Scientific Inquiry and Review (SIR)
Volume 10 Issue 1, 2026
ISSN(P): 2521-2427, ISSN(E): 2521-2435
Homepage: https://journals.umt.edu.pk/index.php/SIR

Title: **A Way to Complete General Relativity**

Author (s): Rodrigo Steinvorth[1] and Syed Ali Mardan[2]

Affiliation (s): [1]Independent Researcher, Germany
[2]Khazar University, Baku, Azerbaijan

Academic Editor Abaid ur Rehman

DOI: https://doi.org/10.32350/sir.101.01

History: Received: December 15, 2026, Revised: January 21, 2026, Accepted: February 25, 2026, Published: March 16, 2026

Citation: Steinvorth R, Mardan SA. Completed general relativity. *Sci Inq Rev.* 2026;10(1):1–32. https://doi.org/10.32350/sir.101.01

Conflict of Interest: Author(s) declared no conflict of interest

A publication of
The School of Science
University of Management and Technology, Lahore, Pakistan

A Way to Complete General Relativity

Rodrigo Steinvorth[1], and Syed Ali Mardan[2*]

[1]Independent Researcher, Germany
[2]Center for Theoretical Physics, Khazar University, Baku, Azerbaijan

ABSTRACT

The current study presents the remodeling of General Relativity (GR) where gravitational effects are explained by phase dynamics in a two-dimensional time space instead of curvature of spacetime. It is based on previously modified formulation of special relativity, where motion of particles is linked to a unitary constraint of temporal velocity, and the relativistic effects have time dependence of an internal time phase. The study generalizes the principle of equivalence by supposing that gravitational interactions only act on the phase degrees of freedom of time and space but the geometry of spacetime is still flat. In this context, the study derives the principles of invariant action of phase evolution as well as the equations of motion of massive particles and photons in the time manifold. Additionally, the research obtained explicit solutions of inertial motion, uniform acceleration, and spherically-symmetric gravitational fields. The theory in the weak-field regime regenerates Newtonian gravity and standard results of GR. Meanwhile, the formulation is fundamentally different to Einstein's gravity in that the curvature of spacetime is not a dynamical variable. This leads to the fact that there are no curvature singularities, and event horizons are well-defined. The findings indicate that relativistic gravitation can be systematically developed in phase evolution instead of spacetime geometry. This offers an alternative theoretical framework with a structure that can be extended appropriately in the future.

Keywords: general relativity (GR), gravitational field, spacetime, spherical symmetry, time

Highlights

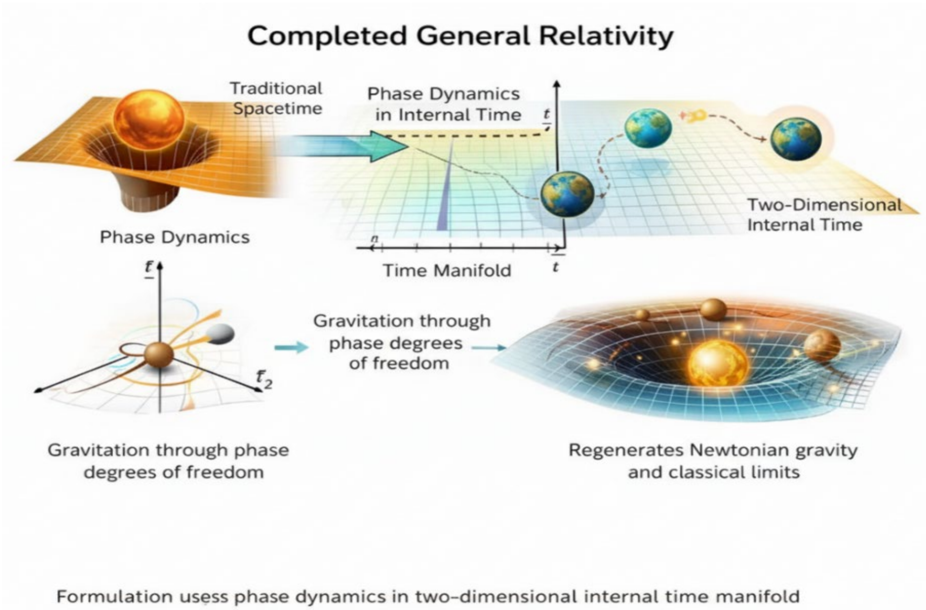
- The current study reformulates GR by replacing spacetime curvature with phase dynamics in a two-dimensional time manifold, where gravitational effects arise from evolution in internal time.
- Gravity is modeled through phase degrees of freedom rather than

*Corresponding Author: syedalimardanazmi@yahoo.com

curved spacetime. This eliminates curvature singularities while preserving key phenomena, such as inertial motion, acceleration, and spherical gravitational fields.

- The theory reproduces Newtonian gravity and standard GR results in the weak-field regime, offering a viable alternative framework with potential for further theoretical extension.

GRAPHICAL ABSTRACT



1. INTRODUCTION

GR is the established classical framework for gravitation, formulating gravitational dynamics as a generally-covariant metric theory. This theory is encoded in the geometry of spacetime, and freely-falling bodies follow geodesic trajectories of a dynamical metric [1,2]. The direct detection of gravitational waves from coalescing compact binaries by the LIGO and Virgo interferometers has provided observational access to the nonlinear regime of the Einstein's field equations, with observed in-spiral, merger, and ringdown waveforms in close agreement with GR predictions [3,4]. The lensing structure is expected from photon trajectories in a Kerr-like spacetime, thereby testing GR on length scales comparable to the gravitational radius [5,6].

Under broad and physically-reasonable assumptions, the Penrose–Hawking singularity theorems demonstrate that classical GR generically predicts geodesic incompleteness. This signals a breakdown of the theory rather than a physically-meaningful endpoint of evolution [7,8]. This tension is reflected most clearly in the so-called problem of time, which highlights the conceptual incompatibility between the dynamical role of time in GR and the external time parameter employed in quantum mechanics [9]. Recent work on two-dimensional time and time-phase geometry has pursued this direction. This was done by seeking a framework in which relativistic invariance and quantum discreteness arise from a common temporal structure rather than from fundamentally-distinct principles [1012].

Phase plays a central and irreducible role in quantum theory, governing interference phenomena, quantization conditions, and dynamical evolution [13]. In the path-integral formulation of quantum mechanics, physical amplitudes are obtained by coherently summing over all possible histories, each weighted by a phase determined by the action, placing phase at the core of quantum dynamics [14]. Closely-related concepts, such as geometric and Berry phases further show that phase accumulation can encode global and topological properties of a system, independent of local forces, with observable consequences across a wide range of physical contexts [15,16].

The current study builds on a completed reformulation of special relativity developed in previous work [10–12], in which relativistic kinematics are described using a two-dimensional time manifold equipped with an internal phase degree of freedom. It is in line with modern approaches that treat time as an operational or relational quantity derived from internal degrees of freedom rather than as a fundamental external parameter [17,18]. The additional temporal structure enters exclusively through phase variables that parameterize how time evolution is represented, compared, and synchronized between inertial frames, without introducing new observable coordinates or violating relativistic covariance [18,19]. The interpretation is consistent with contemporary relational and emergent-time programs in both quantum theory and quantum gravity, where temporal structure arises from correlations, internal clocks, or phase evolution rather than from a priori geometric time coordinate [17, 19–22].

Guided by Einstein's equivalence principle, which asserts the local indistinguishability of inertial motion and free fall [23], the study formulates a generalized version adapted to the phase-based kinematics of the two-time framework. Early operational analyses clarified that the equivalence principle expresses the universal coupling of gravity to matter and the local eliminability of gravitational effects by a suitable choice of frame, independent of any specific geometric realization [24]. Subsequent developments emphasized that this universality need not be tied uniquely to spacetime curvature, particularly when quantum or non-geometric effects are taken into account [25]. These dynamics are formulated in terms of invariant action principles governing phase evolution. The foundational role of symmetry and invariance, in determining conserved quantities, was established by Noether's theorem [26], and has long been central to relativistic field theory [27,28]. Building on these insights, and on later systematic treatments of the equivalence principle in modern relativistic frameworks [29], the study postulates those gravitational interactions couple universally to phase degrees of freedom rather than directly to spacetime geometry. Experimental demonstrations of gravitationally-induced quantum phase shifts further support the idea that gravity can manifest directly through phase rather than geometry alone [30].

Conservation of energy and momentum then follows directly from invariance properties, without the need for an independent geometric field equation [31]. In this sense, the role played by spacetime curvature in GR is replaced by constrained phase dynamics, with relativistic motion emerging from symmetry, unitarity, and variational principles rather than from geometric curvature itself [32]. The study develops explicit dynamical solutions in several benchmark settings: inertial motion, which reproduces the special-relativistic limit; uniformly accelerated motion, serving as an analogue of a constant gravitational field [33,34]. A spherically-symmetric gravitational configuration corresponds to the exterior Schwarzschild solution [35,36]. These cases are chosen to enable direct comparison with textbook predictions of GR in regimes where the theory is well-tested, including weak-field limits, redshift, and test-particle motion [37,38]. In the context of compact objects, event horizons remain well-defined and operationally-meaningful [39] but curvature singularities do not arise, since the formulation does not rely on divergent curvature invariants. This is a viewpoint compatible with horizon-based and operational treatments of black-hole physics [40,41].

The scope of this study is classical. The study proceeds by formulating the gravitational equations in the time dimensions and introducing the associated time metric, deriving conservation laws and field equations on the time manifold, as well as presenting explicit solutions for constant and spherically-symmetric gravitational fields. The formalism is then extended to the space dimensions, including angular momentum, followed by a systematic comparison with Einstein's GR and concluding remarks.

2. GENERAL RELATIVITY (GR) EQUATIONS FOR THE TIME DIMENSIONS

Here, the study develops the gravitational dynamics of the time dimensions of the relativistic structure that has been constructed. The study begins with a short overview of Completed Special Relativity (CSR). Afterwards, it generalizes the equivalence principle of Einstein to a phase-based context and makes two postulates that determine how time and space phases respond to gravitational fields. The following sections are based on these postulates to construct the dynamical equations of Completed General Relativity (CGR).

2.1. Extension of the Equivalence Principle and Review of Completed Special Relativity (CSR)

Einstein's equivalence principle states that, from the perspective of a point-like massive particle, the effects of gravity are locally indistinguishable from those of acceleration. As a consequence, no local experiment performed on such a particle can distinguish gravitational motion from accelerated motion. In the previous works [10–12], a formulation of CSR was introduced, in which the velocity of a particle in time was described by the complex quantity mentioned as follows:

$$T = \rho e^{i\dot{\theta}} \tag{1}$$

This applies uniformly to both massive and massless particles (photons). In this representation, the oscillatory phase variable θ encodes the dynamical state of the particle in the time manifold. For a massive particle, this phase is taken to depend on the spatial velocity v,

$$\dot{\theta} = \tan^{-1}(v) \tag{2}$$

while for a photon, it is determined by the frequency, or equivalently the inverse wavelength,

$$\dot{\theta} = \tan^{-1}(-\lambda) \tag{3}$$

The researchers were only concerned about the special case without acceleration; however, the implication of this symmetry is an extension of Einstein's equivalence principle:

2.2. Gravity Parallel to the Direction of Motion

Postulate A: The component of the gravitational field parallel to the direction of motion of a particle is equivalent to acceleration in that it modifies the phase evolution of the particle in the time manifold, for both massive particles and photons.

To motivate this postulate, consider two particles in free fall separated by a finite distance. For each particle, a locally-comoving coordinate system may be defined such that, at a given instant, the particle's motion is aligned along a single spatial direction, as established in [12]. For a point particle in free fall whose motion is parallel to the gravitational field, the dynamics can be fully specified within the time manifold, since no angular change in spatial direction occurs.

2.3. Parallel and Perpendicular Components of Gravity

Postulate B: The instantaneous component of gravity parallel to the motion of a particle changes the phase of the particle in time, while the instantaneous component of gravity perpendicular to the motion changes the phase of the particle in space.

The study proposed that motion in the time manifold is governed by a principle of least action, determined by a Lagrangian that is an invariant function of the time phase [12],

$$\mathcal{L}_t = m\left(e^{i\dot{\theta}}\right) \tag{4}$$

In the same way, motion in the space manifold is hypothesized to be determined by a Lagrangian that is an invariant function of the phase,

$$\mathcal{L}_t = \beta_i r^2 \dot{\alpha}$$

In the special case, these contributions were combined into a single Lagrangian. In the present formulation of CGR, however, no such assumption is made a priori. Instead, the phase dynamics in time and space are treated separately, with gravitational effects entering through their respective phase responses. This approach differs fundamentally from Einstein's geometric interpretation of gravity in terms of spacetime curvature. Nevertheless, as demonstrated in subsequent sections, it reproduces equivalent predictions for particle motion in gravitational fields in regimes where GR has been experimentally confirmed.

3. TIME METRIC IN COMPLETED SPECIAL RELATIVITY (CSR)

This section introduces the metric structure associated with the time manifold in CSR. By reformulating proper time as a norm in a two-dimensional time space, the study shows how relativistic time dilation and invariant temporal velocity arise geometrically, without invoking spacetime curvature. This construction provides the basis for extending the formalism to gravitational settings in subsequent sections. The study recalls that the Minkowski metric for massive particles in Einstein's spacetime is defined as a function of the space and time coordinates:

$$\tau^2 = t^2 - x^2 - y^2 - z^2, \qquad (5)$$

where τ denotes the invariant proper time. In the CSR framework developed in Refs. [10–12], proper time is instead specified as a function of coordinates in a two-dimensional time manifold. In particular, the study showed that

$$\tau^2 = |S|^2 = \left|\rho t e^{i2\pi\omega_v t}\right|^2 = \left(t e^{i2\pi\omega_v t}\right)^2.$$

Since massive particles move in time-like trajectories, this can be written as

$$\tau = |S| = t e^{i2\pi\omega_v t}.$$

Photons are governed by a symmetric relation, reflecting the massless limit of the theory,

$$\tau = |S| = t e^{-i2\pi\omega t}.$$

A central postulate of CSR is that the magnitude of the velocity of any particle in the time manifold is invariant,

$$|T| = \left|\rho e^{i\dot{\theta}}\right| = 1.$$

In the absence of acceleration or gravitational fields, the phase depends only on the constant velocity or frequency of the particle and therefore,

$$\ddot{\theta} = 0.$$

This situation changes in the presence of a gravitational field with a component parallel to the direction of motion, where the velocity of a massive particle or the frequency of a photon varies with time as the particle propagates through space. In this case, the phase evolution becomes time dependent,

$$\dot{\theta}(t) = \tan^{-1}\big(-\lambda(t)\big) \text{ or } \dot{\theta}(t) = \tan^{-1}\big(v(t)\big),$$

and consequently,

$$\ddot{\theta}(t) \neq 0.$$

The action governing motion in the time manifold is given by,

$$A = m\int_a^b d\tau = m\int_a^b e^{i\dot{\theta}}\, dt.$$

This action may be written in geometric form by introducing a metric on the time manifold,

$$\tau^2 = g_{cd}dY^c dY^d = \begin{bmatrix}\cos^2\dot{\theta} & 0\\ 0 & -\sin^2\dot{\theta}\end{bmatrix}\begin{bmatrix}dY^c\\ dY^d\end{bmatrix}.$$

The two dimensions of time ($c = 1, d = 2$) are orthogonal by construction, and the normalization condition implies,

$$\begin{bmatrix}\cos^2\dot{\theta} & 0\\ 0 & -\sin^2\dot{\theta}\end{bmatrix}\begin{bmatrix}dY^1\\ dY^2\end{bmatrix} = \cos^2\dot{\theta}\ (dY^1)^2 - \sin^2\dot{\theta}(dY^2)^2 = 1$$

The familiar special-relativistic “paradox” in which two inertial observers each perceive the other’s clock as running more slowly finds a natural geometric interpretation in this metric. From the perspective of each observer, motion occurs along a single time direction ($\tau = t$, $\cos^2\dot{\theta} = 1$), while the other observer is perceived as moving in two orthogonal time dimensions,

$$\tau' = \cos\dot{\theta} + i\sin\dot{\theta} = e^{i\dot{\theta}}.$$

In both descriptions, however, the magnitude of the velocity through the time manifold remains invariant. In direct analogy with the constancy of the speed of light as a fundamental principle of relativity, it was previously postulated that the constant magnitude of temporal velocity is a universal law of nature [10]. As a consequence, the time metric must remain diagonal,

$$g_{01} = g_{10} = 0.$$

4. CONSERVATION OF ENERGY/MOMENTUM AND THE TIME MANIFOLD FIELD EQUATIONS

In this section, the study derives the field equations governing particle motion in the time manifold by imposing conservation of energy and momentum as fundamental invariants. Rather than introducing curvature as a dynamical variable, gravity is formulated through phase evolution subject to a unitary constraint, which plays the role of energy–momentum conservation in the time manifold. This leads naturally to a set of field equations and postulates to replace the geometric field equations of Einstein's GR. The first objective is to derive field equations governing the behavior of particles in the time manifold. The study adopts the same conceptual strategy used by Einstein in GR, namely, that the dynamical equations should be determined by conserved quantities but reformulate it within the phase-based structure of the time manifold. Accordingly, the study posits that,

$$\text{Function of the time metric} = \text{function of conserved quantities.} \quad (6)$$

The motion of a particle in the time manifold depends exclusively on its phase in time, which is fully specified by its velocity in the case of massive particles or by its frequency in the case of photons. The energy of the particle is therefore encoded entirely in its time-phase evolution and rest mass. Information associated with angular momentum, which is invariant in space and related to the phase in the space manifold, is not encoded in the time manifold. This leads towards a significant simplification of the right-hand side of Eq. (6). Conservation of energy and momentum in the time manifold is equivalent to enforcing a unitary constraint. Specifically, this corresponds to Postulate 3 of the previous work [10], which states that the magnitude of the velocity of a particle in time is constant. This constraint may be written as,

$$\cos^2\dot{\theta} + \sin^2\dot{\theta} = \cos^2\left(\dot{\theta} + d\dot{\theta}\right) + \sin^2\left(\dot{\theta} + d\dot{\theta}\right).$$

Equivalently, this condition can be expressed as a scalar field equation, where $\dot{\theta}(X)$ denotes the phase of a particle in time as a function of its position in the time manifold,

$$\cos^2\left(\dot{\theta}(X)\right) + \sin^2\left(\dot{\theta}(X)\right) = 1.$$

4.1. Gravitational Coupling to the Time Phase

As postulated earlier, only the component of the gravitational field parallel to the direction of motion in space contributes to the evolution of the time phase. The study therefore, defines the gradient of the gravitational field acting on the time phase as,

$$\nabla\, \phi_{G^{\parallel}}(X) = \left(\frac{\partial \phi_{G^{\parallel}}}{\partial X^0}, \frac{\partial \phi_{G^{\parallel}}}{\partial X^1}\right).$$

The angle between the direction of the gravitational field in space (ϕ) relative to the direction of motion of a particle in space (V) is:

$$\cos^{-1}\left(\frac{\phi \cdot V}{|\phi||V|}\right),$$

which defines a scalar field in spacetime, denoted by χ,

$$\cos(\chi) = \left(\frac{\phi \cdot V}{|\phi||V|}\right).$$

Although the gravitational constant $G$is universal, its contribution to the time phase must be modulated by $\cos\chi$ to enforce Postulate A. Accordingly, the effective gravitational field acting on the time phase is,

$$\phi_{G^{\parallel}} = \cos(\chi)\, \phi.$$

The study introduces the sign parameter ρ, where $\rho = -1$ corresponds to motion towards another particle and $\rho = +1$ otherwise. Including this directional dependence yields,

$$\phi_{G^{\parallel}} = \rho\cos(\chi)\, \phi.$$

At this point, an important conceptual distinction emerges. Distances in space are meaningful only with respect to a specific particle and reference frame [12]. Consequently, $\cos\chi$ has physical meaning only when evaluated along a particular particle trajectory.

4.2. Energy–momentum Structure in the Time Manifold

In our previous work [12], the energy–momentum relations were derived

$$(P^0)^2 + (P^1)^2 = \left(\frac{m\rho\beta_2}{\sqrt{1-v^2}}\right)^2 + \left(\frac{im\rho\beta_2 v}{\sqrt{1-v^2}}\right)^2 = m^2,$$

which may be rewritten as,

$$(P^0)^2 + (P^1)^2 = \left(\frac{m}{\sqrt{1-v^2}}\right)^2 - \left(\frac{mv}{\sqrt{1-v^2}}\right)^2 = m^2.$$

This leads to the familiar relativistic relation

$$E^2 = |P^1|^2 + m^2,$$

which applies to both massive and massless particles. The squared sum of energy and momentum is therefore invariant in time. To formalize this result, we define the time-manifold energy–momentum tensor, denoted $P_{\tau\nu}$ (to distinguish it from the Einstein tensor),

$$P_{\tau v} = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}.$$

Here, P_{00} represents the energy contained in the real dimension of time, while P_{11} represents the energy in the imaginary dimension. The off-diagonal components vanish identically, since the magnitude of the velocity in time is fixed,

$$P_{01} = P_{10} = 0.$$

The tensor is therefore diagonal,

$$P_{\tau v} = \begin{bmatrix} P_{00} & 0 \\ 0 & P_{11} \end{bmatrix}$$

This requires a conceptual shift: the time manifold remains flat and does not curve in the presence of gravity. Instead, it is the phase in time equivalently, the velocity or frequency of the particle that responds to gravity. Imposing invariance of $P_{\tau\nu}$ yields,

$$(P^0)^2 + (P^1)^2 = E^2 + (P^1)^2 = m^2\left[\left(\frac{1}{\sqrt{1-v^2}}\right)^2 + \left(\frac{iv}{\sqrt{1-v^2}}\right)^2\right]$$
$$= m^2.\left[\left(\frac{1}{\sqrt{1-v^2}}\right)^2 + \left(\frac{iv}{\sqrt{1-v^2}}\right)^2\right] = 1,$$

where the bracketed expression is both invariant and tensorial. We therefore define

$$P_{\tau v} = \begin{bmatrix} \dfrac{1}{1-v^2} & 0 \\ 0 & -\dfrac{v^2}{1-v^2} \end{bmatrix}$$

with an analogous expression for photons obtained by replacing v with λ.

$$P_{\tau v} = \begin{bmatrix} \frac{1}{1-\lambda^2} & 0 \\ 0 & -\frac{\lambda^2}{1-\lambda^2} \end{bmatrix}.$$

Since the tensor is diagonal, it may be contracted without loss of information,

$$P_{\tau} = \begin{bmatrix} \frac{1}{1-v^2} \\ -\frac{v^2}{1-v^2} \end{bmatrix}.$$

The tensor is never zero (which is a major difference between Einstein's Tensor):

$$P_{\tau} \neq 0,$$

and its magnitude is always one

$$|P_{\tau}| = 1.$$

also, it is undefined, when $\lim_{v\to 1}(1-v^2)$ and $\lim_{\lambda\to 1}(1-\lambda^2)$.

A long approach was taken to obtain a familiar result; the time energy/momentum tensor is equal to the metric tensor for the time manifold,

$$P_{\tau v} = g_{\tau v}.$$

The metric of the time manifold is therefore identical to its energy–momentum tensor. As a result, Eq. (6) is trivially satisfied, and the dynamical content of the theory is carried entirely by the unitary constraint, restated as a law of nature:

It is a law of nature that the magnitude of the velocity in time of all particles is constant.

4.3. Unitary Trajectories in the Time Manifold

In Einstein's GR, the solution of the field equations is the spacetime metric, and particles move along geodesics of the curved spacetime manifold. In the present framework, a different hypothesis was adopted.

Postulate C. Particles move in the time manifold along trajectories that satisfy the unitary constraint imposed by the constant magnitude of temporal velocity.

Since the metric of the time manifold is known and flat, the equations of motion of particles in time can be computed explicitly with respect to one another in the presence of a gravitational field. The procedure for constructing solutions is as follows:

1. Define the gravitational field being observed. For instance: in the absence of gravity $\nabla\phi = 0, \rho\cos(\chi) = 1$; a particle free falling in a gravitational field of a point mass in a vacuum $\nabla\phi = \frac{MG}{r^2}, \rho\cos(\chi) = -1$; etc.
2. Select a reference frame (can be the reference frame of another particle but must not be). The solution of the equations would always be relative to a reference frame being chosen.
3. Select a particle with an initial state with respect to the selected reference frame (v_0, t_0, ρ_0). There is a slight change in how we think about ρ between the special and general case. When gravity is present, $\rho = -1$ if the direction of motion is in the direction of the gravitational field, otherwise $\rho = 1$.
4. Find the equation for the magnitude of the velocity of the particle between both reference frames as a function of the gravitational field with respect to the observer reference frame:

$$v(t) = F(\phi(t))$$

5. Use the unitary constraint given by the velocity of a particle in the time manifold to calculate the invariant distance between the particle and the reference frame being selected. This is the integral of the real component of time of the particle's reference frame with respect to the observer's reference frame along the trajectory found before.

$$t' = \int \frac{\rho}{\sqrt{1 - \left(v(t)\right)^2}} dt$$

6. Afterwards, the unitary constraint is used to find the equation for the parallel distance travelled in space (not to be confused with radius) which is equal to the imaginary distance in time:

$(x')^2 = 1 + (t')^2$

Step 4 is not coordinate invariant; it is specifically related to the two particles (reference frames) selected. For instance, if a comparison is selected with the inertial reference frame of any particle, there would always be the result that minimizes the action when both phases remain the same, regardless of whether the particle is free falling in a gravitational field or not. When step 5 is used in this trivial case, an invariant is obtained. All reference frames agree that the particle has not moved relative to its own reference frame, or in other words, the proper distance between the particle and the reference frame being selected would be invariant.

The preceding work has focused solely on the time manifold but it can always be linked to the space manifold. This is because by knowing the derivative of the phase in time, the velocity in space is known at all given points of time. Spacetime is a projection of a 2-dimensional time manifold onto a 3-dimensional space manifold. This is because the metric for the time manifold is known, and it is always flat and only two dimensional. The whole process to find solutions in the time manifold is much simpler than in GR. However, the solution to the field equations in time is not trivial for two reasons:

1. The phase in time is a function of the gravitational field (and any other forces acting on the particle) and can be very complex for complex gravitational fields. At the same time, the gravitational field is a function of the phase in time. This is equivalent to the statement made for GR that: "matter tells the phase in time (and space) how to change and the change in the phase in time (and space) determines how matter moves".
2. The change in the phase depends on the change of the angle between the gravitational field and the velocity vector in the trajectory of a particle in space because only their parallel component has an effect in the time dimension

$(\phi_{G^{\parallel}} = \cos(\chi)\, \phi)$.

In most situations, there are multiple significant masses with different velocities, making it difficult to find the equation for χ. Simplifications to the gravitational field, such as Schwarzschild did with a point mass in a vacuum, are needed to obtain generalized solutions.

5. SOLUTION OF THE SPECIAL CASE

Let us verify whether the approach developed in the preceding sections yields the correct description of particle motion in the absence of a gravitational field. The first step is to specify the gravitational field. In this special case, no gravitational field is present and therefore,

$$\phi = 0.$$

Secondly, an observer is selected in an inertial reference frame, that is, a frame undergoing no acceleration.

Thirdly, a particle is considered moving with constant velocity relative to this observer. The state of the particle at time t_0, as measured in the observer's frame, is specified by,

$$[v = v_0, \rho = 1, t = t_0].$$

The objective is to determine the trajectory of the particle from $t = t_0$ to $t = t_1$. Since the velocity is constant, the derivative of the phase in time is given by,

$$\dot{\theta} = \tan^{-1}(v_0),$$

also, $\dot{\theta}$ is a constant and therefore,

$$0 = \ddot{\theta}.$$

The final step is to compute the invariant temporal separation between the particle and the observer after the elapsed time interval. This is obtained by integrating the real component of time along the trajectory,

$$\Delta t' = \int_{t_0}^{t_1} \frac{1}{\sqrt{1 - {v_0}^2}} dt$$

In this case, the integration is trivial because the velocity is constant, yielding,

$$\Delta t' = \frac{t_1 - t_0}{\sqrt{1 - {v_0}^2}}$$

Therefore, the standard special-relativistic time-dilation formula is recovered. This confirms that the time-manifold formulation reproduces the correct inertial limit. In the low-velocity limit $v_0 \ll 1$, the expression reduces to Newton's result,

$$t' = t.$$

6. CONSTANT GRAVITATIONAL FIELD

A constant gravitational field models the physical situation experienced in a uniformly-accelerated reference frame, such as an observer in an accelerating elevator, or approximately by an observer at rest near the surface of the Earth. Within the current framework, this corresponds to a gravitational potential that grows linearly with time,

$$\phi(t) = gt.$$

The corresponding gradient of the gravitational potential is therefore constant,

$$\nabla\phi(t) = \frac{\partial\phi(t)}{\partial t} = g.$$

A reference frame is selected that is not accelerating with respect to the gravitational field. The particle under consideration is initially at rest relative to this frame and propagates in the direction of the gravitational field. Its initial state is specified by,

$$[v_0 = 0, \qquad \rho = -1, \qquad t = t_0 = 0].$$

The derivative of the phase in time is a function of the particle's velocity, which itself evolves due to the gravitational field. For a constant gravitational field, the velocity increases linearly with time,

$$0 \leq v(t) = (v_0 + gt) < 1.$$

Now, the invariant temporal separation is computed between the particle and the reference frame. Using the unitary constraint, this is given by,

$$t' = \rho \int \frac{1}{\sqrt{1 - (gt)^2}} dt$$

Carrying out the integration yields,

$$t' = \frac{\rho}{g} \operatorname{arctanh}(gt)$$

Equivalently, this result may be expressed in logarithmic form as,

$$t' = \frac{\rho}{2g} \ln\left(\frac{1 + gt}{1 - gt}\right)$$

This is precisely the expected result for uniformly-accelerated motion, consistent with the standard relativistic description of constant proper acceleration. The agreement confirms that the time-manifold formalism correctly reproduces known results in the presence of a constant gravitational field.

7. CONSTANT PHASE IN TIME IN A CONSTANT GRAVITATIONAL FIELD

Now, the complementary situation is considered in which a particle remains at a constant phase in time despite the presence of a constant gravitational field. This case illustrates how gravitational effects can act purely through spatial phase evolution, consistent with Postulate B and familiar physical situations, such as circular motion or static support in a gravitational field. In this case, a particle is considered whose phase in time does not change, meaning that its velocity remains constant. Physically, this corresponds either to motion perpendicular to the gravitational field or to a situation in which an external force balances gravity, keeping the particle at rest with respect to the field.

The gravitational field and its gradient are the same as those introduced previously,

$$\phi(t) = gt,$$

with constant gradient,

$$\nabla\phi(t) = \frac{\partial\phi(t)}{\partial t} = g.$$

Again, a reference frame is selected that is not accelerating with respect to the gravitational field. The particle under consideration has the initial state,

$$[v = v_0, \rho = 1, t = t_0 = 0].$$

In this configuration, the particle's velocity does not depend on the gravitational field,

$$v(t) = v_0.$$

As a result, the phase in time remains constant, and the evolution of the time phase is identical to that of the inertial special-relativistic case. The invariant temporal separation between the particle and the reference frame is therefore given by,

$$\Delta t' = \int_{t_0}^{t_1} \frac{1}{\sqrt{1 - {v_0}^2}} dt.$$

Since the velocity is constant, the integration is trivial, yielding,

$$\Delta t' = \frac{t_1 - t_0}{\sqrt{1 - {v_0}^2}}.$$

At first sight, this may appear counterintuitive in the presence of a gravitational field. However, the result follows directly from Postulate B, which states that only the component of gravity parallel to the direction of motion affects the phase in time.

8. GRAVITATIONAL FIELD OF A POINT MASS IN A VACUUM (SCHWARZSCHILD-REFINED SOLUTION)

Consider a test particle freely falling towards a spherically-symmetric massive object, such as a black hole, in vacuum. The gravitational field generated by the test particle itself is neglected, as its mass is assumed to be negligible compared to that of the central object. The gravitational field depends on the mass of the source M and the radial separation r between the particle and the source. For clarity, we begin with the case of a photon, for which the spatial and temporal separations coincide,

$$\Delta r = \Delta t.$$

In this representation, the gravitational field may therefore be expressed as a function of the temporal separation between the source and the photon. For a photon, the relation between wavelengths measured in two reference frames with relative velocity $v_0$at a given instant is

$$\lambda' = (1 - (v_0)^2)\lambda.$$

In the case of a constant gravitational field, as discussed in sections 6 and 7, the velocity is induced by the gravitational potential, yielding

$$\lambda' = (1 - (gt)^2)\lambda.$$

Newton's Law of Universal Gravitation is:

$$F = ma = -\frac{mMG}{r^2} = -m\nabla\phi = \frac{mv^2}{r}.$$

$$\nabla\phi = \frac{MG}{r^2} = \frac{v^2}{r}.$$

$$v^2 = \frac{MG}{r}.$$

Since we are looking at the photon, we can also write:

$$v^2 = \frac{MG}{t}.$$

We can substitute in the frequency transformation to obtain:

$$\lambda' = \left(1 - \left(\frac{MG}{t}\right)\right)\lambda,$$

$$\lambda' = \left(1 - \left(\frac{MG}{t}\right)\right)\lambda.$$

In the weak-field limit, that is, when $t \to \infty$, the expected result is obtained, the frequency of light is constant in the absence of a gravitational field. As the gravitational field becomes large, $t \to MG$, the frequency of the photon decreases. However, the equations for time place a limit for this decrease because the longest allowable wavelength of a photon is $1/2$:

$$\lambda' = \left(1 - \left(\frac{MG}{t}\right)\right)\lambda, \lambda' \geq \frac{1}{2}.$$

Therefore, the smallest time allowed is:

$$r_S = t_S = 2MG,$$

and setting this as the limit for velocity:

$$0 \leq v(t) < \frac{MG}{t_S} = \frac{1}{2}.$$

The researchers selected the observer reference frame as sitting at rest sharing an origin with a massive particle that was being observed. The particle being observed had the following initial state:

$$[v = v_0, \rho = 1, t_0 = t'_0].$$

The motion of the particle in time is governed by the following integral:

$$t' = \int_{t_0}^{t} \frac{\rho}{\sqrt{1 - \frac{MG}{t}}} dt,$$

However, as seen before, t is shifted since its minimum occurs at the event horizon, t_S:

$$t' = \int_{t_0}^{t} \frac{\rho}{\sqrt{1 - \frac{2MG}{t}}} dt,$$

$$t' = \int_{t_0}^{t} \frac{\rho}{\sqrt{1 - \frac{t_S}{t}}} dt,$$

$$t' = \sqrt{t(t - t_S)} + t_S \ln\left(\sqrt{t - t_S} + \sqrt{t}\right) - \sqrt{t_0(t_0 - t_S)} + t_S \ln\left(\sqrt{t_0 - t_S} + \sqrt{t_0}\right).$$

As expected, far away from the event horizon, the impact of the second term in the radical becomes insignificant and the Newtonian time is obtained. The metric is:

$$P_\tau = \begin{bmatrix} \frac{\rho}{1 - \frac{t_S}{t}} \\ -\frac{\rho \frac{t_S}{t}}{1 - \frac{t_S}{t}} \end{bmatrix}$$

$$P_\tau = \begin{bmatrix} \frac{\rho t}{t - t_S} \\ -\frac{\rho t_S}{t - t_S} \end{bmatrix}$$

This result differs structurally from the Schwarzschild metric in GR, which is expected since curvature of spacetime is not a dynamical variable in the current framework. Nevertheless, both descriptions agree on key physical features: the existence of an event horizon, the Newtonian limit at large radius, and the loss of observational access beyond the horizon. In contrast to Einstein gravity, the divergence at the horizon is not attributed to curvature singularities, rather to limitations on phase-based observability in the time manifold. As a result, information beyond the horizon is not dynamically-encoded for external observers, while the underlying geometry remains flat.

9. GENERAL RELATIVITY (GR) EQUATIONS FOR THE SPACE DIMENSIONS

9.1. Angular Momentum and Gravity in the Space Manifold

Up to this point, the analysis focused on the dynamics in the time manifold, from which invariant temporal distances and energy–momentum relations emerge. The study now turns towards the space manifold and shows how angular momentum and gravitational bending arise naturally from phase evolution in space while remaining consistent with the invariants already established in the time sector.

The solutions discussed in previous sections are always defined relative to a chosen reference frame. The study calculated the change in the phase in time interpreted as the velocity or frequency of the observed particle and the invariant distance in the time manifold between the particle and the reference frame. Although different observers may assign different coordinate descriptions, all observers agree on the proper distance in the time manifold, reflecting the invariant structure established earlier.

In direct analogy with proper time in the time manifold, now an invariant quantity in the space manifold is sought that can be used to determine the spatial trajectory of a particle relative to a chosen reference frame. This invariant should be related to angular momentum, in the same way that proper time is related to energy–momentum. Since angular momentum is naturally described in spherical geometry, polar spherical coordinates (r, θ_s, φ) are adopted. The subscript on θ_s is introduced to avoid confusion with the time-phase variable θ. The metric of flat three-dimensional space is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

With a vector of quantum numbers β_i determining the sign (± 1) of the angular momentum between two particles [12]:

$$\begin{bmatrix}\beta_x\\ \beta_y\\ \beta_z\end{bmatrix}$$

The metric in polar spherical coordinates is (to simplify the equation, let's ignore for the time being β):

$$dS^2 = dr^2 + r^2 d\theta_s^2 + r^2 \sin^2\theta_s d\varphi^2$$

This quantity is not invariant under changes of reference frame, since different observers would not generally agree on the value of the radial coordinate r. However, all observers do agree on the instantaneous perpendicular distance between two particles moving relative to one another. Equivalently, for a particle moving with respect to a reference frame, the instantaneous perpendicular separation is observer-independent. This invariant distance is denoted by b,

$$b \;\rightarrow\; \text{invariant.}$$

This invariance also implies that the instantaneous rate of change of this perpendicular distance is invariant,

$$\frac{db}{d\tau} = \frac{db}{dt}\frac{dt}{d\tau} = \frac{\beta db}{\sqrt{1-v^2}dt} \longrightarrow \text{invariant},$$

where b is related to the derivative of the phase in space, and at that time [12]. It has been shown that in CSR the following is an invariant:

$$\beta r^2 \dot{\alpha}.$$

This angle was named as the phase in space. Postulate B, introduced previously, states that only gravity perpendicular to the direction of motion of a particle changes its phase in space.

The minimum perpendicular distance of a particle moving in a straight line with respect to a stationary reference frame is:

- $\mathbf{r}(t)$= position vector of the particle
- $\mathbf{v}$= constant velocity vector (straight-line motion)

The perpendicular distance b from the origin to the line of motion is

$$b = \frac{|\,\mathbf{r}\times\mathbf{v}\,|}{|\,\mathbf{v}\,|} \propto \beta r^2 \dot{\alpha}.$$

This is coordinate-independent and works in any coordinate system. Let's summarize the information that we already have from the analysis in time:

1. The researchers looked at a particle with a set of initial conditions including a given direction in space $(r_0, \theta_{s0}, \varphi_0)$.
2. A reference frame was selected to observe the particle. The reference frame was chosen for calculational convenience (the researchers could have chosen any reference frame they liked but the mathematics involved can be much more complex depending on the choice).
3. There was a gravitational field which was defined for the time dimension $(\phi(t))$.
4. The distance in time was calculated between the reference frame and the particle that was being observed $t' = F(t)$.
5. There was a parallel distance that the particle moved in space relative to the chosen reference frame $((x')^2 = 1 + (t')^2)$.

From this starting point, there is a need to find the trajectory of the particle in space. This trajectory is encoded in some function like:

$\beta r^2 \dot{\alpha}$.

In special relativity, only one angle was selected, since there was no acceleration, and straight-line motion could be described using two dimensions (the radius and one angle). The researchers adjusted this to include an additional degree of freedom when they had acceleration:

$\beta r^2 \sqrt{\dot{\theta}_s^2 + \sin^2(\theta_s)\dot{\varphi}^2}$.

This quantity was identified as the angular momentum per unit mass of the particle with respect to the chosen reference frame. It is therefore an invariant and is valid in any coordinate system. The next postulate is a necessary condition of the constant velocity of particles in time.

9.2. Particles Move in the Space Manifold along a Trajectory that Minimizes the Change in the Phase of Space.

Accordingly, the trajectory of the particle in space depends on the action equation

$$A = m \int_{t_0}^{t} \beta r^2 \sqrt{\dot{\theta}_s^2 + \sin^2(\theta_s)\dot{\varphi}^2}$$

Given Postulate B, both $\dot{\theta}_s$ and $\dot{\varphi}$ depend on $\sin(\chi)\,\phi(t)$, where χ is the angle between the velocity vector and the gravitational field. As in Einstein's field equations, the resulting equations can become highly-nontrivial, and simplified gravitational fields are required to obtain closed-form solutions.

Crucially, the spatial phase metric obtained here coincides with the angular sector of the Schwarzschild metric. As a result, predictions related to spatial bending, such as the deflection of light and the relationship between velocity and radius agree with those obtained from Einstein's GR, even though spacetime curvature plays no dynamical role in the present formulation.

The researchers explained in detail in a previous study [12] how a collision was predicted by our equations when two particles try to occupy the same position in space at the same time. The result is the same for CGR. If two particles are on a collision course, all observers agree that when they approach the collision $r \longrightarrow 0$. However, at that same time, since the particles are on a collision course, by definition, $b\ is\ of\ order\ \varepsilon$, where ε is governed by the uncertainty principle [12]. Therefore, the phase in time becomes undefined because $\sin^{-1}\left(\frac{b}{r}\right) = \sin\left(\frac{0}{0}\right) \longrightarrow undefined$ and the uncertainty principle would be violated, and the action would spike at one point towards ∞.

10. COMPARISON BETWEEN THE COMPLETED THEORY OF GENERAL RELATIVITY (CGR) AND EINSTEIN'S GENERAL RELATIVITY (GR)

Having developed the dynamical framework and explicit solutions of the CGR, the study now provides a direct comparison with Einstein's GR. The purpose of this comparison is twofold: to demonstrate agreement with experimentally-tested predictions of GR, and to clarify the conceptual and structural differences introduced by the present reformulation.

Any viable theory of gravitation must reproduce the empirically-verified predictions of GR in the regimes where GR has been tested. CGR is required to recover established phenomenology of GR while providing a different underlying dynamical interpretation.

Within the framework developed in this work, CGR reproduces following experimentally-confirmed predictions of GR

- Predicts event horizon and formation of a black hole
- Predicts same Schwarzschild Radius: 2GM
- Predicts bending of light
- Predicts the gravitational red shift of light
- It is consistent with Newton's equations in the weak limit

It has major implications and revisions to GR and its results:

- There is no curvature of space time. Gravity is caused by the change in the phase of time and the phase of space. This occurs by the action of gravitons which also travel at the speed of light.
- Space and time are only defined for a particle relative to other particles. The new theory is consistent with the rules of quantum mechanics and therefore provides in this case a better explanation of nature and experimental results.
- There is no singularity present in a black hole. Information about the motion of particles inside the event horizon is not possible to acquire for any observer and the position of all particles is undefined.

11. DISCUSSION AND CONCLUSION

The current study developed a completed reformulation of classical GR in which gravitational phenomena are described through phase dynamics on a two-dimensional time manifold, rather than through the curvature of spacetime. Building on a completed formulation of special relativity subject to a unitary constraint on temporal motion, the study extended the equivalence principle to a phase-based framework and constructed gravitational dynamics from invariant action principles governing the evolution of phase in both time and space.

Explicit equations of motion were derived and analytic solutions were obtained in several benchmark settings, including inertial motion, uniformly-accelerated motion, and a spherically-symmetric gravitational field. In each case, the theory reproduces the established phenomenology of GR in regimes where it has been experimentally tested. In particular, the framework yields the correct Newtonian limit in weak gravitational fields, predicts gravitational redshift and light bending, as well as identifies the same characteristic radius associated with event horizons. As a result, the completed theory is observationally-consistent with Einstein's GR across all currently accessible tests.

Despite this empirical agreement, the underlying dynamical interpretation differs fundamentally from that of Einstein gravity. In the present formulation, spacetime curvature is not treated as a dynamical degree of freedom. Instead, gravitational effects arise from constrained phase evolution, subject to unitarity and invariant action principles, on an underlying flat time manifold. The geometric role played by curvature in GR is thus replaced by phase dynamics that encode gravitational influence without modifying the causal structure of spacetime.

A significant consequence of this reformulation concerns the treatment of black holes and singularities. While event horizons remain well-defined and operationally-meaningful, curvature singularities do not arise. The inaccessibility of information associated with regions beyond the horizon is attributed to limitations of phase-based observability rather than to divergences of geometric invariants. So, the classical singularity problem identified by Penrose-type collapse theorems is addressed through a change in the fundamental dynamical variables rather than through modifications of spacetime topology or causal structure.

Here, no claim is made to provide a complete theory of quantum gravity. Nevertheless, since phase and unitarity are foundational elements of quantum mechanics, the phase-based structure developed here offers a natural and conceptually-economical starting point for future extensions. The framework suggests a setting in which relativistic gravitation and quantum dynamics may be formulated within a common language based on invariant phase evolution.

Besides its formal formulation, the structure formulated in this work also has an obvious physical interpretation whereby gravity is not conceived as a curvature in spacetime but as a consequence of the evolution of particles in a background temporal structure with internal degrees of freedom. Here, every particle has a phase which is the dynamical state of that particle, and the gravitational effects are not due to geometry distortions but to variations in the phase dynamics of the particle. The addition of a two-dimensional time is not to be considered as an additional observable dimension, however, as a revelation of a more profound structure of time, in which one factor is the more familiar flow of proper time and the other is the phase evolution. In this view, the redistribution of phase between time and space of motion is manifested in such phenomena as gravitational redshift, free fall, and the bending of light, as well as in others. The basic restriction which determines this evolution is such that it is consistent with relativistic invariance and conservation laws, and does not require the introduction of curvature as a dynamical variable. Consequently, the theory offers another perspective of explaining gravitation where classical predictions are maintained but singularities are substituted by physical observability limits, and the role of phase, which is already central in quantum theory, is made central in explaining gravitational dynamics. This interpretation indicates that there may be a conceptual gap between relativity and quantum mechanics, where both theories are formulated in the same language, the language of phase evolution and not of geometric structure.

Author Contributions

Rodrigo Steinvorth: conceptualization, investigation, methodology, writing–original draft. **Syed Ali Mardan:** validation, writing – review & editing.

Conflict of Interest

The authors of the study have no financial or non-financial conflict of interest in the subject matter or materials discussed in this study.

Data Availability Statement

Data availability is not applicable as no new data was created.

Funding Details

There are no funding resources dedicated for this research.

Use of Generative AI:

AI tools are not used for this research

REFERENCES

1. Bertotti B, Iess L, Tortora P. A test of general relativity using radio links with the Cassini spacecraft. *Nature.* 2003;425:374-376. https://doi.org/10.1038/nature01997
   Tests GR via Shapiro time delay with high precision, confirming spacetime curvature predictions in the weak-field regime.
2. Will CM. The confrontation between general relativity and experiment.

Living Rev Relativ. 2014;17:e4. https://doi.org/10.12942/lrr-2014-4

3. Thorne KS, Drever RW, Adhikari RX, et al. Observation of gravitational waves from a binary black hole merger. *Phys Rev Lett.* 2016;116:e061102. https://doi.org/10.1103/PhysRevLett.116.061102
4. LIGO Scientific Collaboration; Virgo Collaboration. Tests of general relativity with GW150914. *Phys Rev Lett.* 2016;116(22):e221101. https://doi.org/10.1103/PhysRevLett.116.221101
5. Akiyama K. First M87 event horizon telescope results. I. the shadow of the supermassive black hole. *Astrophys J Lett.* 2019;875:eL1. https://doi.org/10.3847/2041-8213/ab0ec7
6. Psaltis D, Medeiros L, Christian P, et al. Gravitational test beyond the first post-Newtonian order with the shadow of the M87 black hole. *Phys Rev Lett.* 2020;125:e141104. https://doi.org/10.1103/PhysRevLett.125.141104
7. Penrose R. Gravitational collapse and space-time singularities. *Phys Rev Lett.* 1965;14:57-59. https://doi.org/10.1103/PhysRevLett.14.57
8. Hawking SW, Penrose R. The singularities of gravitational collapse and cosmology. *Proc R Soc A.* 1970;314:529-548. https://doi.org/10.1098/rspa.1970.0021
9. Isham CJ. Canonical quantum gravity and the problem of time. In: Ibort LA, Rodríguez MA, eds. *Integrable Systems, Quantum Groups, and Quantum Field Theories.* Kluwer Academic Publishers; 1993. https://arxiv.org/abs/gr-qc/9210011
10. Steinvorth R, Mardan Azmi SA. Two dimensions of time: Reconciling relativistic invariance and quantum discreteness. figshare. https://doi.org/10.6084/m9.figshare.29517641. Updated, 2025. Introduces a two-dimensional internal time manifold in which relativistic kinematics emerge from phase structure.
11. Steinvorth R, Azmi SAM. Quantum principles from time phase geometry: uncertainty, exclusion, and interference. *Sci Inq Rev.* 2025;9(1):37-22. https://journals.umt.edu.pk/index.php/SIR/article/view/7421 Develops quantum phenomena such as uncertainty and interference from phase-based temporal structure.
12. Steinvorth R, Mardan Azmi SA. Foundations of a two-time relativistic

framework: Lorentz symmetry, proper time and matter structure. *J Innov Res Math Comput Sci.* 2025;4:82-110. https://doi.org/10.62270/jirmcs.v4i1.56
Establishes the relativistic structure of the two-time framework, including Lorentz invariance and proper time construction.

13. Aharonov Y, Bohm D. Significance of electromagnetic potentials in the quantum theory. *Phys Rev.* 1959;115:485-491. https://doi.org/10.1103/PhysRev.115.485
14. Feynman RP. Space-time approach to non-relativistic quantum mechanics. *Rev Mod Phys.* 1948;20:367-387. https://doi.org/10.1103/RevModPhys.20.367
15. Berry MV. Quantal phase factors accompanying adiabatic changes. *Proc R Soc A.* 1984;392:45-57. https://doi.org/10.1098/rspa.1984.0023
16. Shapere A, Wilczek F, eds. *Geometric Phases in Physics.* World Scientific; 1989. https://doi.org/10.1142/0613
17. Castro-Ruiz E, Giacomini F, Belenchia A, Brukner C. Quantum clocks and the temporal localisability of events in the presence of gravitating quantum systems. *Nat Commun.* 2020;11:e2672. https://doi.org/10.1038/s41467-020-16013-1
18. Giacomini F, Castro-Ruiz E, Brukner C. Quantum mechanics and the covariance of physical laws in quantum reference frames. *Nat Commun.* 2019;10:e494. https://doi.org/10.1038/s41467-018-08155-0
19. Höhn PA, Vanrietvelde A. How to switch between relational quantum clocks. *New J Phys.* 2018;20:e123048. https://doi.org/10.1088/1367-2630/abd1ac
20. Marchetti L, Oriti D. Effective relational cosmological dynamics from quantum gravity. *J High Energy Phys.* 2021;5:e025. https://doi.org/10.1007/JHEP05(2021)025
Develops relational time and emergent cosmological dynamics from quantum gravity, supporting non-fundamental notions of time.
21. Smith A, Ahmadi M. Quantizing time: interacting clocks and systems. *Quantum.* 2019;3:e160. https://doi.org/10.22331/q-2019-07-08-160
22. Marletto C, Vedral V. Evolution without evolution and the problem of time. *Phys Rev D.* 2017;95:e043510.

https://doi.org/10.1103/PhysRevD.95.043510

23. Damour T. The equivalence principle and the constants of nature. *Space Sci Rev.* 2009;148:191-199. https://doi.org/10.1007/s11214-009-9533-6
Provides a modern theoretical analysis of the equivalence principle and its role in fundamental physics.
24. Ehlers J, Pirani FAE, Schild A. The geometry of free fall and light propagation. *Gen Relativ Gravit.* 2012;44:1587-1609. https://doi.org/10.1007/s10714-012-1353-4
25. Lämmerzahl C. On the equivalence principle in quantum theory. *Gen Relativ Gravit.* 1996;28:1043-1070. https://doi.org/10.1007/BF02113157
26. Noether E. Invariant variation problems. *Transp Theory Stat Phys.* 1971;1:186-207. https://doi.org/10.1080/00411457108231446
27. Das S, Fridman M, Lambiase G. General formalism of the quantum equivalence principle. *Commun Phys.* 2023;6:e198. https://doi.org/10.1038/s42005-023-01306-w
Develops a relativistic formulation of the quantum equivalence principle, extending its role in modern gravity and quantum frameworks.
28. Padmanabhan T. Gravity as an emergent phenomenon: a conceptual description. *AIP Conf Proc.* 2007;939:114-123. https://doi.org/10.1063/1.2803795
29. Williams J, Chiow SW, Yu N, Müller H. Quantum test of the equivalence principle and space-time aboard the International Space Station. *New J Phys.* 2016;18:e025018. https://doi.org/10.1088/1367-2630/18/2/025018
Proposes experimental tests of the equivalence principle in quantum regimes using space-based interferometry.
30. Colella R, Overhauser AW, Werner SA. Observation of gravitationally induced quantum interference. *Phys Rev Lett.* 1975;34:1472-1474. https://doi.org/10.1103/PhysRevLett.34.1472
31. Husain V, Mahmood H. General covariance and dynamics with a Gauss law. *Phys Rev D.* 2024;109:e064016. https://doi.org/10.1103/PhysRevD.109.064016
Explores gravitational dynamics derived from symmetry and constraint

principles, emphasizing general covariance beyond standard formulations.

32. Baydin A, Zhu H, Bamba M, Hazzard K, Kono J. Perspective on the quantum vacuum in matter. *Opt Mater Express.* 2025;15:e1833. https://doi.org/10.1364/OME.569339
Reviews vacuum fluctuations and their physical effects in matter, highlighting the role of quantum vacuum structure in observable phenomena.
33. Rindler W. Hyperbolic motion in curved space-time. *Phys Rev.* 1960;119:e2082. https://doi.org/10.1103/PhysRev.119.2082
34. Deser S, Levin O. Accelerated detectors and temperature in (anti) de Sitter spaces. *Class Quantum Grav.* 1997;14:L163-L168. https://doi.org/10.1088/0264-9381/14/9/003
35. Schwarzschild K. On the gravitational field of a mass point according to Einstein’s theory. *arXiv.* 1999. https://doi.org/10.48550/arXiv.physics/9905030
36. Cardoso V, Pani P. Testing the nature of dark compact objects: a status report. *Living Rev Relativ.* 2019;22:e4. https://doi.org/10.1007/s41114-019-0020-4
37. Blanchet L. Gravitational radiation from post-Newtonian sources and inspiralling compact binaries. *Living Rev Relativ.* 2002;17:e2. https://doi.org/10.12942/lrr-2002-3
38. Provides a comprehensive formulation of gravitational dynamics and relativistic motion in strong-field systems.
39. Jacobson T. Thermodynamics of spacetime: the Einstein equation of state. *Phys Rev Lett.* 1995;75:1260-1263. https://doi.org/10.1103/PhysRevLett.75.1260
40. Hayward SA. General laws of black-hole dynamics. *Phys Rev D.* 1994;49:6467-6474. https://doi.org/10.1103/PhysRevD.49.6467
41. Bousso R. The holographic principle. *Rev Mod Phys.* 2002;74:825-874. https://doi.org/10.1103/RevModPhys.74.825
42. Unruh WG, Wald RM. Information loss. *Rep Prog Phys.* 2017;80:e092002. https://doi.org/10.1088/1361-6633/aa778e