
Zameer Abbas  Sohaib Abdal  Fayyaz Hussain  Nasir Hussain  Muhammad Adnan  Bagh Ali  Rana Muhammad  Liaqat Ali  Zulqarnain  Saba Younas

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MHD Boundary Layer Flow and Heat Transfer of Nanofluid over a Vertical Stretching Sheet in the Presence of a Heat Source

Zameer Abbas¹, Sohaib Abdal²*, Fayyaz Hussain¹, Nasir Hussain¹, Muhammad Adnan¹, Bagh Ali³, Rana Muhammad Zulqarnain², Liaqat Ali⁴, Saba Younas¹

¹Department of Mathematics, National College of Business Administration & Economics, Lahore, Pakistan
²School of Mathematics, Northwest University, Xi’an, China
³Department of Applied Mathematics, Northwestern Polytechnical University, Xi’an, China
⁴School of Energy and power, Xi’an Jiaotong University, Xi’an, China

*sohaib@stumail.nwu.edu.cn

Abstract

The nanoparticles used in nanofluid are prepared for carbides and oxides. In this paper, a nanofluid flow over a stretching sheet in the presence of viscous dissipation, heat source, and magnetic field was numerically explored with the help of the Runge-Kutta shooting technique and the effects of various parameters were analyzed using graphical representation.

Keywords: buoyancy assisting, buoyancy opposing, MHD Boundary Layer Flow, nanofluid, Runge-Kutta shooting technique

Introduction


According to the best knowledge of the author, the numerical study of the effect of heat and boundary layer flow on steady convection flow and heat transfer past a vertical stretching sheet is not available. Energy and momentum equations are obtained with the help of similarity variables. The governing partial differential equations are transmuted into ordinary differential equations and numerically solved by using Runge-Kutta shooting technique. The effects of various parameters are analyzed using their graphical representation.

2. Problem Description

In this study, the vertical stretching sheet is in the direction of the x-axis
and the \( y \)-axis is orthogonal to the sheet. \( u \) is the velocity component in the \( x \)-direction and \( v \) is the velocity component in the \( y \)-direction. Taking into consideration \( c \) which is a positive constant; \( u = u_e(x) = ax \) represents the unrestricted stream velocity, while \( u = u_w(x) = cx \) represents the velocity when there is stretching on the sheet. When a heat source / sink is present, \( H_0 \) is an external magnetic field that is practically perpendicular to the sheet. The principal equations of continuity, momentum, and energy are written as follows,

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \quad (1) \\
{u} \frac{\partial u}{\partial x} + {v} \frac{\partial u}{\partial y} &= \frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e H_0^2}{\rho} u + g \beta (T - T_\infty) \quad (2) \\
{u} \frac{\partial T}{\partial x} + {v} \frac{\partial T}{\partial y} &= \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p} (T - T_\infty) + \tau \left( D_B \frac{\partial C}{\partial y} + \frac{D_T (\partial T)}{T_\infty} \right)^2 \quad (3) \\
{u} \frac{\partial C}{\partial x} + {v} \frac{\partial C}{\partial y} &= D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T \partial^2 T}{T_\infty} \quad (4)
\end{align*}
\]

where \( D_B \) is Brownian diffusion, \( D_T \) is Thermophoresis diffusion, \( \sigma \) is electrical conductivity, \( \mu_e \) is magnetic permeability, \( T_\infty \) is the temperature of free stream, \( g \) is the acceleration due to gravity, \( \beta \) is the volumetric coefficient of thermal expansion, \( k \) is thermal conductivity, \( \nu = \frac{\mu}{\rho} \) is the kinematic viscosity, and \( T_w = T_\infty + bx \) is the temperature of the sheet. \( C_w = C_\infty + bx \), \( \tau \) is the ratio of heat capacities.

The boundary conditions pertaining to the horizontally moving boundary and convective heat transfer at the wall are formulated below.

\[
v = 0, u = u_w(x) = cx, -k \frac{\partial T}{\partial y} = h_f (T_f - T), C = C_w \text{ at } y = 0 \quad (5)
\]

\[
u = u_e(x) = ax, T = T_\infty, C = C_\infty \text{ as } y \to \infty,
\]

3. Similarity Analysis

The subsequent change and dimensionless quantities are used into equations while taking into account the boundary conditions.

We have

\[
\eta = \sqrt{\frac{a}{\nu}} y, \psi = x \sqrt{\nu} a f(\eta), \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and}
\]
\[ u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \]  
\[ u = x a f'(\eta), \quad v = -\sqrt{av} f(\eta). \]

The equation of continuity satisfies equations (2-4) and transforms them as shown below.

\[ f''' + ff'' - f'^2 + 1 + H_a^2 \left(1 - f'\right) + \lambda \theta = 0 \]  
\[ \theta'' + pr \left[ f \theta' - f' \theta + \delta \theta + N_b \theta \Phi' + N_c \theta^2 \right] = 0 \]
\[ \Phi'' + Le \left( f \Phi' - f' \Phi \right) + \frac{N_t}{Nb} \theta'' = 0 \]

The boundary conditions (5) become

\[ f = 0, f' = \frac{c}{a} = A, \theta' = B(\theta - 1), \phi = 1 \text{ at } \eta = 0 \]  
\[ f' = 1, = 0, \phi = 0 \text{ as } \eta \rightarrow \infty. \]

\[ H_a = \mu_e H_o \sqrt{\frac{\sigma}{\rho a}} \] is the Hartmann number, \( \lambda = \frac{Gr_x}{Re_x^2} \) is the mixed convection parameter,

\[ Gr_x = g \beta (T_w - T_x) \frac{x^3}{v^2} \] is the local Grashof number, \( Re_x = u_e(x) \frac{x}{v} \) and \( p_r = \frac{v}{\alpha} \).

\[ \delta = \frac{Q}{\rho a C_p} \] represents the factor of heat generation or absorption. \( L_e = \frac{v}{D_B} \) is the Lewis number parameter, \( N_b = \tau D_B \frac{b x}{x} \) is the Brownain motion parameter, and \( N_t = \tau DT_x \frac{b x}{T_x} \) is the Thermophoresis parameter.

### 4. Results and Discussion

A steady laminar flow above a vertical stretching sheet with the existence
of viscous dissipation, heat sink or source, and magnetic field was explored numerically with the help of the Runge-Kutta shooting technique. Physical parameter effects $L_e$, $N_b$, $N_t$, Hartmann number, stretching velocity ratio, Biot number and velocity distribution along with skin friction and heat transfer coefficient.

In Table 1, numerical comparison of the values of Pr for heat transformation is obtained by using the Runge-Kutta shooting scheme. The skin friction coefficient is compared with previously studied results in Table 2. In Figure 1, the effect of $Ha$ on velocity for opposing and assisting flow is shown ($N_b = N_t = L_e = 0$). Figure 2 shows the effects of $A$ when stretching is in flow in the opposite direction ($N_b = N_t = L_e = 0$). In Figure 3, the effects of $Bi$ in the presence of a heat source on the dimensionless temperature for assisting and opposing flow can be seen. A similar effect for $Bi$ can be seen in Figure 4 which illustrates the result of the Biot number in the company of heat sink on dimensionless temperature for assisting and opposing flow at $Pr = Ha = A$ ($N_b = N_t = L_e = 0$). Figure 5 displays the effects of stretching velocity ratio and mixed convection parameter on dimensionless skin friction for both assisting and opposing flow. It is found that skin friction increases with the mixed convection parameter and $Ha$, even though it drops with the stretching velocity ratio for mutually opposing and assisting flow ($N_b = N_t = L_e = 0$). The effects of Biot number $Bi$, heat generation / absorption coefficient $\delta$, and mixed convection parameter $\lambda$ on the dimensionless heat transfer rate for both assisting and opposing flow are illustrated respectively in Figure 6 ($N_b = N_t = L_e = 0$). The effects of Brownian motion $N_b$ in the presence of heat source on the dimensionless temperature for assisting and opposing flows are shown in Figure 7, respectively ($N_t = 0.1, 0.2$) ($L_e = 0.1, 0.5$). Figure 8 illustrates the result of Thermophoresis $N_t$ in the company of heat sink on the dimensionless temperature for assisting and opposing flow at $Pr = Ha = A$ ($N_b = 0.1, 0.2$) ($L_e = 0.1, 0.5$). The effects of Lewis number $Le$ and $\lambda$ for both assisting and opposing flow are illustrated respectively in Figure 9 ($N_t = 0.1, 0.2$) ($N_b = 0.1, 0.5$).
Table 1. Numerical Comparison of $\theta'(0)$ for Pr

<table>
<thead>
<tr>
<th>Pr</th>
<th>Liaqat et al. [29]</th>
<th>Sohaib et al. [28]</th>
<th>Our Results</th>
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Table 2. Numerical Comparison of $-f''(0)$ for $\lambda$

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<th>M</th>
<th>Bagh et al. [31]</th>
<th>Liaqat et al. [30]</th>
<th>Our Results</th>
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Figure 1. Impact of Ha and Bi on velocity for supporting and conflicting flow
**Figure 2.** Impact of A and Bi on velocity for supporting and conflicting flow

**Figure 3.** Impact of Bi and $\delta$ on temperature for supporting and conflicting flow

**Figure 4.** Impact of Bi and $\lambda$ on temperature for supporting and conflicting flow
Figure 5. Impact of $A$ and $\lambda$ on temperature for supporting and conflicting flow

Figure 6. Impact of $\lambda$ and $\delta$ on temperature for supporting and conflicting flow

Figure 7. Impact of Nb and $\lambda$ on temperature for supporting and conflicting flow
5. Conclusion

A nanofluid flow over a stretching sheet in the presence of viscous dissipation, heat source, and magnetic field was numerically explored with the help of the Runge-Kutta shooting technique. The effects of the specific parameters that influence the temperature and velocity distribution were noticed. Some notable observations are outlined as follows.

- The increase in $H_a$ and $\lambda$ causes an increase in the velocity profile; however, an opposite behavior is demonstrated for $A$.
- The heat transfer coefficient increases buoyancy assisting and it decreases buoyancy opposing.
• The non-dimensional velocity decreases by increasing the Hartmann number for buoyancy assisting but an opposite effect is seen for buoyancy opposing flow.
• Temperature profile increases with the increase in $N_t$.
• Temperature rises as $Bi$ increases.

References


