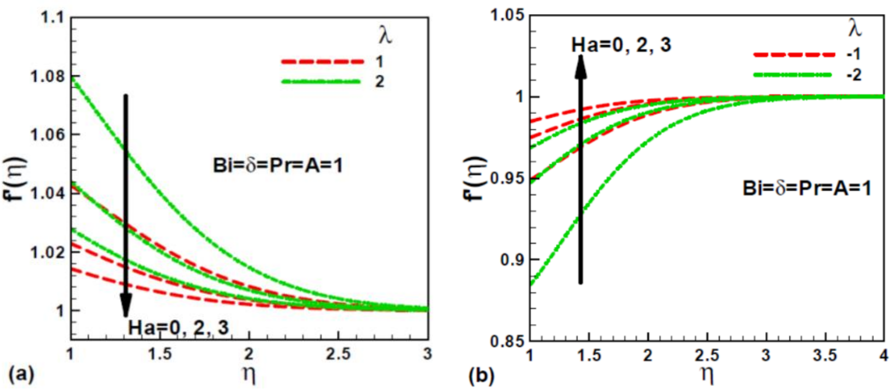

Figure 1. Impact of Ha and Bi on velocity for supporting and conflicting flow

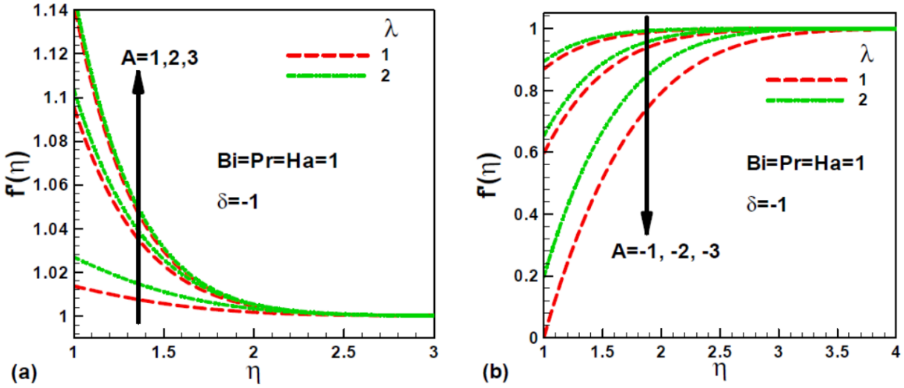

Figure 2. Impact of A and Bi on velocity for supporting and conflicting flow

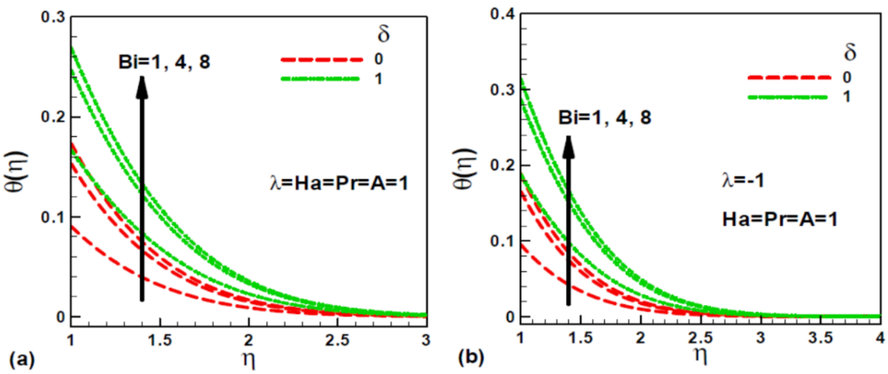

Figure 3. Impact of Bi and δ on temperature for supporting and conflicting flow

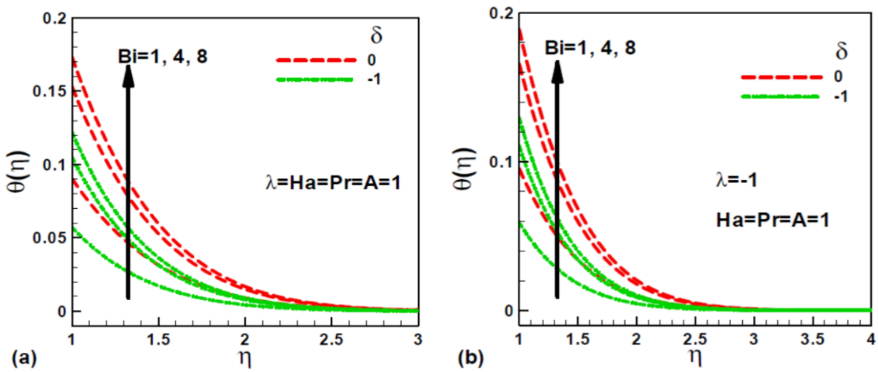

Figure 4. Impact of Bi and λ on temperature for supporting and conflicting flow

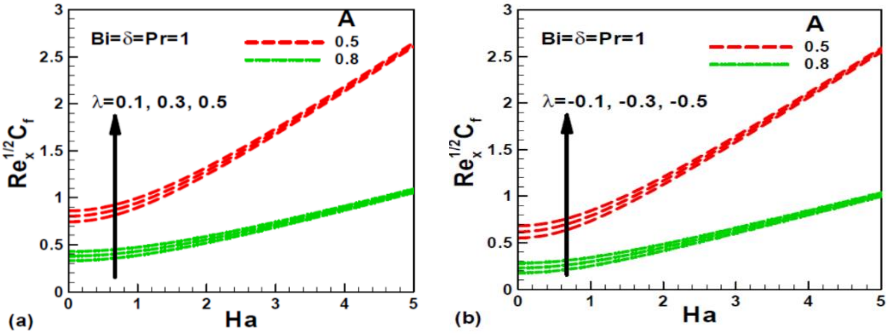

Figure 5. Impact of A and λ on temperature for supporting and conflicting flow

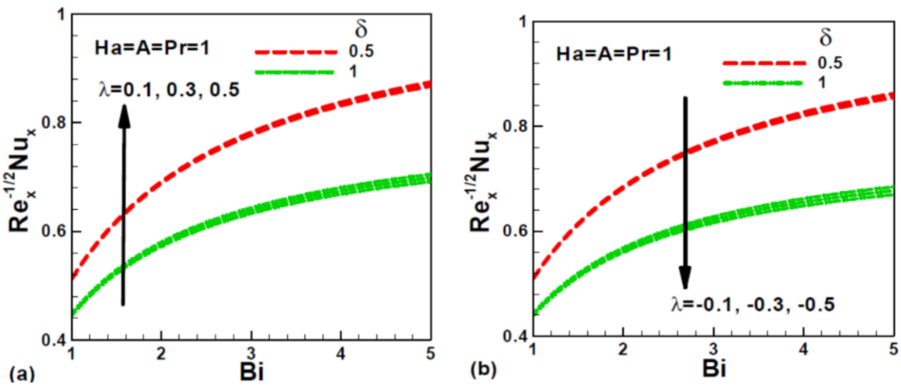

Figure 6. Impact of λ and δ on temperature for supporting and conflicting flow

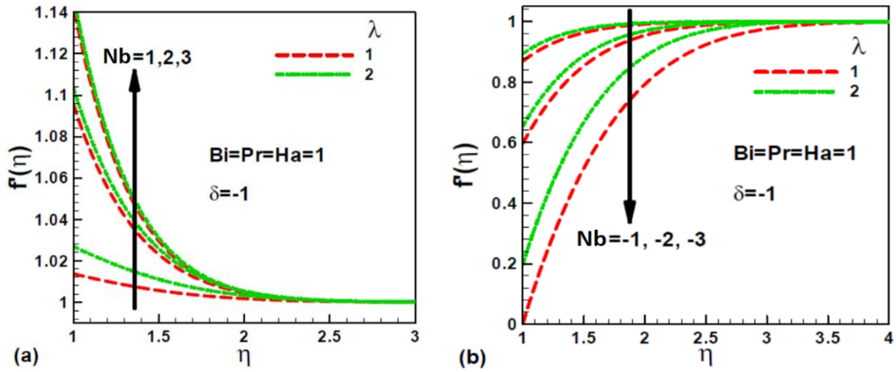

Figure 7. Impact of Nb and λ on temperature for supporting and conflicting flow

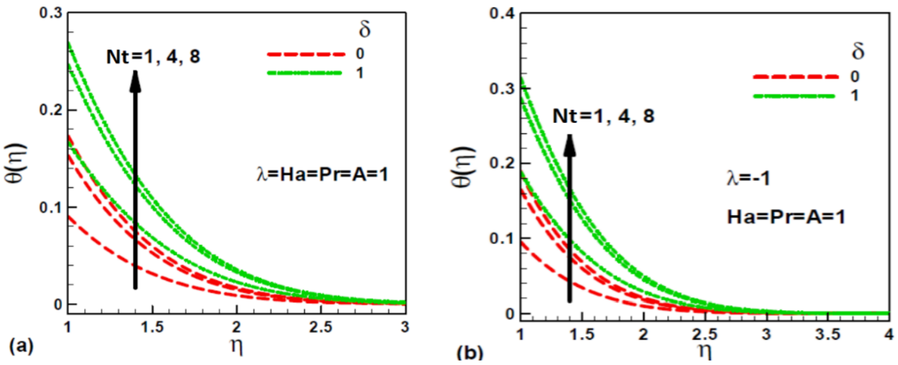

Figure 8. Impact of Nt and δ on temperature for supporting and conflicting flow

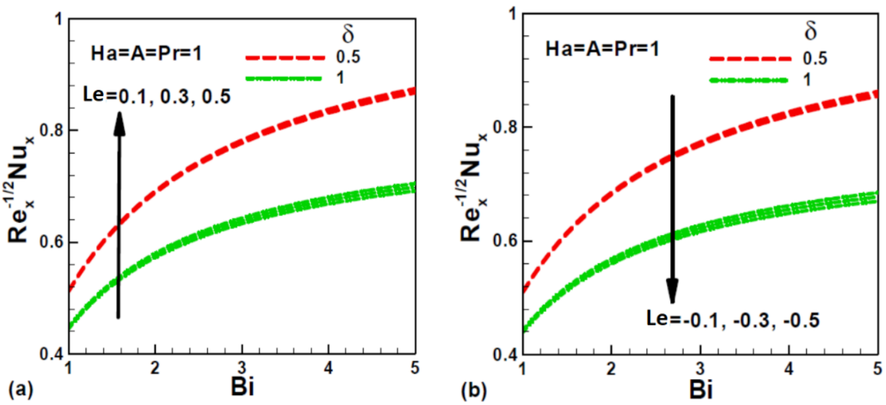

Figure 9. Impact of Le and δ on temperature for supporting and conflicting flow

5. Conclusion

A nanofluid flow over a stretching sheet in the presence of viscous dissipation, heat source, and magnetic field was numerically explored with the help of the Runge-Kutta shooting technique. The effects of the specific parameters that influence the temperature and velocity distribution were noticed. Some notable observations are outlined as follows.

- The increase in H_a and λ causes an increase in the velocity profile; however, an opposite behavior is demonstrated for A.
- The heat transfer coefficient increases buoyancy assisting and it decreases buoyancy opposing.

- The non-dimensional velocity decreases by increasing the Hartmann number for buoyancy assisting but an opposite effect is seen for buoyancy opposing flow.
- Temperature profile increases with the increase in N_t.
- Temperature rises as Bi increases.

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