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Buoyancy Effect on MHD Slip Flow and Heat Transfer of a Nanofluid Flow Over a Vertical Porous Plate

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Abstract

This study investigated the boundary layer flow and heat transfer aspects of a nanofluid over a porous plate with thermal radiation. Using suitable similarity transformations, partial differential equations were converted into ordinary differential equations and then solved numerically with the help of the Runge-Kutta scheme. The effects of various parameters were analyzed such as Prandtl number P_r , Lewis number L_e , Thermophoresis N_t , Mixed convection parameter λ , Brownian motion N_b , Magnetic parameter M , and Suction/Blowing parameter S . The results were depicted with the help of graphs.

Keywords: MHD, mixed convection, nanofluid, suction/blowing, thermal slip, velocity slip

Introduction

A fluid containing nanometer sized particles which are known as nanoparticles are called nanofluid and is typically prepared for metal, oxide, or carbon nanotubes. The common base fluid contains water and oil.

The study of the magnetic properties and behavior of electrically conducting fluid is called magneto-hydro-dynamics. There are many examples of magneto fluids including plasma, liquid metal, salt water, and electrolyte. Alawiet al. [1] focused on determining and modelling the dynamic thermal conductivity of nanofluids. Abbas et al. [2] discussed the critical view of the influence of nanofluids on the improvement of the PV/T system. Ahmed et al. [3] considered

transverse magnetic field the transient free convective flow of nanofluids with generalized thermal transport between two vertical and parallel plates. Akbar et al. [4] discussed the peristaltic transport of fluid in human body.

Ahmadloo and Azizi [5] compared the neural network model for nanofluids with base fluids. Arulprakasajothi et al. [6] discussed the flux conditions for the Nusselt number. Choi et al. [7] investigated the thermal conductivity of water-based nanofluid. Das et al. [8] explored a nanoparticle that has attracted much attention due to the increase in heat. Devendiran et al. [9] investigated the heat transfer flow of the heat exchanging system. Esfe et al. [10] discussed the thermal conductivity of ferromagnetic nanofluids. Haque et al. [11] analyzed the effect of biting treatment with the help of the physical properties. Hayat et al. [12] analyzed the flow of Carreau nanofluid over a stretching sheet. Kundan and Mallick [13] explored experimentally by using volume fraction based nanofluids. Mansour et al. [14] investigated the proposition that MHD localizes heat sources. Mohammad et al. [15] discussed the improvement resulting from the combination of nanoparticles. Mahmood et al. [16] analyzed the heat transfer of an incompressible fluid flow on a disc. Nicolas et al. [17] established the theory of thermal condition for obligatory temperature at the wall. Prakash et al. [18] postulated that all fluids are nanofluids because they contain nanometer-sized particles known as nanoparticles. Ramzan et al. [19] analyzed the boundary layer fluid flow of nanofluid on a moving surface.

Rao et al. [20] analyzed the flow of Casson fluid on an inclined plate. Saleem et al. [21] studied the effect of gyrotactic microorganism on MHD flow of Jeffrey nanofluids. Thaku et al. [22] investigated the proposition that nanofluids can be used as heat transfer fluids due to their exceptional thermal properties. Uddin et al. [23] investigated the magneto-hydro-dynamic boundary layer flow over a permeable vertical surface. Uysal and Korkmaz [24] studied the numerically hybrid nanofluid flux. Abdal et al. [25] studied numerically the effects of viscous dissipation on MHD. Abdal et al. [26] also discussed the multi-slip effects on MHD. Ali et al. [27] investigated the solutal boundary conditions on bio-convective micropolar nanofluid. Ali et al. [28] also investigated the multi-slip effects on unsteady Casson nanofluid. Saba

et al. [29] studied the viscous fluid buoyancy flow along with a porous plate.

According to the author’s best knowledge, the study of the MHD buoyancy effect of a nanofluid flow over a vertical porous plate has not been undertaken yet. Moreover, the effects of thermal and velocity slip have not been analyzed. The physical interpretation for several parameters is inspected with the support of graphs in this study. Using similarity transformations, non-linear differential equations are solved numerically employing Runge-Kutta shooting technique.

2. Formulation and Problem

We considered the buoyancy effect on MHD slip flow and heat transfer of a nanofluid flow over a vertical porous plate. The governing equation’s continuity, momentum, and heat transfer are written as follows,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho} (u - u_\infty) + g\beta^*(T - T_\infty) \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \tau \left(D_B \frac{\partial c}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right) \tag{3}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial c}{\partial y} = D_B \frac{\partial^2 c}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

where u and v are velocity components, $B = \frac{B_0}{\sqrt{x}}$ is the non-uniform magnetic field with B_0 as a constant, μ is fluid viscosity coefficient, ρ is fluid density, σ is electric conductivity, k is thermal diffusivity, u_∞ is free stream velocity, $\nu = \mu / \rho$ is kinematic viscosity, β^* is volumetric coefficient of thermal expansion, T is the temperature, while T_∞ is the free stream temperature. D_B depicts Brownian diffusion, D_T depicts Thermophoresis diffusion, and τ is the ratio of heat capacities.

Boundary conditions for this problem are given by the following equation,

$$u = L_1 \frac{\partial u}{\partial y}, v = -v_w, T = T_w + D_1 \frac{\partial T}{\partial y} \text{ at } y = 0 \tag{5}$$

$$u = u_\infty, T = T_\infty \text{ as } y \rightarrow \infty.$$

Where T_w is variable with $T_w > T_\infty$,

$$T_w = T_\infty + \frac{T_0}{x}, C_w = C_\infty + \frac{C_0}{x}, \text{ where } C_0 \text{ and } T_0 \text{ are constants.}$$

$L_1 = L(\text{Re}_x)^{\frac{1}{2}}$ is the velocity slip factor and $D_1 = D(\text{Re}_x)^{\frac{1}{2}}$ is the thermal slip factor with L and D being the initial values of velocity and thermal slip factors, while $\text{Re}_x = u_\infty \frac{x}{\nu}$ is the local Reynolds number.

3. Similarity Analysis

We also introduce the following dimensionless variable,

$$\theta = \frac{T-T_\infty}{T_w-T_\infty}, \phi = \frac{C-C_\infty}{C_w-C_\infty} \tag{7}$$

$$\psi = \sqrt{u_\infty \nu x} f(\eta), \eta = y \sqrt{\frac{u_\infty}{\nu x}} \tag{8}$$

where ψ is the stream function such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{9}$$

$$u = u_\infty f'(\eta), v = \frac{1}{2x} u_\infty \eta f'(\eta) - \frac{1}{2\sqrt{x}} \sqrt{u_\infty \nu} f(\eta)$$

The boundary conditions are

$$\frac{\partial \psi}{\partial y} = L_1 \frac{\partial^2 \psi}{\partial y^2}, \frac{\partial \psi}{\partial x} = v_w, \theta = 1 + D \frac{u_\infty}{\nu} \theta' \text{ at } y = 0, \phi = 1 + D \frac{u_\infty}{\nu} \phi'$$

$$\frac{\partial \psi}{\partial y} = u_\infty, \theta = 0 \text{ as } y \rightarrow \infty$$

By using similarity transformation equation (7,8), the nonlinear partial differential Equations (2)–(4) transform into the system of nonlinear ODE’s,

$$f''' + \frac{1}{2} f f'' - M(f' - 1) + \lambda \theta = 0 \tag{10}$$

$$\theta'' + P_r f' \theta + \frac{1}{2} P_r f \theta' + P_r N_b \theta' \phi' + P_r N_t \theta'^2 = 0 \tag{11}$$

$$\phi'' + L_e (f' \phi + f \phi') + \frac{N_t}{N_b} \theta'' = 0 \tag{12}$$

The new boundary conditions are

$$f' = \delta f'', f = S, \theta = 1 + \beta \theta' \text{ at } \eta = 0 \tag{13}$$

and

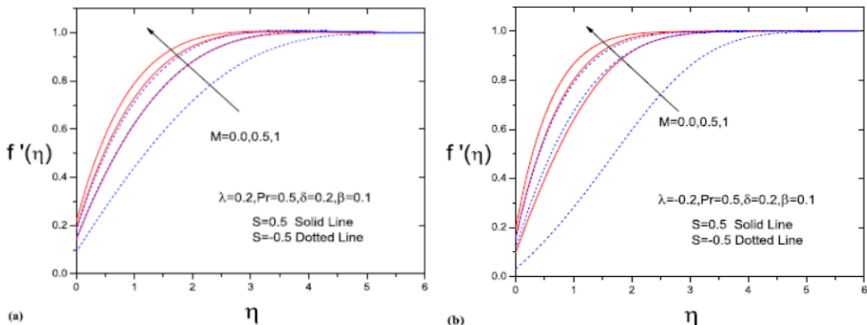
$$f' = 1, \theta = 0 \text{ at } \eta \rightarrow \infty$$

Where $\lambda = g\beta^* \frac{T_0}{u_\infty^2}$ is the mixed convection parameter, $M = \frac{\sigma B_0^2}{\rho u_\infty}$ is the magnetic parameter, $P_r = \frac{v}{k}$ is the Prandtl number, $N_t = \frac{T_0 D T_i}{T_\infty v x}$ is the thermophoresis parameter, $N_b = T \left[\frac{D_B C_0}{v x} \right]$ is the Brownian motion parameter, and $L_e = \frac{v}{D_B}$ is the Lewis number parameter.

4. Results and Discussion

In order to get a clear insight into the physical problem, numerical computations were carried out using Runge-Kutta method with shooting technique for various values of different parameters, such as the magnetic parameter M , mixed convection parameter λ , velocity slip parameter δ , thermal slip parameter β , suction/blowing parameter S , Prandtl number P_r , Lewis number L_e , Brownian motion N_b , and thermophoresis N_t .

Fig.1(a) shows that velocity profile increases when the value of magnetic parameter M increases for buoyancy aided flow in the presence of S . A similar effect shows for buoyancy opposed flow in (b). Also, temperature profile decreases as the value of magnetic parameter M increases for buoyancy aided flow in the presence of S as shown in (c). A similar behavior shows for buoyancy opposed flow in (d).



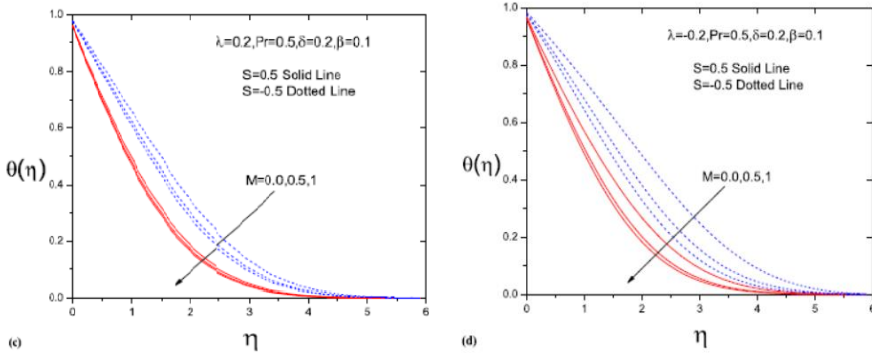


Figure 1. Velocity profile increases when the value of magnetic parameter M increases

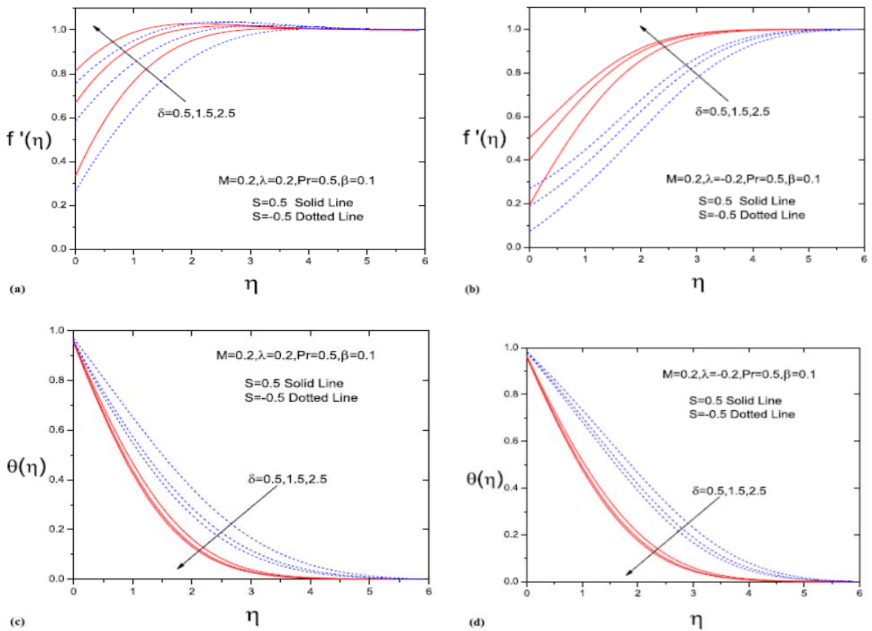


Figure 2. The effect of δ ($N_b = N_t = L_e = 0$)

Fig. 2(a) shows that velocity profile increases with the increasing value of δ for buoyancy aided flow in the presence of S. A similar effect shows for buoyancy opposed flow in (b). Also, temperature profile decreases as the value of δ increases for buoyancy aided flow in the presence of S as shown in (c). A similar behavior shows for buoyancy opposed flow in (d).

Effects of mixed convection parameter λ on velocity and temperature are displayed in Figs. 3.

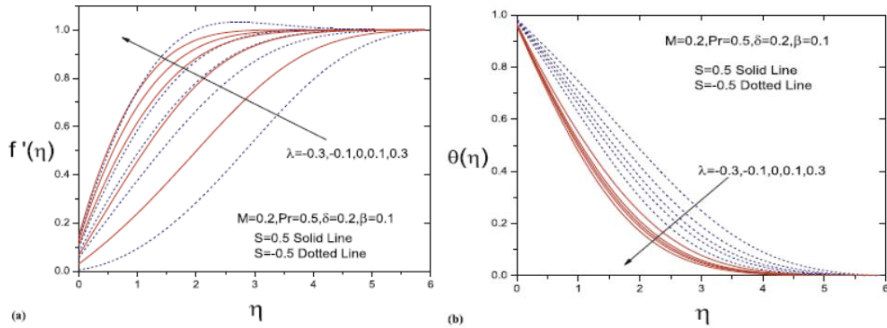


Figure 3. The effect of δ ($N_b = N_t = L_e = 0$)

Fig. 3(a) shows that velocity profile increases with the increasing value of mixed convection parameter λ in the presence of S . Similarly, temperature profile increases as the value of mixed convection parameter λ increases in the presence of S . A similar behavior shows for S , with and without λ , regarding velocity and temperature profiles as shown in Figs. 4.

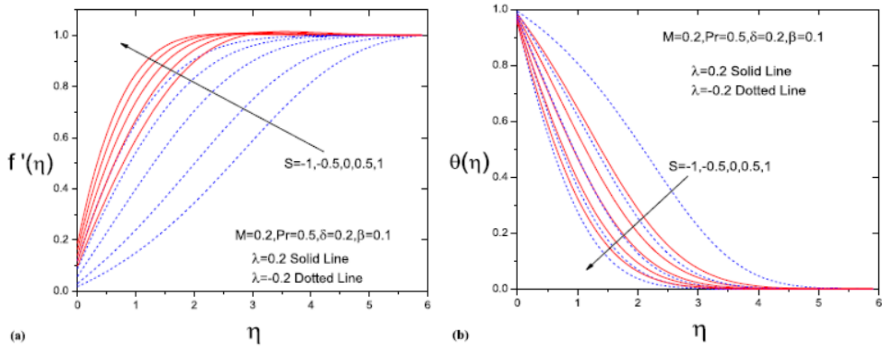


Figure 4. ($N_b = N_t = L_e = 0$)

Figs. 5(a)-5(d) show the effect of thermal slip parameter β on velocity and temperature profiles for buoyancy aided and opposed flow ($N_b = N_t = L_e = 0$). Fig.5(a) shows that velocity profile decreases as the value of β increases for buoyancy aided flow in the presence of S . Velocity profile shows a similar behavior for β without S in buoyancy opposed flow. The opposite behavior shows for temperature profile as shown in 5(c) and 5(d).

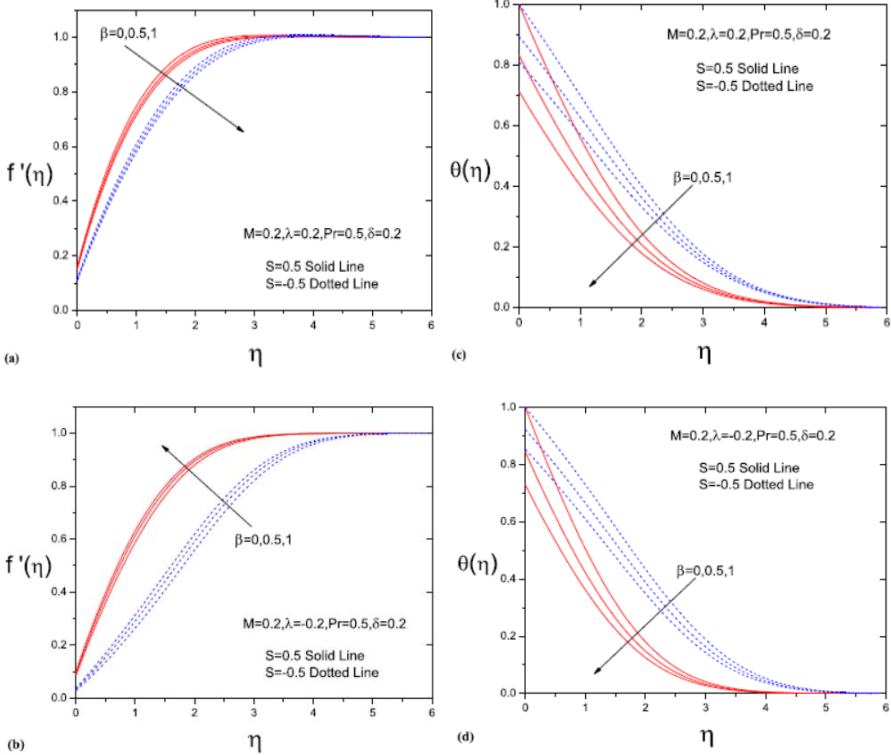
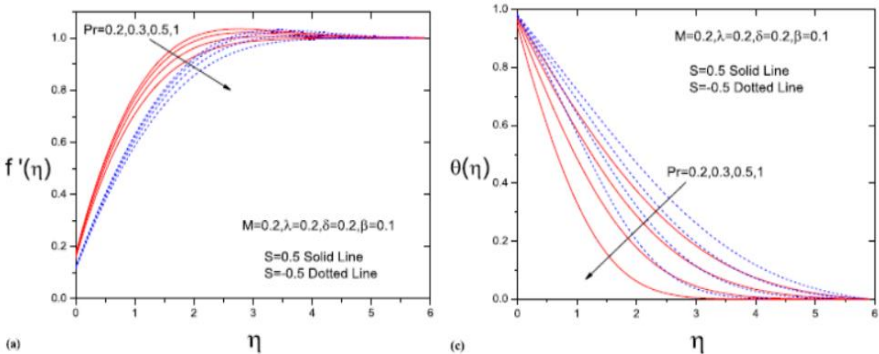


Figure 5. The effect of thermal slip parameter β on velocity and temperature profiles

Fig. 6(a) shows that velocity profile decreases with the increasing value of Prandtl number P_r for buoyancy aided flow in the presence of S . The opposite behavior shows for buoyancy opposed flow as shown in 6(b). A similar behavior shows for P_r in 6(c) and 6(d).



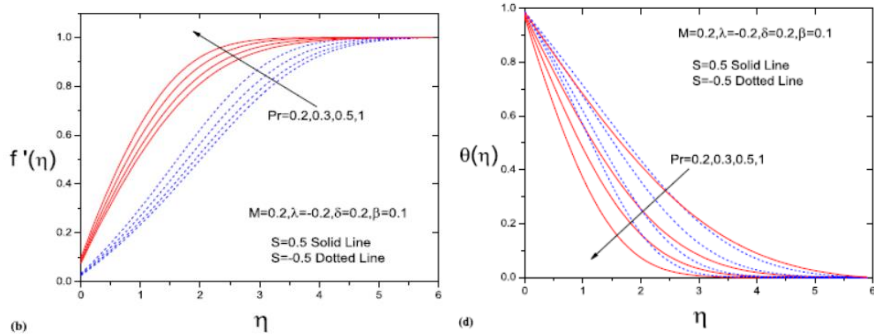


Figure 6. The effect of Prandtl number P_r with and without buoyancy flow

Figs. 6(a)-6(d) show the effect of Prandtl number P_r with and without buoyancy flow in the presence of S .

Fig. 7(a) shows the behavior of the skin friction coefficient. It was observed that $f''(0)$ increases with the increasing value of M . On the other hand, $\theta'(0)$ decreases with the increasing value of M ($N_b = N_t = L_e = 0$). An opposite behavior shows for δ in 7(c) and 7(d).

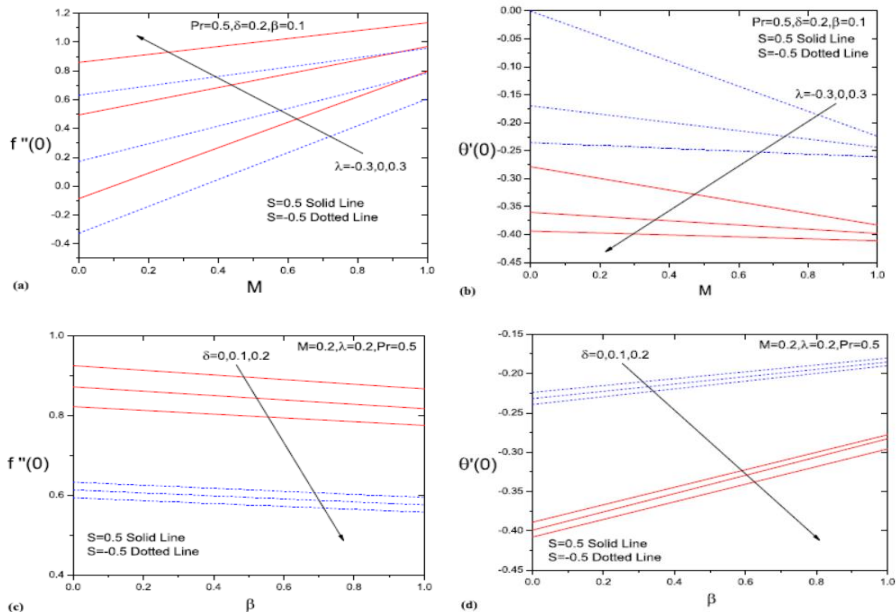


Figure 7. The behavior of the skin friction coefficient

Figs. 8(a)-8(b) show the effect of Brownian motion on velocity and temperature profiles, respectively.

($N_t = 0.1, 0.2, L_e = 0.1, 0.5$) for Fig. 8(a), $M = 0.2, P_r = 0.5, \delta = 0.2, \beta = 0.1, \lambda = 0.2$ solid lines, $\lambda = -0.2$ dotted lines, and for Fig 8(b) $M = 0.2, P_r = 0.5, \delta = 0.2, \beta = 0.1, \lambda = 0.2$ solid lines, $\lambda = -0.2$ dotted lines. It was observed that velocity profile increases with the increasing value of N_b . An opposite behavior shows for temperature profile. A similar effect shows for thermophoresis as shown in Figs. 9(a)-9(d).

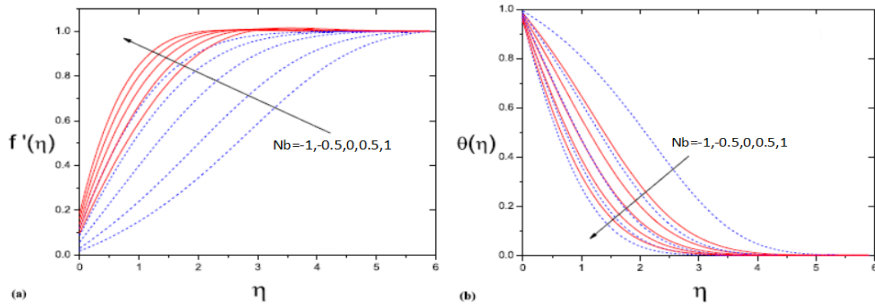


Figure 8. The effect of Brownian motion on velocity and temperature profiles

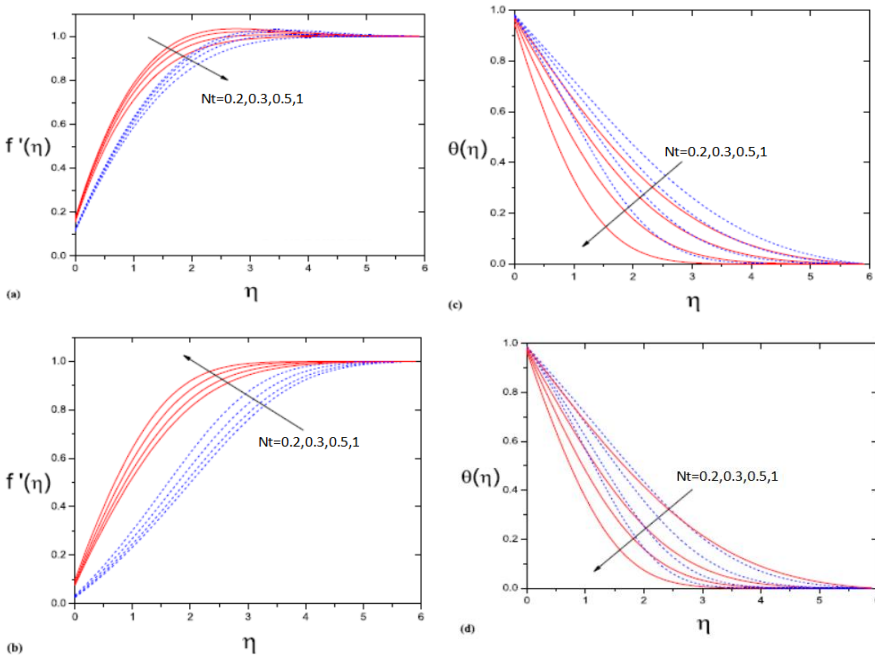


Figure 9. The effect of Brownian motion on thermophoresis

Figs. 10(a)-10(d) show the effect of Lewis number L_e , on velocity and temperature profiles for buoyancy aided and opposed flows. It was observed that velocity profile increases as the value of L_e increases, with and without buoyancy flow. An opposite behavior shows for temperature profile.

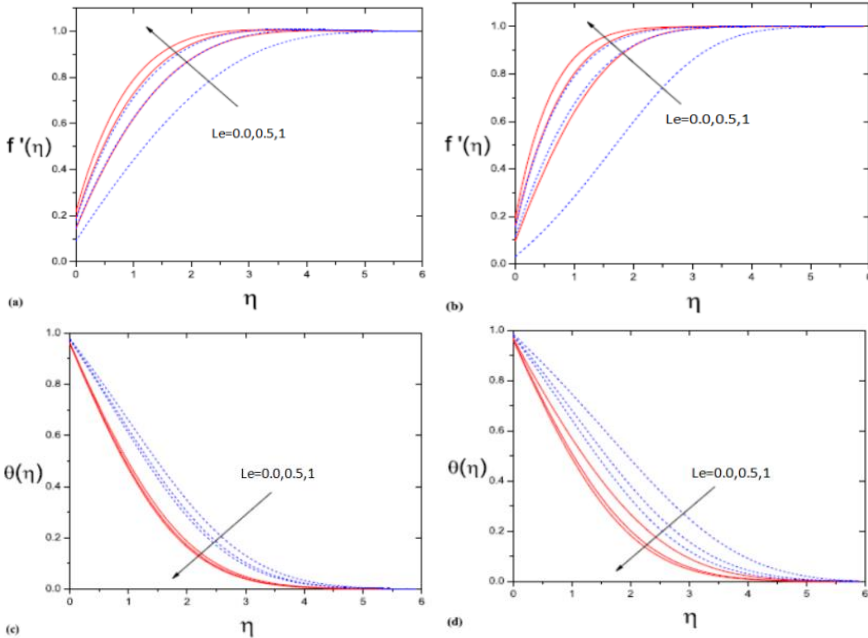


Figure 10. The effect of Lewis number L_e , on velocity and temperature profiles

5. Findings

The buoyancy effect on MHD slip flow and heat transfer of a nanofluid flow over a vertical porous plate were studied numerically. The impact of different physical parameters on velocity profile, temperature profile, Nusselt number, and skin friction coefficient were detected. The main outcomes are given below.

1. Velocity profile and mixed convection parameter both stretched within and opposite direction of flow.
2. Temperature profile decreased when magnetic parameter increased. Also, an opposite behavior showed for λ and S , for both assisting and opposing flows.

3. Skin friction coefficient increased for Hartmann number and mixed convection parameter.
4. The consequence of λ on heat transfer was appreciable. Due to the increase in Hartmann number, Nusselt number also increased. However, it decreased according to the heat generation/absorption coefficient.

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