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Stretching a Surface in a Rotating Fluid through Porous Medium

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ABSTRACT

This study examines the rotating fluid flow of a viscous fluid originated by the stretching of the surface over which the fluid exists. The study focuses on the effects of slip velocity and the porosity of the medium. The Homotopy Analysis Method (HAM) is utilized to obtain the analytical expressions of the flow variables. Similarity transformations are used to convert the involved partial differential equations into ordinary differential equations. The effect of porosity and slip velocity parameters are presented through graphs. It is found that the parameter of porosity increases the similarity velocity profiles of the rotating fluid.

Keywords**:** homotopy analysis method, porosity parameter, rotating fluid flow, slip velocity

Highlights

- Analytical HAM (homotopy analysis method) solutions reveal porosity boosts velocity profiles in rotating fluid flow over stretching surfaces
- Slip parameter reduces x-direction velocity but enhances y-direction velocity in porous medium flows
- The current study advances low-Re laminar flow modeling for filtration and seepage applications

1. INTRODUCTION

Flow through porous media is an important class of small Reynolds number (Re) laminar flow. This type of flow is found in the filtration of fluids and the seepage of water in canal and river banks. Some other examples of this flow are the movements of underground water and oils [1-3].

The slip condition is also an important aspect which has not been given

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proper attention in the study of fluid dynamics. Navier [4] described shear stress-based slip boundary condition. Saqib et al. [5] used fractional derivatives in Caputo sense. In another paper, Saqib et al. [6] discussed $Cu - Al2O3 - H2O$ hybrid nanofluid. Hussan et al. [7] investigated a viscoplastic Casson fluid in a two dimension flow, with a stretching surface taken into account. Some of the recent advancements regarding flow over a stretching sheet and slip effects have been referenced in the literatue [8-14]. Nadeem et al. [15] developed the Caputo fractional model for Casson fluid with the help of Flick's and Fourier's laws. Farhad et al. [16] analyzed blood flow using Casson fluid model through a horizontal cylinder in the presence of magnetic particles. Nadeem et al. [17] discussed the Brinkman type fluid flow in a channel.

We used the Homotopy Analysis Method (HAM) [18-22] to obtain the analytic series outcomes in this paper. Crane [23] explored the stretching of a surface. Brady and Acrivos [24] and Wang [25] provided deep insight into axisymmetric and three-dimensional cases. They expressed the effects of different parameters in to and three dimensional flows. Wang [26] discussed the case of stretching a surface in rotating fluid.

Keeping all the above-mentioned ponts, the arrangement of the paper is as fallows.

Section 1 includes introduction, section 2 includes mathematical formulation equations and Homotopy Analysis Method, section 3 includes discussion, and finally, section 4 includes graphical representation.

2. MATHEMATICAL FORMULATION

The velocity field is defined as

$$\mathbf{V} = \left[u(x,y,z),\ v(x,y,z),\ w(x,y,z)\right] \tag{1}$$

The Navier-Stokes equation in this frame is [26]

$$\rho\left[\frac{d}{dt}\mathbf{V} + 2\mathbf{\Omega}\times\mathbf{V}+\mathbf{\Omega}\times\left(\mathbf{\Omega}\times\mathbf{r}\right)\right] = -\nabla p + \mu\nabla^2\mathbf{V} - \left(\mu\,\Phi/k\right)\mathbf{V} \tag{2}$$

The Navier-Stokes equations in component form when the flow is steady, incompressible, and in rotating frame through porous media are

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) + w\left(\frac{\partial u}{\partial z}\right) - 2\mathbf{\Omega} v - \mathbf{\Omega}^2 x = \quad (3)$$

$$-\frac{1}{\rho}P_x + \nu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\} - \frac{\mu\Phi}{k}u$$

$$u\left(\frac{\partial v}{\partial x}\right) + v\left(\frac{\partial v}{\partial y}\right) + w\left(\frac{\partial v}{\partial z}\right) + 2\mathbf{\Omega} u - \mathbf{\Omega}^2 y = \quad (4)$$

$$-\frac{1}{\rho}P_y + \nu\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\} - \frac{\mu\Phi}{k}v$$

$$u\left(\frac{\partial w}{\partial x}\right) + v\left(\frac{\partial w}{\partial y}\right) + w\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho}P_z + \nu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\} \quad (5)$$

Using

$$p^* = p - \frac{1}{2}\rho\Omega^2 r^2 \quad (6)$$

we get

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) + w\left(\frac{\partial u}{\partial z}\right) - 2\mathbf{\Omega} v = -\frac{1}{\rho}P_x^* + \nu\left\{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right\} - \frac{\mu\Phi}{k}u \quad (7)$$

$$u\left(\frac{\partial v}{\partial x}\right) + v\left(\frac{\partial v}{\partial y}\right) + w\left(\frac{\partial v}{\partial z}\right) + 2\mathbf{\Omega} u = -\frac{1}{\rho}P_y^* + \nu\left\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right\} - \frac{\mu\Phi}{k}v \quad (8)$$

$$u\left(\frac{\partial w}{\partial x}\right) + v\left(\frac{\partial w}{\partial y}\right) + w\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho}P_z^* + \nu\left\{\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right\} \quad (9)$$

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (10)$$

with boundary conditions

$$u - \gamma\frac{du}{dz} = ax, \quad v - \gamma\frac{dv}{dz} = 0, \quad w = 0 \; at \; z = 0, \quad (11)$$
$$u \to 0, \; v \to 0 \; as \; z \to \infty,$$

where γ is the slip parameter.

Suppose the surface is expanded in a horizontal line, then the components of velocity are

$$u = ax, v = w = 0 \quad (12)$$

In the above equation, the dimensions of 'a' show the rate of stretch. Using the dimensionless quantities

$$u = axf'(\eta), \; v = axh(\eta), \; w = -\sqrt{a\nu}\, f(\eta), \quad \eta = z\sqrt{\frac{a}{\nu}} \quad (13)$$

With the help of eq. (13), eqs. (7) to (10) are reduced to

$$(f')^2 - ff'' - 2\lambda h = f''' - Rf' \quad (14)$$

$$f'h - fh' + 2\lambda f' = h'' - Rh \quad (15)$$

and BCs are reduced to

$$f(0) = 0, f'(0) = 1 + \beta f''(0), f'(\infty) = 0, \quad (16)$$

$$h(0) - \beta h'(0) = 0, \; h(\infty) = 0 \quad (17)$$

$$\beta = \sqrt{\frac{\gamma^2 a}{\nu}} \quad (18)$$

where β is a dimensionless slip parameter, while λ is a dimensionless parameter given by $\lambda = \mathbf{\Omega}/a$, and $R = \mu\left(\Phi/k\right)$ is a porosity parameter.

2.1. Solution of the Considred Problem by HAM

In this method, we use the initial guesses, satisfying the given boundary conditions as

$$f_0(\eta) = \frac{\beta}{2}(1 - e^{\frac{-\eta}{\beta}}) \quad (19)$$

and

$$h_0(\eta) = e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}, \quad (20)$$

while the corresponding auxiliary linear operators are

$$\mathrm{L}_1(f) = f''' - f'' \tag{21}$$

and

$$\mathrm{L}_2(h) = h'' + h', \tag{22}$$

possessing the conditions

$$\mathrm{L}_1(c_1 + c_2\eta + c_3 e^{-\eta}) = 0, \tag{23}$$

$$\mathrm{L}_2(c_4 + c_5 e^{-\eta}) = 0, \tag{24}$$

and $c_{i,}$ *(, i from 1 to 5)* are the constants.

2.2. The Problem of Zero-Order

The equations of zeroth order are defined below,

$$(1-p)\mathrm{L}_1[\overline{f}(\eta, p) - f_0(\eta)] = p\hbar_1 N_1[\overline{f}(\eta, p), \overline{h}(\eta, p)] \tag{25}$$

$$(1-p)\mathrm{L}_2[\overline{h}(\eta, p) - h_0(\eta)] = p\hbar_2 \; N_2[\overline{f}(\eta, p), \overline{h}(\eta, p)] \tag{26}$$

where N_1 and N_2 are non-linear auxiliary operators defined below as

$$N_1[\overline{f}(\eta,p),\, \overline{h}(\eta,p)] = \frac{\partial^3 \overline{f}}{\partial \eta^3}(\eta,p) - \left(\frac{\partial}{\partial \eta}\overline{f}(\eta,p)\right)^2 + \overline{f}(\eta,p)\left(\frac{\partial^2}{\partial \eta^2}\overline{f}(\eta,p)\right) + 2\lambda \overline{h}(\eta,p) - Rf', \tag{27}$$

$$N_2[\overline{f}(\eta,p),\, \overline{h}(\eta,p)] = \frac{\partial^2}{\partial \eta^2}\overline{h}(\eta,p) - \overline{h}(\eta,p)\frac{\partial}{\partial \eta}\overline{f}(\eta,p) + \overline{f}(\eta,p)\frac{\partial}{\partial \eta}\overline{h}(\eta,p) - 2\lambda\frac{\partial}{\partial \eta}\overline{f}(\eta,p) - Rh, \tag{28}$$

where $\overline{f}(\eta, p)$ and $\overline{h}(\eta, p)$ are functions of η and p. Putting $p = 0$ and $p = 1$, we have

$$\overline{f}(\eta,0) = f_0(\eta), \qquad \overline{h}(\eta,0) = h_0(\eta) \tag{29}$$

$$\overline{f}(\eta,1) = f(\eta), \qquad \overline{h}(\eta,1) = h(\eta) \tag{30}$$

provided conditions

$$f(0) = 0, f'(0) = 1 + \beta f''(0), \qquad f'(\infty) = 0, \tag{31}$$

$$h(0) - \beta h'(0) = 0, \qquad h(\infty) = 0. \tag{32}$$

We note that deformation equations of the zero-order contain the auxiliary parameters $\hbar_1$, $\hbar_2$. Note that $\hbar_1$ and $\hbar_2$ are assumed, so that the problem of zero-order may have a solution for all $p \in [0, 1]$

Expand $\bar{f}(\eta, p)$ and $\bar{h}(\eta, p)$ in the following power series,

$$\bar{f}(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \tag{33}$$

$$\bar{h}(\eta, p) = h_0(\eta) + \sum_{m=1}^{\infty} h_m(\eta) p^m, \tag{34}$$

where

$$f_m(\eta) = \frac{1}{m!} \frac{\partial^m}{\partial p^m} \bar{f}(\eta, p)|_{p=0} \tag{35}$$

and $h_m(\eta) = \dfrac{1}{m!} \dfrac{\partial^m}{\partial p^m} \bar{h}(\eta, p)|_{p=0}$ (36)

The series (33) and (34) converge upon $\hbar_1$ and $\hbar_2$. We choose $\hbar_1$ and $\hbar_2$ so that these series may converge at p = 1, so the above equations become

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta), \tag{37}$$

$$h(\eta) = h_0(\eta) + \sum_{m=1}^{\infty} h_m(\eta), \tag{38}$$

2.3. Deformation Equations of Higher Order

Now taking mth derivative of zero order deformation equations with respect to p, then putting p = 0 and dividing it by m!, we get

$$L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar_1 R^1{}_m(\eta) \tag{39}$$

$$L_2[h_m(\eta) - \chi_m h_{m-1}(\eta)] = \hbar_2 R_m^2(\eta) \quad (40)$$

and the given conditions are

$$f_m(0) = 0, \; f_m'(0) = 1 + \beta_m f''(0), \quad f_m'(\infty) = 0, \quad (41)$$

$$h_m(0) - \beta\, h_m'(0) = 0, \quad h_m(\infty) = 0 \quad (42)$$

where

$$R_m^1(\eta) = \frac{d^3 f_{m-1}}{d\eta^3} - \sum_{k=0}^{m-1} \frac{df_{m-1-k}}{d\eta} \cdot \frac{df_k}{d\eta} + \sum_{k=0}^{m-1} f_{m-1-k} \frac{d^2 f_k}{d\eta^2} + 2\lambda h_{m-1} - R\frac{df_{m-1}}{d\eta} \quad (43)$$

$$R_m^2(\eta) = \frac{d^2 h_{m-1}}{d\eta^2} - \sum_{k=0}^{m-1} h_{m-1-k} \cdot \frac{df_k}{d\eta} + \sum_{k=0}^{m-1} f_{m-1-k} \frac{dh_k}{d\eta} - 2\lambda \frac{df_{m-1}}{d\eta} - Rh_{m-1} \quad (45)$$

and $$\chi_m = \begin{cases} 0 & if\ m \le 1, \\ 1 & if\ m > 1. \end{cases} \quad (46)$$

Put m = 1 in equation (39), we get

$$L_1[f_1(\eta) - \chi_1 f_0(\eta)] = \hbar_1 R_1^1(\eta). \quad (47)$$

Since $\chi_1 = 0$

$$L_1[f_1(\eta)] = \hbar_1 R_1^1(\eta). \quad (48)$$

where

$$R_1^1(\eta) = \frac{d^3 f_0}{d\eta^3} - \left(\frac{df_0}{d\eta}\right)^2 + f_0 \frac{d^2 f_0}{d\eta^2} + 2\lambda h_0 - R\, f_0 \quad (49)$$

So the problem is

$$f_1''' + f_1'' = \hbar_1[f_0''' - (f_0')^2 + f_0'' f_0 + 2\lambda h_0 - R\, f_0] \quad (50)$$

Substituting h_0, f_0, and their derivatives, the following equation is obtained

$$f_1''' + f_1'' = 2\lambda\eta\hbar_1\left(e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}\right) \quad (51)$$

The complementary solution of equation (51) is

$$f_{1c} = c_1 + c_2\eta + c_3 e^{-\eta} \tag{52}$$

and the particular solution is

$$f_{1p} = 2\lambda\hbar_1[\frac{\beta^6}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}\eta] + \frac{\beta^3}{-1+\beta}\hbar_1[\frac{1}{2\beta^2} - \frac{1}{4}\frac{R}{2} -]e^{\frac{-\eta}{\beta}} \tag{53}$$

So, the general solution is

$$f_1 = f_{1c} + f_{1p} \tag{54}$$

$$f_1 = c_1 + c_2\eta + c_3 e^{-\eta} + 2\lambda\hbar_1[\frac{\beta^6}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}\eta] + \frac{\beta^3}{-1+\beta}\hbar_1[\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2} -]e^{\frac{-\eta}{\beta}} \tag{55}$$

$$f_1' = c_2 - c_3 e^{-\eta} - \frac{\beta^2}{-1+\beta}\hbar_1[\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2} -]e^{\frac{-\eta}{\beta}} + 2\lambda\hbar_1[\frac{-\beta^4}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}(1-\eta)] \tag{56}$$

$$f_1'' = c_3 e^{-\eta} + \frac{\beta^2}{1-\beta}\hbar_1[\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2} -]e^{\frac{-\eta}{\beta}} + 2\lambda\hbar_1[\frac{-\beta^2}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta}(\eta - 2)]. \tag{57}$$

Using the boundary conditions

$$f_1(0) = 0, \; f_1'(0) = 1 + \beta f_1''(0), \quad f_1'(\infty) = 0, \tag{58}$$

we get

$$c_1 = -\frac{1}{1+\beta}\{-1 - \frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2} - \frac{1}{\beta} + \frac{\beta^2}{-1+\beta} + \frac{2}{\beta})\} + 2\lambda\hbar_1(\frac{\beta^6}{-1+\beta^2}) + \frac{\beta^3}{-1+\beta}\hbar_1(\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2}) \tag{59}$$

$$c_2 = 0 \tag{60}$$

$$c_3 = \frac{1}{1+\beta}\{-1 - \frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2} - \frac{1}{4} - \frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2} - \frac{1}{\beta} + \frac{\beta^2}{-1+\beta} + \frac{2}{\beta})\}. \tag{61}$$

Putting the values of c_1, c_2, and c_3 in equation (55), we get

$$f_1 = -\frac{1}{1+\beta}\{-1-\frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2}-\frac{1}{\beta}+\frac{\beta^2}{-1+\beta}+\frac{2}{\beta})\} + 2\lambda\hbar_1(\frac{\beta^6}{-1+\beta^2}) + \frac{\beta^3}{-1+\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + \frac{1}{1+\beta}\{-1-\frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2}-\frac{1}{\beta}+\frac{\beta^2}{-1+\beta}+\frac{2}{\beta})\}e^{-\eta} + 2\lambda\hbar_1[\frac{\beta^6}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}}-\frac{e^{-\eta}}{\beta}\eta] + \frac{\beta^3}{-1+\beta}\hbar_1[\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}]e^{\frac{-\eta}{\beta}} \tag{62}$$

Therefore,

$$f = f_0(\eta) + f_1(\eta) \tag{63}$$

$$f = \frac{\beta}{2}(1-e^{\frac{-\eta}{\beta}}) - \frac{1}{1+\beta}\{-1-\frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2}-\frac{1}{\beta}+\frac{\beta^2}{-1+\beta}+\frac{2}{\beta})\} + 2\lambda\hbar_1(\frac{\beta^6}{-1+\beta^2}) + \frac{\beta^3}{-1+\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + \frac{1}{1+\beta}\{-1-\frac{2\beta^2}{1-\beta}\hbar_1(\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}) + 2\lambda\hbar_1(\frac{-\beta^4}{1+\beta^2}-\frac{1}{\beta}+\frac{\beta^2}{-1+\beta}+\frac{2}{\beta})\}e^{-\eta} + 2\lambda\hbar_1[\frac{\beta^6}{-1+\beta^2}e^{\frac{-\eta}{\beta^2}}-\frac{e^{-\eta}}{\beta}\eta] + \frac{\beta^3}{-1+\beta}\hbar_1[\frac{1}{2\beta^2}-\frac{1}{4}-\frac{R}{2}]e^{\frac{-\eta}{\beta}} \tag{64}$$

Now, to solve equation (14) put m = 1 in equation (40), we get

$$L_2[h_1(\eta) - \chi_1 h_0(\eta)] = \hbar_2 R^2{}_1(\eta) \tag{65}$$

Since $\chi_1 = 0$,

$$L_2[h_1(\eta)] = \hbar_2 R_1^2(\eta) \tag{66}$$

where

$$R_1^2(\eta) = \frac{d^2h_0}{d\eta^2} - h_0\frac{df_0}{d\eta} + f_0\frac{dh_0}{d\eta} - 2\lambda\frac{df_{m0}}{d\eta} - R\,h_0 \tag{67}$$

So, we have the problem

$$h_1'' + h_1' = \hbar_2[h_0'' + f_0h_0' - f_0'.h_0 - R\,h_0 - 2\lambda f_0'] \tag{68}$$

Substituting h_0, f_0, and their derivatives, the following equation is obtained

$$h_1'' + h_1' = \hbar_2[(\tfrac{1}{\beta^4} - \tfrac{1}{2\beta} - R)e^{\frac{-\eta}{\beta^2}} + (\tfrac{1}{2} - \tfrac{1}{\beta} + \tfrac{R}{\beta})e^{-\eta} + (\tfrac{1}{2\beta} - \tfrac{1}{2})e^{\frac{-\eta}{\beta^2} - \frac{\eta}{\beta}} + (\tfrac{1}{2\beta} - \tfrac{1}{2})e^{-\eta} - \tfrac{\eta}{\beta} - \lambda e^{\frac{-\eta}{\beta}}] \tag{69}$$

$$h_1(\eta) = c_1 + c_2e^{-\eta} + \hbar_2[(\frac{2-\beta^2-2\beta^4R}{2(1-\beta^2)})e^{\frac{-\eta}{\beta^2}} - (\frac{\beta-2+2R}{2\beta})\,\eta e^{-\eta} + (\tfrac{\beta^3(1-\beta)}{2(1+\beta)(1+\beta-\beta^2)})e^{\frac{-(1+\beta)}{\beta^2}\eta} + \tfrac{\beta(1-\beta)}{2(1+\beta)}e^{-(\frac{1+\beta}{\beta})\eta} - \tfrac{\beta^2}{1-\beta}\lambda e^{\frac{-\eta}{\beta}}] \tag{70}$$

Now, using the boundary conditions

$$h(0) - \beta h'(0) = 0, \qquad h(\infty) = 0 \tag{71}$$

we get the general solution

$$\begin{aligned} h_1(\eta) = {} & \frac{-1}{1+\beta}e^{-\eta}\hbar_2[\{\frac{2-\beta-2\beta^4R}{2(1-\beta^2)} + \frac{\beta^3(1-\beta)}{2(1+\beta)(1+\beta-\beta^2)} + \frac{\beta(1-\beta)}{2(1+\beta)} - \frac{\beta^3}{1-\beta}\lambda\} \\ & + \beta\{\frac{-2+\beta^2+2\beta^4R}{2\beta^2(1-\beta^2)} - \frac{\beta-2+2R}{2\beta} - \frac{(1-\beta)\beta}{2(1+\beta-\beta^2)} - \frac{1-\beta}{2} - \frac{\beta\lambda}{1-\beta}\}] \\ & + \hbar_2[(\frac{2-\beta^2-2\beta^4R}{2(1-\beta^2)})e^{\frac{-\eta}{\beta^2}} - (\frac{\beta-2+2R}{2\beta})\,\eta e^{-\eta} \\ & + (\tfrac{\beta^3(1-\beta)}{2(1+\beta)(1+\beta-\beta^2)})e^{\frac{-(1+\beta)}{\beta^2}\eta} + \tfrac{\beta(1-\beta)}{2(1+\beta)}e^{-(\frac{1+\beta}{\beta})\eta} - \tfrac{\beta^2}{1-\beta}\lambda e^{\frac{-\eta}{\beta}}]. \end{aligned} \tag{72}$$

As

$$h = h_0(\eta) + h_1(\eta) \tag{73}$$

Substituting the values of $h_0(\eta)$ and $h_1(\eta)$ in the above equation , we get

$$\begin{aligned} h = {} & e^{\frac{-\eta}{\beta^2}} - \frac{e^{-\eta}}{\beta} - \frac{1}{1+\beta} e^{-\eta} \hbar_2 [\{ \frac{2-\beta-2\beta^4 R}{2(1-\beta^2)} + \frac{\beta^3(1-\beta)}{2(1+\beta)(1+\beta-\beta^2)} + \frac{\beta(1-\beta)}{2(1+\beta)} - \frac{\beta^3}{1-\beta}\lambda \} \\ & + \beta \{ \frac{-2+\beta^2+2\beta^4 R}{2\beta^2(1-\beta^2)} - \frac{\beta-2+2R}{2\beta} - \frac{(1-\beta)\beta}{2(1+\beta-\beta^2)} - \frac{1-\beta}{2} - \frac{\beta\lambda}{1-\beta} \}] \\ & + \hbar_2 [(\frac{2-\beta^2-2\beta^4 R}{2(1-\beta^2)}) e^{\frac{-\eta}{\beta^2}} - (\frac{\beta-2+2R}{2\beta}) \eta e^{-\eta} \\ & + (\frac{\beta^3(1-\beta)}{2(1+\beta)(1+\beta-\beta^2)}) e^{\frac{-(1+\beta)}{\beta^2}\eta} + \frac{\beta(1-\beta)}{2(1+\beta)} e^{-(\frac{1+\beta}{\beta})\eta} - \frac{\beta^2}{1-\beta} \lambda e^{\frac{-\eta}{\beta}}]. \end{aligned} \tag{74}$$

3. DISCUSSION

In this study, the analytical solution for stretching a surface in a rotating fluid through a porous medium with partial slip is constructed. Figures (1) to (3) show the effect of the porosity parameter R, keeping slip parameter β and variation parameter λ fixed on the similarity velocity profile in the x-direction. The effect of the porosity parameter remains negligible. Figures (1), (4), and (5) show the effect of the slip parameter β, keeping porosity parameter R and variation parameter λ fixed on the similarity velocity profile in the x-direction. Velocity decreases as the value of β increases. Figures (6) to (8) show the effect of slip parameter β, keeping porosity parameter R and variation parameter λ fixed on the similarity velocity profile in the y-direction. Velocity increases with an increase in β. Figures (9) and (10) show the effect of porosity parameter R, keeping slip parameters β and variation parameter λ fixed on the similarity velocity profile in the y-direction. Noticeably, an increase in R causes an increase in h.

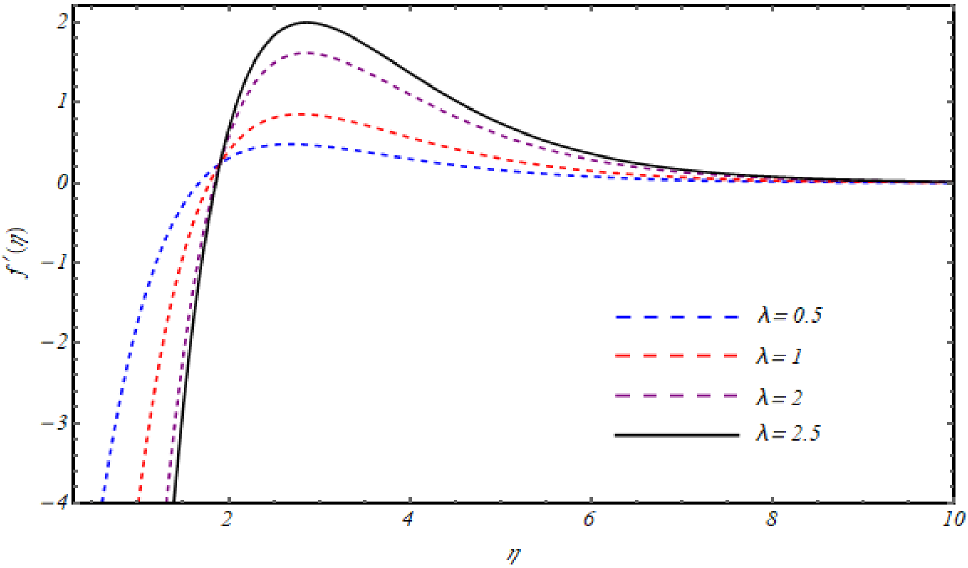

Figure 1. Effects of R, β, and λ on f ′(η) taking $\hbar_1 = 0.7$, $\beta = 0.1$, and R = 0.0.

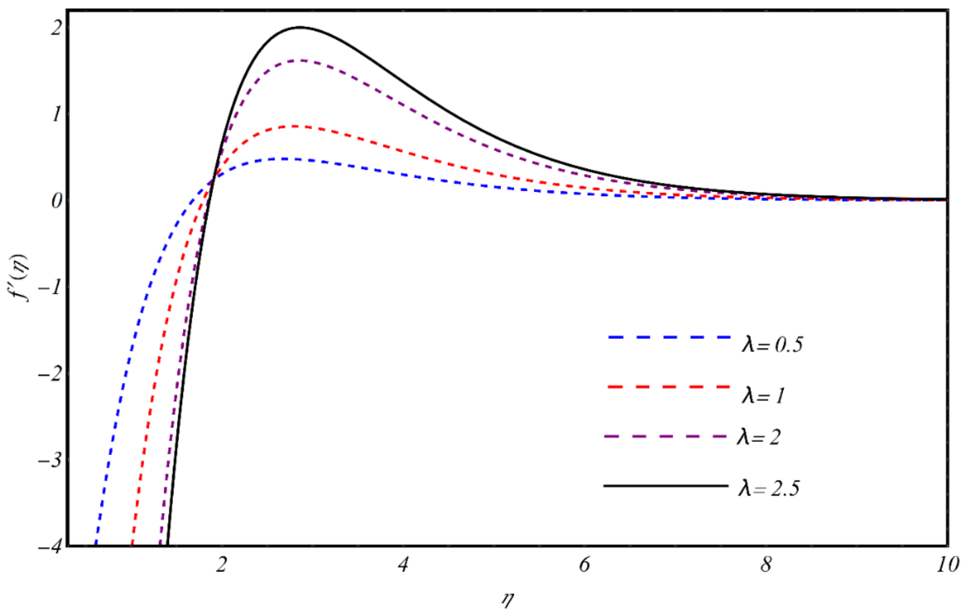

Figure 2. Effects of R, β, and λ on f ′(η) taking $\hbar_1 = 0.7$, $\beta = 0.1$, and R = 0.3.

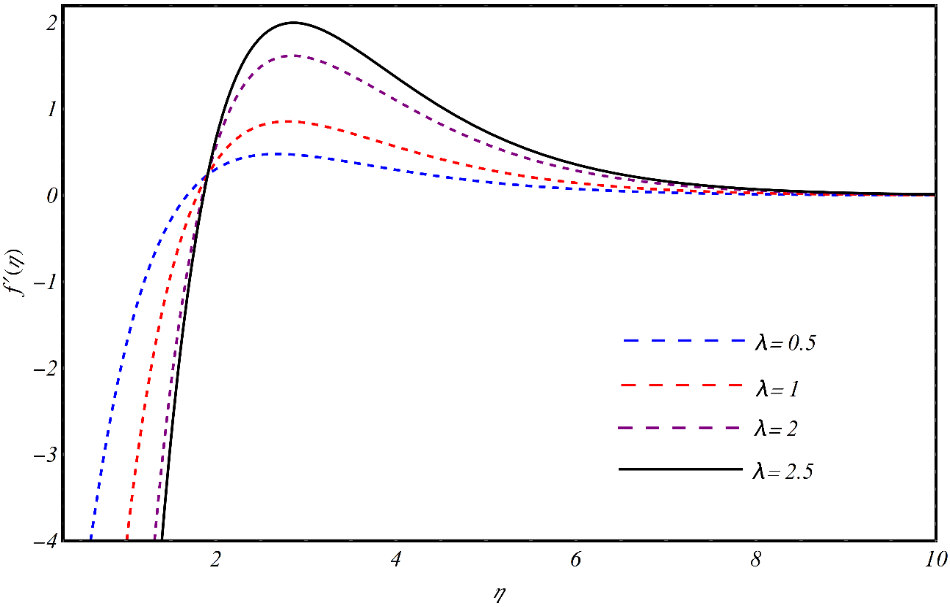

Figure 3. Effects of R , β, and λ on f ′(η) when $\hbar_1 =$ 0.7, β = 0.1, and R = 0.5.

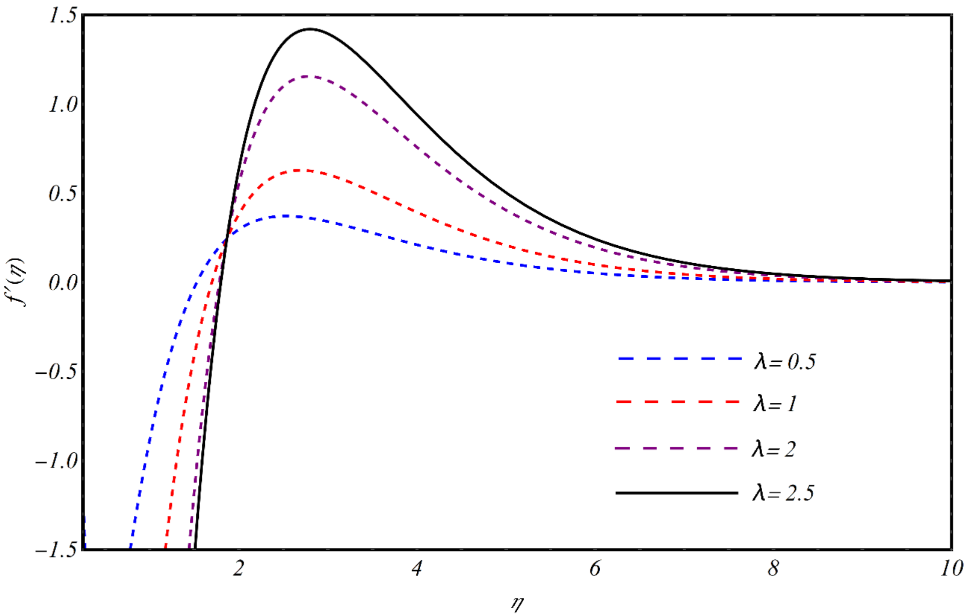

Figure 4. Effects of β, R, and λ on f ′(η) when $\hbar_1 =$ 0.7, β = 0.3, and R = 0.1.

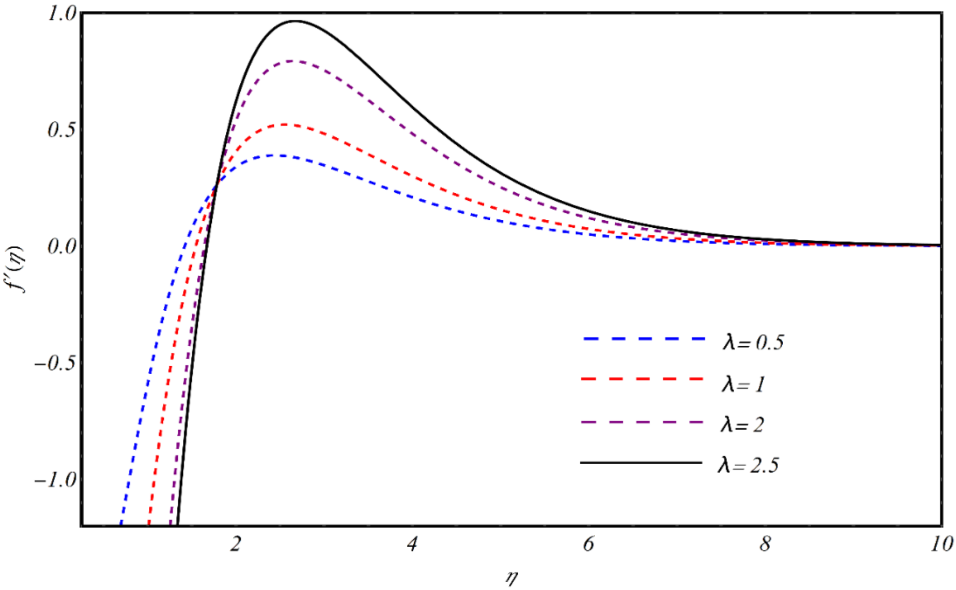

Figure 5. Effects of β, R, and λ on $f'(\eta)$ when $\hbar_1 = 0.7$, $\beta = 0.5$, and $R = 0.1$.

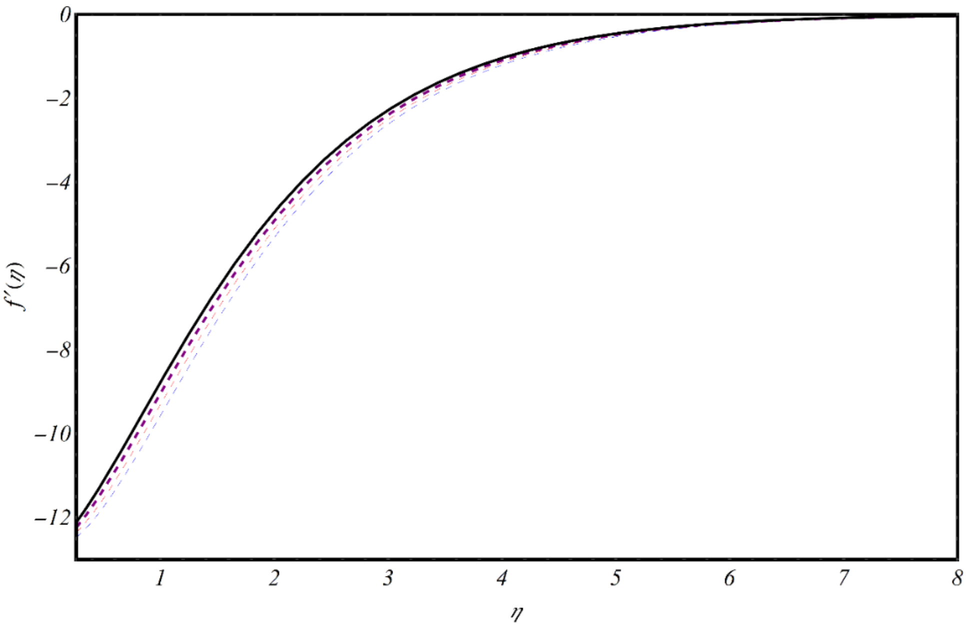

Figure 6. Effects of β, R, and λ on $h(\eta)$ when $\hbar_2 = -0.3$, $\beta = 0.1$, and $R = 0.2$.

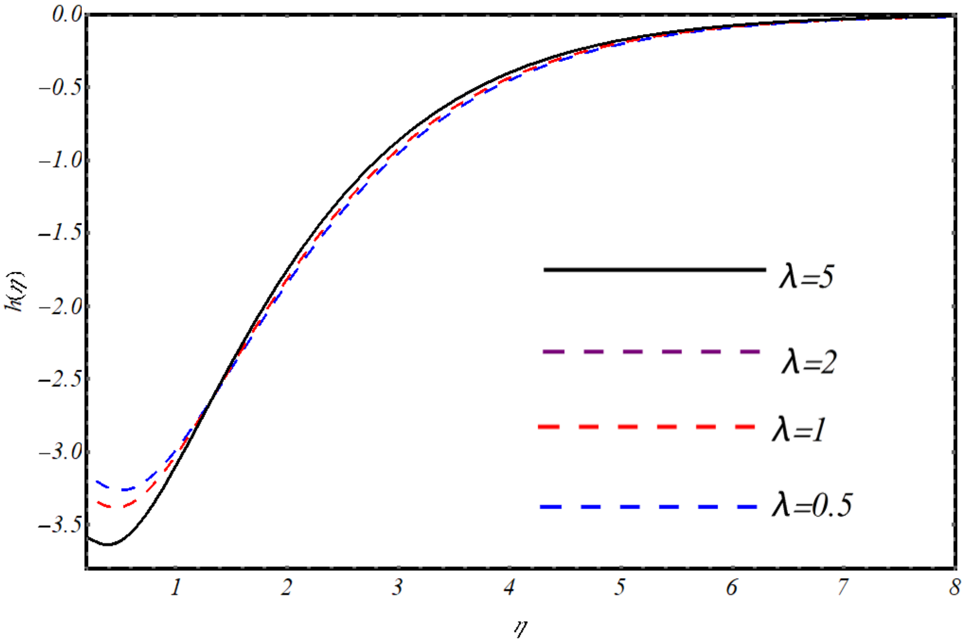

Figure 7. Effects of β, R, and λ on h(η) when $\hbar_2 = -0.3$, $\beta = 0.3$, and $R = 0.2$.

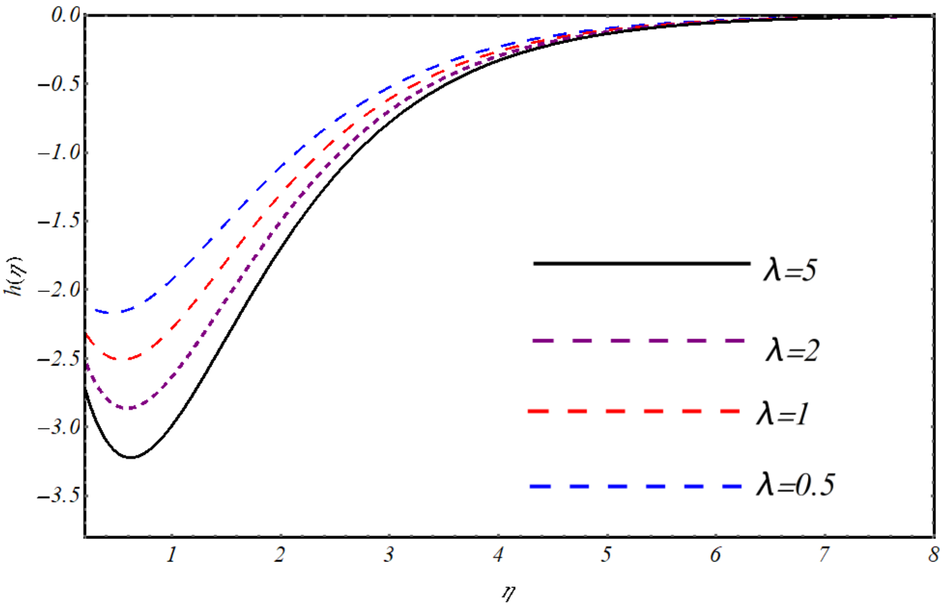

Figure 8. Effects of β, R, and λ on h(η) when $\hbar_2 = -0.3$, $\beta = 0.6$, and $R = 0.2$.

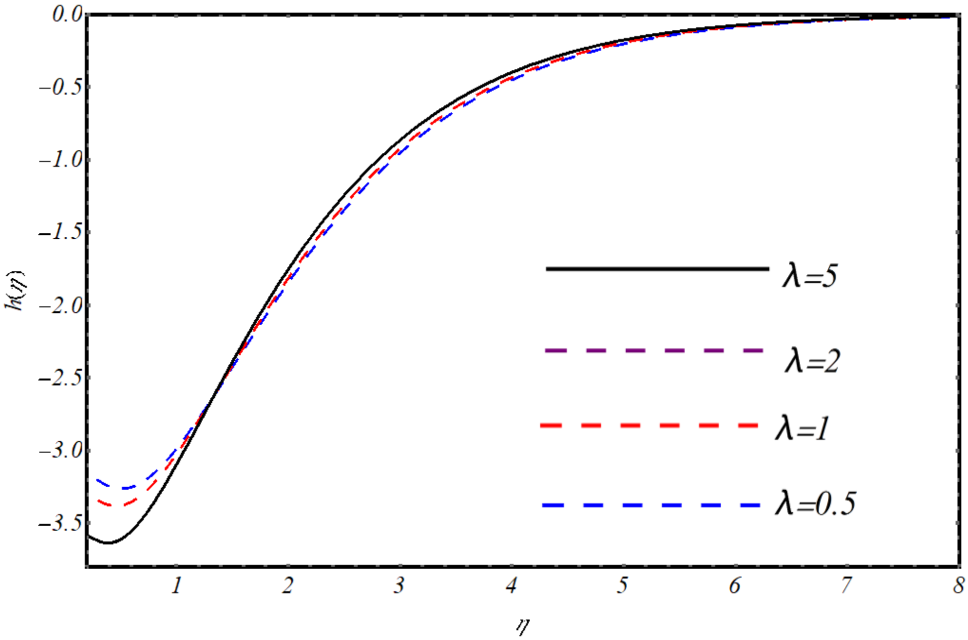

Figure 9. Effects of R, β, and λ on h(η) when $\hbar_2$ = – 0.3, β = 0.3, and R = 0.4.

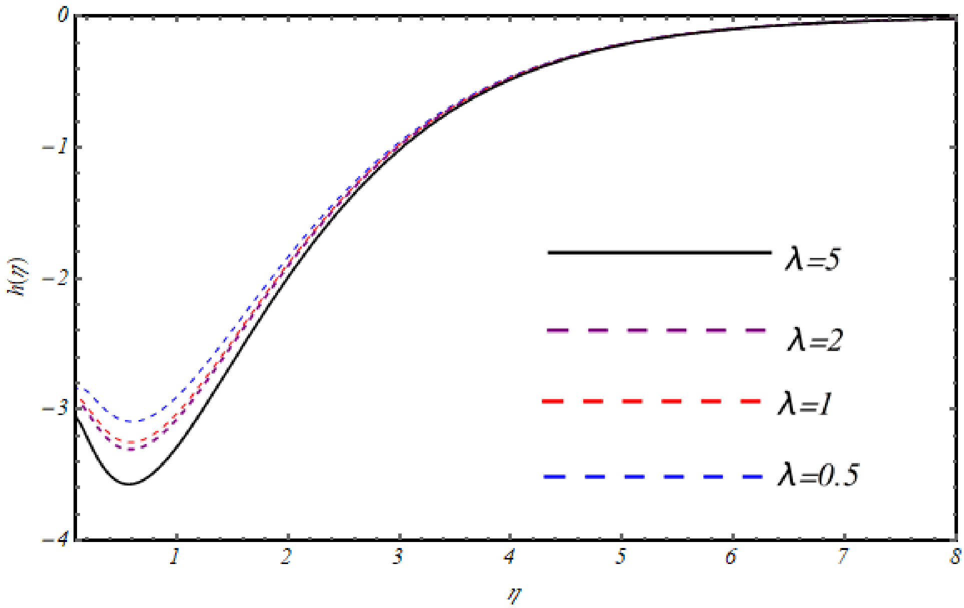

Figure 10. Effects of R , β, and λ on h(η) when $\hbar_2$ = – 0.3, β = 0.3, and R = 0.7.

In this study, the rotating flow of viscous fluid caused by the stretching of the surface is investigated. The governing equations after reducing into ODEs are solved by using HAM. The results are presented by employing graphs and the influence of the involved parameters is discussed in detail. It is noticed that the velocity of the rotating fluid increases with the corresponding increase in the porosity parameter.

Author Contribution

Shafqat Ali: conceptualization & supervision. **Muhammad Shahzad Shabbir:** formal analysis, methodology. **Sajid Hussain:** project administration, formal analysis, methodology, visualization, investigation. **Ayesha Mahmood:** writing - original draft. **Samer Perveen:** validation. **Muhammad Sajid Rashid:** writing - review & editing

Conflict of Interest

The authors of the manuscript have no financial or non-financial conflict of interest in the subject matter or materials discussed in this manuscript.

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Generative AI Disclosure Statement

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